

UNIT 2

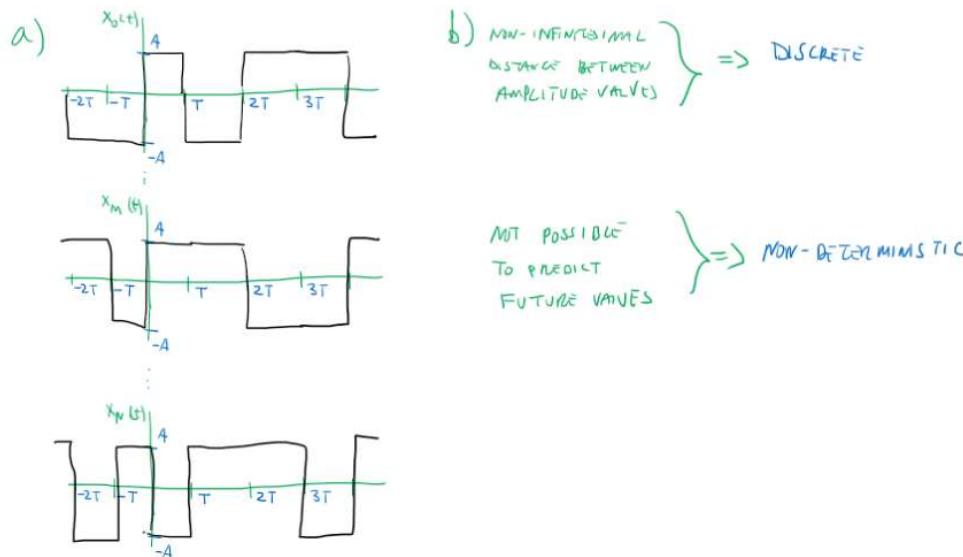
Exercises of UNIT 2- Part I: Temporal Characteristics

2.1 A two-level random binary process is defined by

$X(t) = A$ or $-A$, $(n-1)T < t < nT$ where the levels A and $-A$ occur with equal probability, T is a positive constant, and $n = 0, \pm 1, \pm 2, \dots$

- Sketch a typical sample function
- Classify the process

$$X(t) = A \text{ or } -A, (n-1)T < t < nT$$

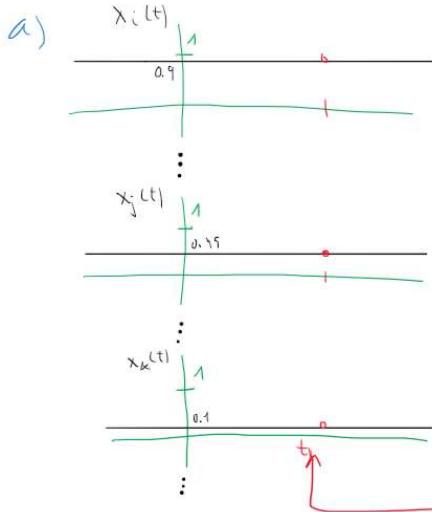


2.2 A random process is defined by $X(t) = A$, where A is a continuous random variable uniformly distributed on $[0,1]$.

- Determine the form of the sample functions
- Classify the process

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70



b) INFINITE VALUES WITH NO GAPS \Rightarrow CONTINUOUS

ONCE YOU KNOW $X(t=t_0)$ YOU
KNOW THAT
 $X(t)=X(t=t_0), \forall t$

c) EXTRA QUESTION

$$E[X(t)]? \rightarrow E[X(t)] = E[A] = 0.5$$

2.3 Given the random process $X(t) = A\cos(\omega_0 t + \Theta)$ where A and ω_0 are constants and Θ is a uniformly distributed random variable on the interval $[0, 2\pi]$.
Find

- (a) The mean
- (b) The autocorrelation
- (c) The time average
- (d) The time autocorrelation

