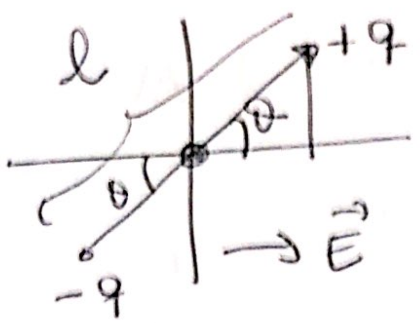
Hoja 2b

1)



F de $\vec{E}$ sobre q

$$\vec{F} = q\vec{E}$$

$U = -qE\cdot x$ (para que $\vec{F} = -\frac{\partial U}{\partial x}$)

ó $U = -\int F\,dx = -qE\int dx = \underline{-qEx}$

Si $x=0$ en el pto medio entre las cargas

$x_1 = l/2 \cos\theta$ y $x_2 = -\frac{l}{2}\cos\theta$

Como $q_1 = q_2 = q$

$$U = -q_1 E\cdot\frac{l}{2}\cos\theta - q_2 E\frac{l}{2}\cos\theta$$

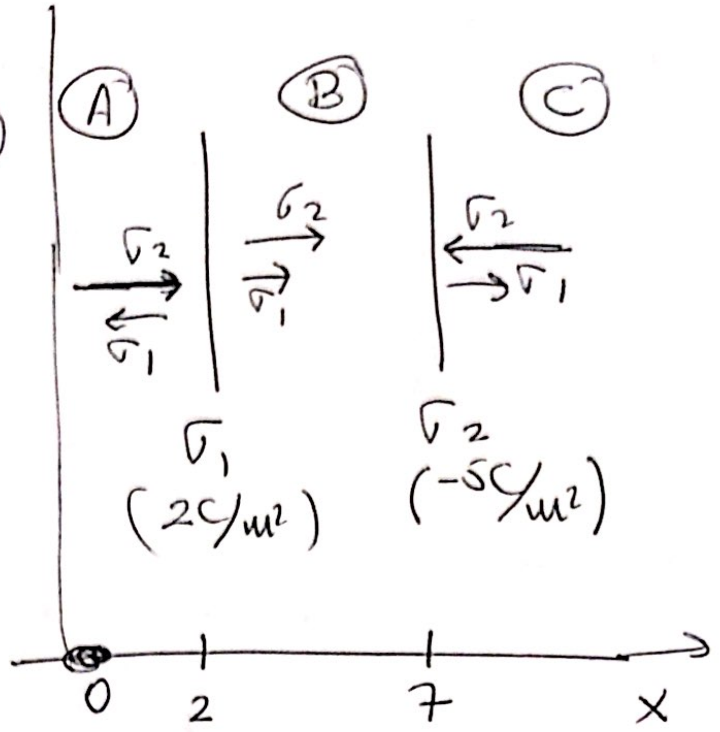
$$U = -qE\,l/2\cos\theta - qE\,l/2\cos\theta = -qEl\cos\theta$$

$$|\vec{p}| = ql$$

$\boxed{U = -p\cdot E}$ mínima U cuando $\theta = 0$ (lo más negativa posible)

Cuando $\theta = 0$ se minimiza la energía →

2b.2)



a) $E_{A,B,C}$?

b) ΔV ?

c) q (7C) en (4,0,0)
F ?

d) W mover q
de (4,0,0) a (6,0,0)
?

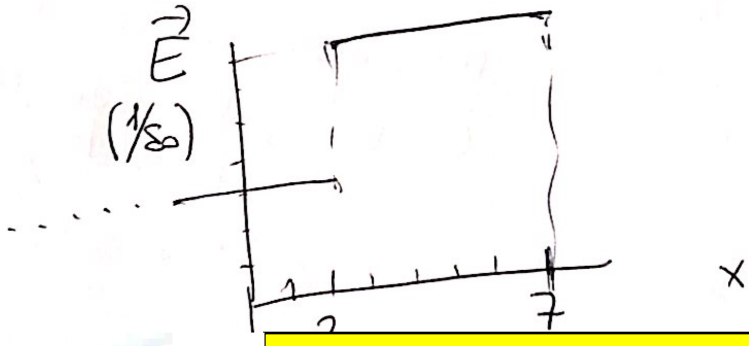
a) E_A ? E_B ? E_C ? $E \perp x$

$$\vec{E}_\sigma = \frac{\sigma}{2\varepsilon_0}$$

(A) $-\infty < x < 2$ $\quad \vec{E}_{\sigma A} = \left(\frac{-\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}\right)\hat{i} = \boxed{\frac{3}{2\varepsilon_0}\hat{i}\ N/C}$

