

# Fundamental Concepts of Statistics

## Exercise session 2

1. The speed of a molecule in a uniform gas at equilibrium is a random variable  $V$  whose density function is given by

$$f_V(v) = av^2 e^{-\frac{m}{2kT}v^2}, \quad v > 0,$$

with  $a$  a constant,  $k$  denoting Boltzmann's constant,  $T$  the absolute temperature and  $m$  the mass of a molecule.

- Derive the distribution of the kinetic energy  $W = \frac{mV^2}{2}$
- Compute  $E(W)$

2. Find the density of  $Y = X^2$  where  $X$  is uniformly distributed on the interval  $[-1, 1]$ .

3. Let  $f(x) = \alpha x^{-\alpha-1}$  for  $x > 1$  and  $f(x) = 0$  otherwise, with  $\alpha > 0$ . Show how to generate random variables from this density from a uniform random number generator.

4. The Weibull cdf is given by

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, \quad x > 0, \quad \alpha, \beta > 0.$$

- Find the density function.
  - Show that if  $W$  follows a Weibull distribution then  $X = (W/\alpha)^\beta$  follows an exponential distribution.
  - How to generate random variables from this density from a uniform random number generator ?
5. Compute  $Cov(X + Y, X - Y)$ , where  $X$  and  $Y$  are random variables with equal variances.



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We assume  $E(X) = E(Y) = \mu$ , but  $\sigma_X$  and  $\sigma_Y$  are unequal. The two

measurements are combined in a weighted average

$$Z = \alpha X + (1 - \alpha)Y,$$

where  $0 \leq \alpha \leq 1$ .

Show that  $E(Z) = \mu$ .

Find  $\alpha$  in terms of  $\sigma_X$  and  $\sigma_Y$  to minimize  $Var(Z)$ .

The logo for Cartagena99 features the text 'Cartagena99' in a stylized, blue, serif font. The '99' is significantly larger and more prominent than the rest of the text. The logo is set against a light blue background with a white swoosh underneath.

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