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70**

$$x(t) = A \cos(\omega_0 t + \Theta)$$

A, ω_0 are constant

$$\Theta \sim U(0, 2\pi)$$

a) Mean $\rightarrow E[x(t)] = E[A \cos(\omega_0 t + \Theta)] = A E[\cos(\omega_0 t + \Theta)] = A \cdot E[g(\Theta)]$

$$= A \cdot \int_{-\infty}^{\infty} g(\theta) f_{\Theta}(\theta) d\theta = A \int_0^{2\pi} \cos(\omega_0 t + \theta) \frac{1}{2\pi} d\theta = \frac{A}{\omega_0} \sin(\omega_0 t + \theta) \Big|_0^{2\pi} = 0$$

$$f_{\Theta}(\theta) = \frac{1}{2\pi}, 0 \leq \theta \leq 2\pi$$

Also $\rightarrow E[A \cos(\omega_0 t + \Theta)] = 0 \text{ IF } \begin{cases} \Theta \sim U(0, \Omega_0) \\ \Omega_0 - \Omega_1 = 2\pi \cdot k, k \in \{1, 2, 3, \dots\} \end{cases}$

b) Auto-correlation $\rightarrow R_{xx}(t_1, t_2) = R_{xx}(t, t+\tau) = E[x(t)x(t+\tau)]$

$$= A^2 E \left[(\cos(\omega_0 t + \Theta)) \cos(\omega_0(t+\tau) + \Theta) \right]$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$= \frac{A^2}{2} \left[E \left[\cos(\omega_0(2t + \tau) + 2\Theta) \right] + E \left[\cos(-\omega_0\tau) \right] \right]$$

o since $2\Theta \sim U(0, 4\pi)$

$$= \frac{A^2}{2} \cos \omega_0 \tau$$

c) Time average $\rightarrow A[x(t)] = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \cos(\omega_0 t + \Theta) dt$

BOUNDED VALUE

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} = 0$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

d) Time autocorrelation $\rightarrow A[x(t)x(t+\tau)] = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t)x(t+\tau) dt$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T A^2 \cos(\omega_0(2t+\tau) + 2\Theta) dt + \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt$$

BY J NEED VALUE

$$= \frac{A^2}{2} \cos \omega_0 \tau$$

e) Some conclusions $\rightarrow X(t)$ is w.s.s. since $E[X(t)] = 0 = \text{constant}$ 1st order

$$R_{XX}(t, t+\tau) = \frac{A^2}{2} \cos \omega_0 \tau = R_{XX}(\tau) \text{ 2nd order}$$

- $E[X(t)] = A[x(t)] \Rightarrow \text{ERGODIC IN THE MEAN}$
- $R_{XX}(\tau) = |R_{XX}(\tau)| \Rightarrow \text{ERGODIC IN THE AUTOCORRELATION}$

2.4 Given the random process

$X(t) = A \sin(\omega_0 t + \Theta)$ where A and ω_0 are constants and Θ is a uniformly distributed random variable on the interval $[-\pi, \pi]$. Define a new random variable $Y(t) = X^2(t)$.

- Find the autocorrelation function of $Y(t)$
- Find the cross-correlation function of $X(t)$ and $Y(t)$
- Are $X(t)$ and $Y(t)$ wide-sense stationary?
- Are $X(t)$ and $Y(t)$ jointly wide-sense stationary?

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

$$\begin{aligned}
 a) R_{yy}(t, t+\tau) &= E[Y(t)Y(t+\tau)] = E\left[\frac{A^2}{2} [1 + \cos(2w_0(t+2\tau))] \right] \\
 &= \frac{A^4}{4} \left[1 + E\left[\cos(w_0t+2\theta)\right] + E\left[\cos(w_0(t+\tau)+2\theta)\right] + E\left[\cos(w_0t+2\theta) \cos(w_0(t+\tau)+2\theta)\right] \right] \\
 &\quad \xrightarrow{\text{2}\theta \approx V(-2n, 2n)} \quad \xrightarrow{\text{2}\theta \approx V(-2n, 2n)} \\
 &= \frac{A^4}{4} \left[1 + \frac{1}{2} E\left[\cos(2w_0(2t+\tau)+4\theta)\right] + \frac{1}{2} E\left[\cos(2w_0\tau)\right] \right] \\
 &\quad \xrightarrow{4\theta \approx V(-4n, 4n)} \quad \xrightarrow{\cos 2w_0\tau} \\
 &= \frac{A^4}{4} \left[1 + \frac{\cos 2w_0\tau}{2} \right]
 \end{aligned}$$