(B) $2 < x < 7$ $\quad \vec{E}_{\sigma B} = \left(\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}\right)\hat{i} = \left(\frac{2}{2\varepsilon_0} + \frac{5}{2\varepsilon_0}\right)\hat{i} = \underline{\frac{7}{2\varepsilon_0}\hat{i}\ N/C}$

(C) $7 < x < \infty$ $\quad \vec{E}_{\sigma C} = \left(\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}\right)\hat{i} = \left(\frac{2}{2\varepsilon_0} - \frac{5}{2\varepsilon_0}\right)\hat{i} = \underline{\frac{-3}{2\varepsilon_0}\hat{i}\ N/C}$



26.2

b) $\vec{E} - \nabla V$

$$(E_x, E_y, E_z) = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

$$E_x = -\frac{\partial V}{\partial x} \rightarrow E_x = -\frac{dV}{dx}, \quad V = -\int E_x dx$$

$$V = -\int_2^7 E_{\sigma B}\, dx = -\int_2^7 \frac{7}{2\varepsilon_0} dx = -\frac{7}{2\varepsilon_0} x \Big]_2^7 = -\left(\frac{7\cdot 7}{2\varepsilon_0} - \frac{7\cdot 2}{2\varepsilon_0}\right)$$

$$\Delta V = \frac{1}{2\varepsilon_0}(49-14) = \boxed{-\frac{35}{2\varepsilon_0} V}$$

c) $q = 7C$ en $(4,0,0)$

En $(4,0,0)$ estamos en la región B

$$\vec{E}_B = \frac{7}{2\varepsilon_0} \hat{i} \text{ N/C}$$

$$\boxed{\vec{F} = q\vec{E} = 7 \cdot \frac{7}{2\varepsilon_0} \hat{i} = \frac{49}{2\varepsilon_0} \hat{i} \text{ N}}$$

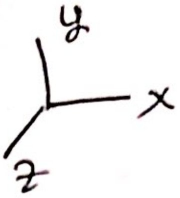
d) $W_{(4,0,0)} \rightarrow (6,0,0)$

$$W = q\Delta V = q(V_{x=6} - V_{x=4})$$

ó

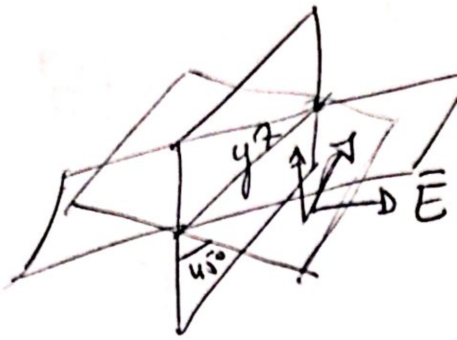
$$\boxed{W = \int F \cdot dr = F \cdot \Delta x = qE\Delta x = 7 \cdot \frac{7}{2\varepsilon_0} \cdot 2 = \frac{49}{\varepsilon_0} J}$$

3)



$\vec{E}$ de $\bar{E}\,\bar{i}$

Flujo de E en $yz = 2\ \mathrm{Vm}^2$



$$\phi_{yz} = \vec{E} \cdot \vec{S} = ES = \boxed{2}$$

a) Si giramos 45°

$\bar{E}$ y $\bar{S}$ 45°

$$\phi = E \cdot S \cdot \cos 45^\circ = 2\frac{\sqrt{2}}{2} = \sqrt{2}$$

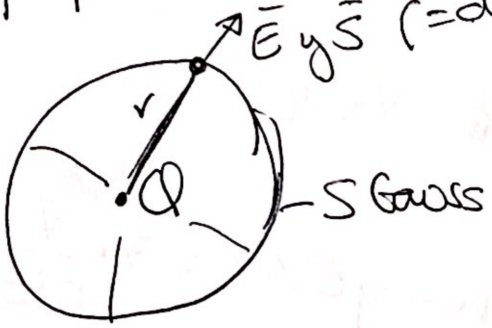
b) Si giramos 90°

$\bar{E}$ y $\bar{S}$ 90°

$$\phi = E \cdot S \cos 90^\circ = \underline{0}\ \mathrm{W}$$

4) Gauss

a) q puntual Q



$$\oint E \cdot dS = E \int dS = \frac{Q}{\varepsilon_0}$$

$$\cancel{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

b) Esfera uniformemente cargada R, Q.
Ahora $q(r)$, cuando $r = R \rightarrow q = Q$ total
Hay que ver la carga proporcional a cualquier radio

$$\boxed{q(r) = \frac{Q V_r}{V_R} = Q \frac{4/3 \pi r^3}{4/3 \pi R^3} = \frac{Q r^3}{R^3}}$$

ya que $\frac{Q}{4/3 \pi R^3} = \frac{q(r)}{4/3 \pi r^3}$ porque $\rho = \frac{Q}{V}$ cte

