UNIT 4 Exercises: Optimal filtering

4.1 Given an 2nd order FIR filter:

a) Draw the block diagram of the filter.

b) Write the expression of y[0], y[1] and y[2] (y[n] is the output of the filter), given that the initial values for the delays are all zero. Provide the general expression and particularize for the following values of x[n]:
x(0)=0.1
x(1)=0.8
x(2)=0.2

4.2 Check that $\vec{w}^T \overrightarrow{R_{XD}} = \overrightarrow{R_{XD}}^T \vec{w}$ for M=3.

4.3. Design a 2-coefficient optimal filter given the following statistics:

$R_{XX}(-2)$ =0.24649
$R_{XX}(-1)$ =0.24671
$R_{XX}(0)$ =0.27136
$R_{XX}(1)$ =0.24671
$R_{XX}(2)$ =0.24649

$R_{XD}(-2)$ =0.24616
$R_{XD}(-1)$ =0.24634
$R_{XD}(0)$ =0.24640
$R_{XD}(1)$ =0.24634
$R_{XD}(2)$ =0.24616

4.4. Check that $\frac{d(\vec{w}^T \overrightarrow{R_{XD}})}{d\vec{w}} = \overrightarrow{R_{XD}}$ for M=3.

4.5. Prove that $E[\vec{x}(n)d(n)] = E[d(n)\vec{x}(n)^T] = \overrightarrow{\boldsymbol{R_{XD}}}$