$$\begin{aligned}
 b) R_{xy}(t, t+\tau) &= E[X(t)Y(t+\tau)] = E\left[A \cos(w_0t+\theta) \frac{A^2}{2} [1 + \cos(2w_0(t+\tau)+2\theta)]\right] \\
 &= \frac{A^3}{2} \left[E\left[\cos(w_0t+\theta)\right] + E\left[\cos(w_0t+\theta) \cos(w_0(t+\tau)+2\theta)\right] \right] \\
 &= \frac{A^3}{2} \frac{1}{2} \left[E\left[\cos(w_0(3t+\tau)+3\theta)\right] + E\left[\cos(-w_0t+2\tau-\theta)\right] \right] \\
 &= 0 \quad \Rightarrow \quad X, Y \text{ ORTHOGONAL}
 \end{aligned}$$

$$\begin{aligned}
 c) X(t) \text{ w.s.s.?} \quad X(t) \rightarrow E[X(t)] &= E\left[A \cos(w_0t+\theta)\right] = 0 \Rightarrow \text{1st ORDER} \\
 Y(t) \text{ w.s.s.?} \quad Y(t) \rightarrow E[Y(t)] &= E[X^2(t)] = R_{xx}(0) = \frac{A^2}{2} \Rightarrow \text{1st ORDER} \\
 R_{xx}(t, t+\tau) &= E\left[A \cos(w_0t+\theta) A \cos(w_0(t+\tau)+2\theta)\right] \\
 &= \frac{A^2}{2} \left[E\left[\cos(w_0t+\theta)\right] + E\left[\cos(w_0\tau)\right] \right] \quad \xrightarrow{\text{w.s.s.}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{A^2}{2} \cos w_0\tau \quad \xrightarrow{\text{R}_{xx}(\tau)} \quad \xrightarrow{\cos w_0\tau} \quad \Rightarrow \text{2nd ORDER} \\
 R_{yy}(t, t+\tau) &= R_{yy}(\tau) \quad \Rightarrow \text{2nd ORDER} \quad \xrightarrow{\text{w.s.s.}}
 \end{aligned}$$

d) $X(t)$ and $Y(t)$ are w.s.s. and $R_{xy}(t, t+\tau) = 0$.

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70**

(t_1, t_2) does not affect the outcome of R_{XY} . Thus, we can perfectly state that $X(t)$ and $Y(t)$ are j.w.s.s.

2.5 Given the random process

$Y(t) = X(t)\cos(\omega_0 t + \Theta)$ where $X(t)$ is a wide-sense stationary random process that amplitude-modulates a carrier of constant angular frequency ω_0 with a random phase Θ independent of $X(t)$ and uniformly distributed on $[-\pi, \pi]$

- (a) Find $E[Y(t)]$
- (b) Find the autocorrelation of $Y(t)$
- (c) Is $Y(t)$ wide-sense stationary?

$$Y(t) = X(t) \cos(\omega_0 t + \Theta)$$

- $X(t)$ w.s.s.
 - ω_0 constant
 - $\Theta \sim U(-\pi, \pi)$
 - $X(t), \Theta$ are INDEPENDENT

a) $E[Y(t)] = E[X(t) \cos(\omega_0 t + \Theta)]$

$$= E[X(t)] E[\cos(\omega_0 t + \Theta)]$$

$$\Theta \sim U(-\pi, \pi)$$

$$= 0$$

b) $R_{YY}(t, t+\tau) = E[Y(t)Y(t+\tau)] = E[X(t)\cos(\omega_0 t + \Theta) X(t+\tau)\cos(\omega_0(t+\tau) + \Theta)]$

$$= E[X(t)X(t+\tau)] \cos(\omega_0 t + \Theta) \cos(\omega_0(t+\tau) + \Theta)$$

$$= \underbrace{E[X(t)X(t+\tau)]}_{R_{XX}(\tau)} E[\cos(\omega_0 t + \Theta) \cos(\omega_0(t+\tau) + \Theta)]$$

$$= R_{XX}(\tau) \cdot \frac{1}{2} \left[E[\cos(\omega_0(2t+\tau) + 2\Theta)] + E[\cos(-\omega_0\tau)] \right]$$

$$\Theta \sim U(-\pi, \pi)$$

$$2\pi$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

c) $Y(t)$ is w.s.s. since $E[Y(t)]$ is constant and its autocorrelation is a function of τ .