Si $r > R$, superficie gaussiana esfera de radio R,

$$\oint E ds = 4\pi r^2 E = \frac{q(r)}{\varepsilon_0} \quad \text{en general}$$

Si $r \geq R$ $\quad 4\pi R^2 E = \frac{Q}{\varepsilon_0} \rightarrow \boxed{E(r \geq R) = \frac{Q}{4\pi\varepsilon_0 R^2}}$

Si $r \leq R$ $\quad 4\pi r^2 E = \frac{Q r^3 / R^3}{\varepsilon_0} \rightarrow$

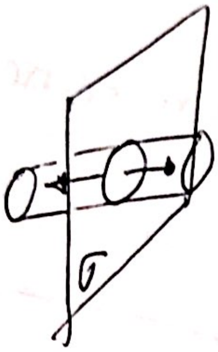
$$E(r) = \frac{Q r^3/R^3}{\varepsilon_0} \Big/ 4\pi r^2 = \frac{Q r^3}{4\pi r^2 R^3 \varepsilon_0} = \boxed{\frac{Q r}{4\pi R^3 \varepsilon_0}}$$

c) Si es una esfera hueca y toda la carga Q está en la superficie

$$q(r)\begin{cases} 0 & \text{si } r<R \\ Q & \text{" } r\geq R \end{cases}$$

$$E(r)\begin{cases} 0 & \text{si } r<R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r\geq R \end{cases}$$

d)



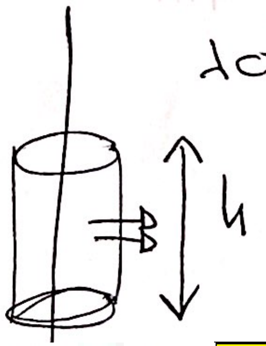
σ cte $\quad Q=\sigma S$

$$\oint E\,ds = 2SE = \frac{Q}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$$

$$2\cancel{S}E = \frac{\sigma \cancel{S}}{\varepsilon_0}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}$$

e)



λ cte , $Q=\lambda h$

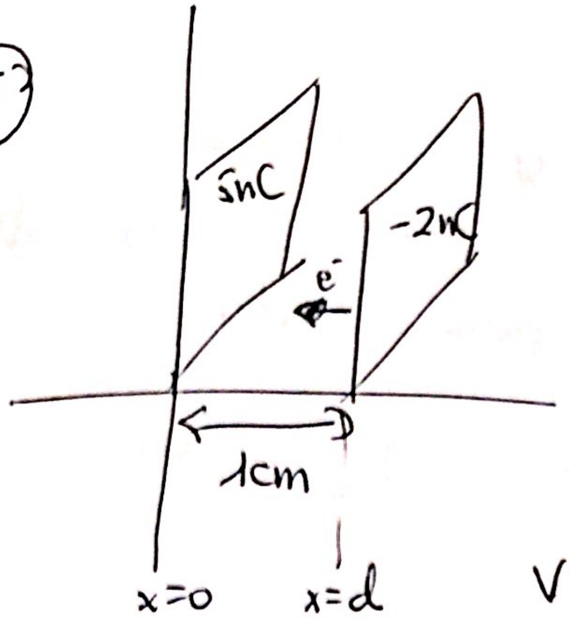
$\vec{E} \parallel \vec{S}$ en las tapas $\vec{E} \perp \vec{S}$

$$\oint E\,dS = E\cdot S = \frac{Q}{\ldots}$$

$$E = \frac{\ldots}{2\pi\varepsilon_0 r} \;;$$ (a distancia r del hilo)

5)

2b.4



$S = 20\,cm^2$

$V_f\, e^-$?

$V(x) = \dfrac{\sigma x}{2\varepsilon_0}$ de un plano

$$V(x) = V_1(x) + V_2(x) = \frac{\sigma_1 |x|}{2\varepsilon_0} + \frac{\sigma_2 |d-x|}{2\varepsilon_0}$$

$$\Delta V = V(0) - V(d) = \frac{\sigma_2 d}{2\varepsilon_0} - \frac{\sigma_1 d}{2\varepsilon_0} = \frac{(\sigma_2 - \sigma_1)d}{2\varepsilon_0}$$

