LINEAR SYSTEMS AND CIRCUIT APPLICATIONS

BIOMEDICAL ENGINEERING

Part I - Linear Systems

19th November 2018

Surname: ______________________

Name: ______________________

1. (2 points) Let be

$$x(t) = \begin{cases} -t+1 & ; \ t \in [0,1] \\ 0 & ; \ \text{otherwise} \end{cases}$$

And let be the following signals:

$$r(t) = \sum_{k=-\infty}^{+\infty} (-1)^k \delta(t-2k); \ z(t) = x(t) * r(t)$$

$$s(t) = -2x(-t/2-1)+2; \ y(t) = \frac{dz(t)}{dt} + 1$$

(a) (1 point) Sketch the signal $z(t)$ and give an analytical expression for $x(t)$. Is $z(t)$ a periodic signal? If it is, what is its period? What is the average value of $z(t)$.

(b) (1 point) Sketch the signals $s(t)$ and $y(t)$

2. (1 point) Consider the following system, whose output can be represented as:

$$y(t) = |x(t)| + \frac{dx(t)}{dt}$$

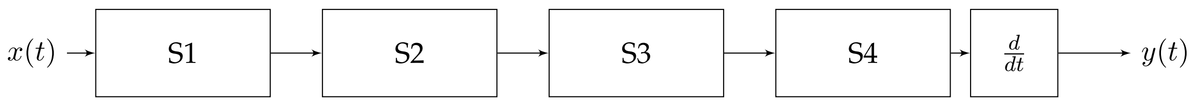
Study the following properties: invertibility, time-invariance and linearity.

3. (1 point) Compute the following convolution, $x(t) * h(t)$:

$$h(t) = e^{-5(t-1)}u(t-1); x(t) = u(t) - u(t-1)$$

4. (1 point) Let be an LTIS, which is the interconnection of different subsystems. We know the following data:

- S1: $h_1(t) = u(t-1)$
- S2: $s_2(t) = u(t-3)$
- S3: $h_3(t) = e^{-5(t-2)}u(t-2)$
- S4: $h_4(t) = \sqcap(\frac{t}{2T})$



where $s_3(t)$ is the **step response** of the system S2, answer the following questions:

(a) (0.5 points) Compute the equivalent impulse response $h_{eq}(t)$, for the whole system.

(b) (0.5 points) Study system properties depending on the value of T: memory, causality and stability.

Note: you may find useful:

$$\sqcap(\frac{t}{T}) = \begin{cases} 1; & \frac{-T}{2} \leq t < \frac{+T}{2} \\ 0; & otherwise \end{cases}$$

5. (1 point) Let be a LTIS, whose step response is $s(t) = e^{-t}u(t)$. Find the output of the system when the input is:

$$x(t) = \begin{cases} 1; & 1 < t < 3 \\ 0; & \text{otherwise} \end{cases}$$

6. (2 points) Let be

$$x(t) = \begin{cases} -t+1 & ;\ t \in [0,1] \\ 0 & ;\ \text{otherwise} \end{cases}$$

And let be:

$$r(t) = \sum_{k=-\infty}^{+\infty} (-1)^k \delta(t-2k);\ z(t) = x(t) * r(t)$$

Compute the FS coefficients of $y(t) = \frac{dz(t)}{dt}$

7. (2 points) Compute the Fourier Transform of the signal $x(t)$, represented in the following figure