2.6 The random processes $X(t)$ and $Y(t)$ are statistically independent, have zero mean and have autocorrelation functions

$$R_{XX}(\tau) = e^{-|\tau|}$$

$$R_{YY}(\tau) = \cos(2\pi\tau)$$

- (a) Find the autocorrelation function of $W_1(t) = X(t) + Y(t)$
- (b) Find the autocorrelation function of $W_2(t) = X(t) - Y(t)$
- (c) Find the cross-correlation function of $W_1(t)$ and $W_2(t)$

$$\begin{aligned} R_{XX}(\tau) &= e^{-|\tau|} & \bar{X} = 0 \\ R_{YY}(\tau) &= \cos 2\pi\tau & \bar{Y} = 0 \\ X, Y \text{ are INDEPENDENT} & & \left. \begin{array}{l} \text{INDEPENDENT} \\ + \\ \bar{X} = \bar{Y} = 0 \end{array} \right\} \Rightarrow \text{ORTHOGONAL} \Rightarrow R_{XY}(t, t+\tau) = 0 \\ & & R_{YX}(t, t+\tau) = 0 \end{aligned}$$

a) $W_1(t) = X(t) + Y(t)$

$$\begin{aligned} R_{W_1 W_1}(t, t+\tau) &= E[W_1(t)W_1(t+\tau)] = E[(X(t) + Y(t)) \cdot (X(t+\tau) + Y(t+\tau))] \\ &= \underbrace{E[X(t)X(t+\tau)]}_{R_{XX}(\tau)} + \cancel{E[Y(t)X(t+\tau)]}^0 + \cancel{E[X(t)Y(t+\tau)]}^0 + \underbrace{E[Y(t)Y(t+\tau)]}_{R_{YY}(\tau)} \\ &= R_{XX}(\tau) + R_{YY}(\tau) = e^{-|\tau|} + \cos 2\pi\tau = R_{W_1 W_1}(\tau) \end{aligned}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70**

$$b) W_2(t) = X(t) - Y(t)$$

$$\begin{aligned} R_{W_2 W_2}(t, t+\tau) &= E[W_2(t) W_2(t+\tau)] = E[(X(t) - Y(t))(X(t+\tau) - Y(t+\tau))] \\ &= E[X(t)X(t+\tau)] - \cancel{E[Y(t)X(t+\tau)]}^{\circ} - \cancel{E[X(t)Y(t+\tau)]}^{\circ} + E[Y(t)Y(t+\tau)] \\ &= R_{XX}(\tau) + R_{YY}(\tau) = e^{-\tau\varepsilon_1} + \cos 2\pi\tau = R_{W_2 W_2}(\tau) = R_{WW_2}(\tau) \end{aligned}$$

$$\begin{aligned} c) R_{W_1 W_2}(t, t+\tau) &= E[W_1(t) W_2(t+\tau)] = E[(X(t) + Y(t))(X(t+\tau) - Y(t+\tau))] \\ &= R_{XX}(\tau) - \cancel{R_{XY}(\tau)}^{\circ} + \cancel{R_{YX}(\tau)}^{\circ} - R_{YY}(\tau) \\ &= R_{XX}(\tau) - R_{YY}(\tau) = e^{-\tau\varepsilon_1} - \cos 2\pi\tau = R_{W_1 W_2}(\tau) \end{aligned}$$

2.7 Given two w.s.s. random processes $X(t)$ and $Y(t)$. Find expressions for the autocorrelation function of $W(t) = X(t) + Y(t)$ if:

- (a) $X(t)$ and $Y(t)$ are correlated
- (b) $X(t)$ and $Y(t)$ are uncorrelated
- (c) $X(t)$ and $Y(t)$ are uncorrelated with zero mean