$\hookrightarrow$ Diferencia de potencial entre las 2 placas

La diferencia de energía potencial de un e^- entre las placas será

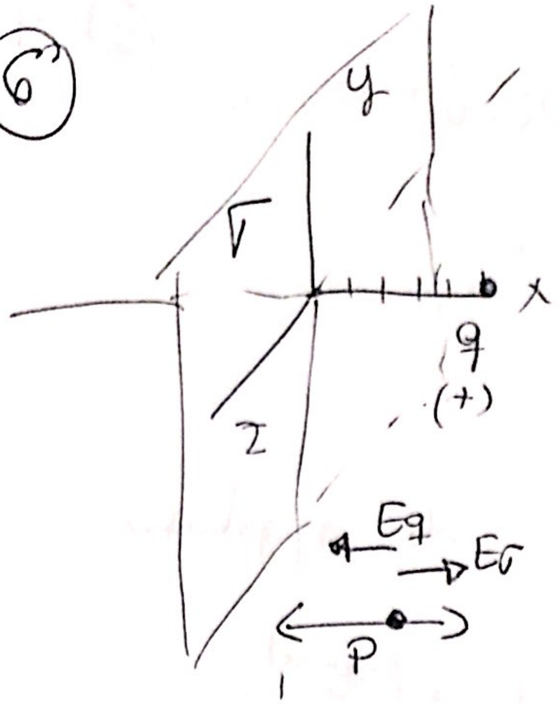
$$\Delta U = q\Delta V = -|e|\Delta V$$

$$\frac{1}{2} m_e v_e^2 = |e| \frac{(\sigma_1 - \sigma_2)d}{2\varepsilon_0} \Rightarrow v = \sqrt{\frac{2|e|(\sigma_1 - \sigma_2)d}{2\varepsilon_0 m_e}}$$

$$\sigma_1 = \frac{Q_1}{S} = \frac{5\cdot 10^{-9}\,C}{20\cdot 10^{-4}\,m^2} = 2'5 \cdot 10^{-6}\,C/m^2$$

$$\sigma_2 = \frac{Q_2}{\ } = \frac{-2\cdot 10^{-9}\,C}{\ } = -10^{-6}\,C/m^2$$

6



q (3C) (5,0,0)
σ 7C/m²
plano infinito

a) P(x,0,0) y (3,0,0)
$\vec{E}_P$?

b) V_P ?

c) $\vec{F}_{q\sigma}$?

a) $\vec{E}(x) = \vec{E}_{plano} + \vec{E}_{carga}$

$$\vec{E}(x) = \frac{\sigma}{2\varepsilon_0}\hat{u}_x - \frac{q}{4\pi\varepsilon_0 (x-x_0)^2}\hat{u}_x \quad x_0 = 5\,m$$

$$\vec{E}(x) \left(\frac{7}{2\cdot 8'85\cdot 10^{-12}} - \frac{3}{4\pi\cdot 8'85\cdot 10^{-12}(x-5)^2}\right)\hat{i}$$

$$\vec{E}(x) = 3'95\cdot 10^{11} - \frac{2'7\cdot 10^{10}}{(x-5)^2}\ \ N/C$$

En (3,0,0), $|\vec{E}| = 3'95\cdot 10^{11} - \frac{2'7\cdot 10^{10}}{4} = 3'88\cdot 10^{11}\ N/C$

sentido + (1,0,0)

2b.5

b) V_P?

$V(0,0,0) = 0$!! Origen de potenciales!

$V = V_{plano} + V_{carga\ q}$

$$V_{plano} = -\frac{\sigma}{2\varepsilon_0}|x| + C_1$$

$$V_q = \frac{q}{4\pi\varepsilon_0 |x - x_0|} + C_2$$

Si queremos que $V(0,0,0) = 0 \longrightarrow \boxed{C_1 = 0}$

$$V_q = 0 = \frac{q}{4\pi\varepsilon_0 |-5|} + C_2 = 0 \qquad \frac{q}{4\pi\varepsilon_0 5} + C_2 = 0$$

$$\Rightarrow \boxed{C_2 = -\frac{q}{4\pi\varepsilon_0 5}}$$

$$V(\vec{r}) = -\frac{\sigma}{2\varepsilon_0}|x| + \frac{q}{4\pi\varepsilon_0 |x-5|} - \frac{q}{4\pi\varepsilon_0 5}$$

Para $(3,0,0)$

$$\boxed{V(3,0,0) = -\frac{\sigma}{2\varepsilon_0} 3 + \frac{q}{4\pi\varepsilon_0 \cdot 2} - \frac{q}{4\pi\varepsilon_0 \cdot 5}}$$

$$= -1'19 \cdot 10^{12} + 1'35 \cdot 10^{10} - 5'4 \cdot 10^{9}\,V = \boxed{-1'18 \cdot 10^{-12}\,V}$$

c) F que ejerce el plano sobre la carga

$F_{q\ sobre\ el\ plano} = -F_{plano\ sobre\ carga}$

$\vec{F}_{\pi \to q} = -1'19 \cdot 10^{12}\,\hat{u}_x\,N$