

Problem 3.2.5 Use the Taylor expansion of the function $\operatorname{arctg} x$ to study the convergence of the series

$$\sum_{n=1}^{\infty} \left(\operatorname{arctg} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}} \right).$$

$$\operatorname{arctan} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad -1 < x < 1$$

Luego: $\operatorname{arctan} \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{n}} - \frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{5} \left(\frac{1}{\sqrt{n}} \right)^5 - \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots$

$$\operatorname{arctg} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}} = -\frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{5} \left(\frac{1}{\sqrt{n}} \right)^5 - \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots$$

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\operatorname{arctg} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}} \right) &= \sum_{n=1}^{\infty} \left(-\frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{5} \left(\frac{1}{\sqrt{n}} \right)^5 - \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots \right) \\ &= - \sum_{n=1}^{\infty} \underbrace{\left(\frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots \right)}_{a_n} + \sum_{n=1}^{\infty} \underbrace{\left(\frac{1}{5} \left(\frac{1}{\sqrt{n}} \right)^5 + \frac{1}{9} \left(\frac{1}{\sqrt{n}} \right)^9 + \dots \right)}_{c_n} \end{aligned}$$

Sean las series:

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \rightsquigarrow b_n = \frac{1}{n^{3/2}} \quad p = \frac{3}{2} \rightsquigarrow \text{convergente}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{5/2}} \rightsquigarrow d_n = \frac{1}{n^{5/2}} \quad p = \frac{5}{2} \rightsquigarrow \text{convergente}$$

Por criterio de comparación

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{\frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots}{\frac{1}{n^{3/2}}} = \frac{1}{3} \rightsquigarrow \sum_{n=1}^{\infty} a_n \text{ converge}$$

Cartagena99

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

$$\operatorname{arctg} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n}} = -\frac{1}{3} \left(\frac{1}{\sqrt{n}} \right)^3 + \frac{1}{5} \left(\frac{1}{\sqrt{n}} \right)^5 - \frac{1}{7} \left(\frac{1}{\sqrt{n}} \right)^7 + \dots$$