Try to use means over cross-correlations if possible.

$$\text{W.s.s.} \Rightarrow E[X(t)] = \bar{X}, \quad R_{XX}(t, t+\tau) = R_{XX}(\tau) \\ E[Y(t)] = \bar{Y}, \quad R_{YY}(t, t+\tau) = R_{YY}(\tau)$$

$$W(t) = X(t) + Y(t)$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70**

a) $X(t), Y(t)$ are correlated

$$R_{WW}(t, t+\tau) = E[W(t)W(t+\tau)] = E[(X(t)+Y(t)) \cdot (X(t+\tau)+Y(t+\tau))] \\ = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau)$$

b) uncorrelated $\Rightarrow R_{XY}(\tau) = E[X(t)Y(t+\tau)] = E[X(t)]E[Y(t+\tau)] = \bar{X} \cdot \bar{Y}$

$$R_{YX}(\tau) = \bar{Y} \bar{X}$$

$$R_{WW}(\tau) = R_{XX}(\tau) + R_{YY}(\tau) + 2\bar{X} \cdot \bar{Y}$$

c) uncorrelated \Rightarrow orthogonal $\Rightarrow R_{XY}(\tau) = R_{YX}(\tau) = 0$
with zero mean

$$R_{WW}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$$

2.8 Consider random processes

$$Y_1(t) = X(t)\cos(\omega_0 t)$$

$$Y_2(t) = Y(t)\cos(\omega_0 t + 2\Theta)$$

where $X(t)$ and $Y(t)$ are jointly wide-sense stationary processes.

- (a) If $\Theta \sim U(\theta_0, \theta_1)$ and independent of $X(t)$ and $Y(t)$, are there any conditions on Θ that will make $Y_1(t)$ and $Y_2(t)$ orthogonal?



CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

$$Y_1(t) = X(t) \cos w_0 t$$

$$Y_2(t) = Y(t) \cos(w_0 t + 2\Theta)$$

$Y_1(t), Y_2(t)$ are j.w.s.s. \Rightarrow

$$E[X(t)] = \bar{x}$$

$$E[Y(t)] = \bar{y}$$

$$R_{XX}(z)$$

$$R_{YY}(z)$$

$$R_{XY}(z)$$

$$\Theta \sim U(\bar{\theta}_0, \bar{\theta}_1)$$

$X(t), Y(t)$ are independent

$\Theta, Y(t)$ are independent

looking for $\sigma_0, \sigma_1 / R_{Y_1 Y_2}(t, t+\tau) = 0$

$$\begin{aligned} R_{Y_1 Y_2}(t, t+\tau) &= E[Y_1(t) Y_2(t+\tau)] = E[X(t) \cos w_0 t \cdot Y(t+\tau) \cos(w_0(t+\tau) + 2\Theta)] \\ &= E[\overbrace{X(t) Y(t+\tau)}^{\text{INDEPENDENT}} \underbrace{\cos w_0 t \cdot \cos(w_0(t+\tau) + 2\Theta)}_{\text{NOT RANDOM}}] \\ &= E[X(t) Y(t+\tau)] E[\cos(w_0(t+\tau) + 2\Theta)] \cdot \cos w_0 t \\ &= \cos w_0 t \cdot R_{XY}(t, t+\tau) \cdot E[\cos(w_0(t+\tau) + 2\Theta)] \end{aligned}$$

$$R_{Y_1 Y_2}(t, t+\tau) = 0 \quad \text{if} \quad E[\cos(w_0(t+\tau) + 2\Theta)] = 0$$

$$\Theta' = 2\Theta \sim U(2\bar{\theta}_0, 2\bar{\theta}_1)$$

$$\downarrow \quad k \in \mathbb{N}, \dots$$

$$E[\cos(w_0(t+\tau) + 2\Theta)] = 0 \quad \text{if} \quad 2\bar{\theta}_1 - 2\bar{\theta}_0 = k \cdot 2\pi$$

$$\bar{\theta}_1 - \bar{\theta}_0 = k \cdot 2\pi$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

2.9 Consider the random processes

$$X(t) = A \cos(\omega_0 t + \Theta)$$

$$Y(t) = B \sin(\omega_0 t + \Theta)$$

where A , B , and ω_0 are constants while Θ is a random variable uniform on $[0, 2\pi]$. $X(t)$ and $Y(t)$ are zero-mean, wide-sense stationary with autocorrelation functions:

$$R_{XX}(\tau) = (A^2/2)\cos(\omega_0\tau)$$

$$R_{YY}(\tau) = (B^2/2)\cos(\omega_0\tau)$$

$X(t)$ and $Y(t)$ jointly wide-sense stationary?

$$X(t) = A \cos(\omega_0 t + \Theta) \rightarrow \text{w.s.s.}$$

$$Y(t) = B \sin(\omega_0 t + \Theta) \rightarrow \text{w.s.s.}$$

$$\Theta \sim U(0, 2\pi)$$

$A, B \rightarrow \text{constant}$

$$X(t), Y(t) \text{ j.w.s.s.?} \Rightarrow X(t) \text{ w.s.s. } \checkmark$$

$$Y(t) \text{ w.s.s. } \checkmark$$

$$R_{XY}(t, t+\tau) = R_{XY}(\tau)$$

$$R_{XY}(t, t+\tau) = E[X(t)Y(t+\tau)]$$

$$= E[A \cos(\omega_0 t + \Theta) B \cos(\omega_0(t+\tau) + \Theta)]$$

= WE CAN FOLLOW THE TRADITIONAL DERIVATION OR ...

$$= AB E[\cos(\omega_0 t + \Theta) \cos(\omega_0(t+\tau) + \Theta)]$$

$$= AB \frac{R_{XX}(\tau)}{A^2} = \frac{B}{A} R_{XX}(\tau) = R_{XY}(\tau)$$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

Exercises to UNIT 2- Part II: Spectral Characteristics

2.10

Given that $X(t) = \sum_{i=1}^N \alpha_i X_i(t)$ where α_i are real constants, show that
 $S_{XX}(\omega) = \sum_{i=1}^N \alpha_i^2 S_{X_i X_i}(\omega)$
if

- (a) the processes $X_i(t)$ are orthogonal
- (b) the processes are independent with zero mean

2.11

If $X(t)$ is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in terms of the power spectrum of $X(t)$ if A_0 and B_0 are real constants.

2.12 The autocorrelation function of a random process $X(t)$ is

$$R_{XX}(\tau) = 3 + 2e^{-4\tau^2}$$

Find

- (a) The power spectrum of $X(t)$
- (b) The average power of $X(t)$
- (c) The fraction of power that lies in the frequency band $\sqrt{-1/2} \leq \omega \leq 1/2$

2.13

Given a random process with autocorrelation $R_{XX}(\tau) = P \cos^4(\omega_0 \tau)$, find

- (a) $S_{XX}(\omega)$
- (b) P_{XX} from $S_{XX}(\omega)$ (c) P_{XX} from $R_{XX}(\tau)$

2.14

Given a random process with autocorrelation

$$R_{XX}(\tau) = Ae^{-\alpha|\tau|} \cos(\omega_0 \tau) \text{ where } A > 0, \alpha > 0, \text{ and } \omega_0 \text{ are real constants,}$$

**CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70**

**ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70**



$W(t) = AX(t) + BY(t)$ where A and B are real constants and $X(t)$ and $Y(t)$ are jointly wide-sense stationary processes. Find

- The power spectrum of $W(t)$ as a function of $S_{XX}(\omega)$, $S_{YY}(\omega)$, $S_{XY}(\omega)$ and $S_{YX}(\omega)$
- The power spectrum of $W(t)$ as a function of $S_{XX}(\omega)$, $S_{YY}(\omega)$, \bar{X} and \bar{Y} , if $X(t)$ and $Y(t)$ are uncorrelated
- $S_{XW}(\omega)$ and $S_{WX}(\omega)$ as functions of $S_{XX}(\omega)$, $S_{YY}(\omega)$, $S_{XY}(\omega)$ and $S_{YX}(\omega)$

2.16 A wide-sense stationary $X(t)$ is applied to an ideal differentiator, so that $Y(t) = dX(t)/dt$. The cross-correlation of the input-output processes is known to be

$$R_{XY}(\tau) = dR_{XX}(\tau)/dt$$

- Determine $S_{XY}(\omega)$ in terms of $S_{XX}(\omega)$
- Determine $S_{YX}(\omega)$ in terms of $S_{XX}(\omega)$

2.17 The cross-correlation of jointly wide-sense stationary processes $X(t)$ and $Y(t)$ is assumed to be

$$R_{XY}(\tau) = Be^{-W\tau}u(\tau) \text{ where } B > 0 \text{ and } W > 0$$

are constants. Find

- $R_{YX}(\tau)$
- $S_{XY}(\omega)$ (use appendix C from Peebles' book)
- $S_{YX}(\omega)$ (use cross-power density properties)

2.18 Consider two random process $X_1(t)$ and $X_2(t)$. The mean of $X_1(t)$ is equal to A ($A > 0$) and $X_2(t)$ is a white noise with power density 5 W/(rad/s) . Given an LTI system with impulse response $h(t) = e^{-\alpha t}u(t)$.

with $\alpha > 0$. Find

(a) The mean value of the response of the LTI system if the input is $X_1(t)$

CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70

- (b) The average power (second-order moment) of the response of the system if the input is $X_2(t)$.

2.19 A random process $X(t)$ with known mean \bar{X} is the input of an LTI system with impulse response

$$h(t) = te^{-Wt}u(t).$$

Find

- (a) The mean value of the response of the LTI system
- (b) The average power (second-order moment) of the response of the system if $X(t)$ is a white noise with power density 5 W/(rad/s).

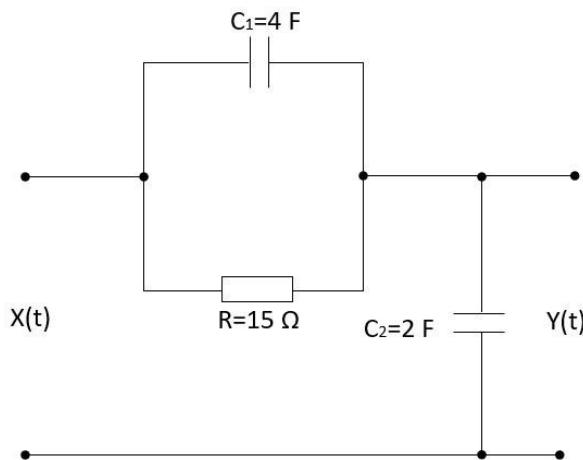
2.20 A white noise with power density $N_0/2$ is applied to a network with impulse response of a system with impulse response

$$h(t) = Wte^{-Wt}u(t)$$

where W is a real positive constant. Find the cross-correlation of the response of input and the output of the system.

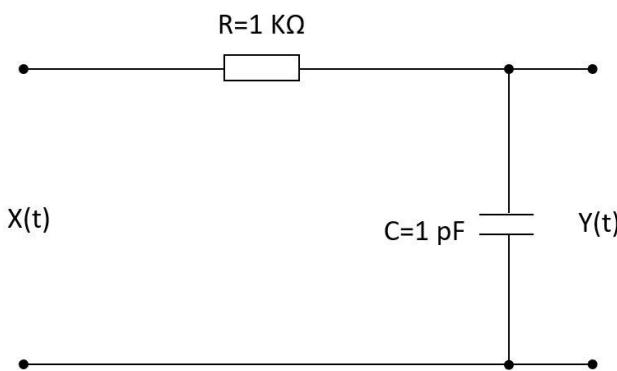
2.21 A stationary random process $X(t)$, having an autocorrelation function

$R_{XX} = 2e^{-4|\tau|}$ is applied to the network of the figure below. Find the power spectrum of the output of the system.



CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70



CLASES PARTICULARES, TUTORÍAS TÉCNICAS ONLINE
LLAMA O ENVÍA WHATSAPP: 689 45 44 70

ONLINE PRIVATE LESSONS FOR SCIENCE STUDENTS
CALL OR WHATSAPP: 689 45 44 70