

Problems

Problem 9.1 Obtain the following immediate (or nearly so) primitives:

$$\begin{array}{lll}
 \text{(i)} \int \frac{dx}{\cos^2 x}; & \text{(iv)} \int \frac{1 + \sin x}{1 + \cos x} dx; & \text{(vii)} \int \frac{1 + \sqrt{1 - \sqrt{x}}}{\sqrt{x}} dx; \\
 \text{(ii)} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx; & \text{(v)} \int \frac{dx}{1 - \sin x}; & \text{(viii)} \int \frac{\cos^3 x}{\sin^4 x} dx; \\
 \text{(iii)} \int \frac{x}{(x^2 + 1)^{5/2}} dx; & \text{(vi)} \int \frac{x}{\sqrt{1 + x^2}} dx; & \text{(ix)} \int x^3 \sqrt{1 - x^2} dx.
 \end{array}$$

HINTS: (iv) multiply and divide by $1 - \cos x$ and expand; (v) idem with $1 + \sin x$; (vii) alternatively $t = \sqrt{1 - \sqrt{x}}$; (viii) $\cos^3 x = (1 - \sin^2 x) \cos x$ and expand; (ix) write $x^3 = x(x^2 - 1) + x$ and expand.

Problem 9.2 Obtain the primitives of the following rational functions:

$$\begin{array}{lll}
 \text{(i)} \int \frac{x^2}{(x-1)^3} dx; & \text{(iii)} \int \frac{2x^2 + 3}{x^2(x-1)} dx; & \text{(v)} \int \frac{4x^2 - x^3 - 46x^2 - 20x + 153}{x^3 - 2x^2 - 9x + 18} dx; \\
 \text{(ii)} \int \frac{dx}{(x-1)^2(x^2 + x + 1)}; & \text{(iv)} \int \frac{2}{x^2 - 2x + 2} dx; & \text{(vi)} \int \frac{x^5 - 2x^3}{x^4 - 2x^2 + 1} dx.
 \end{array}$$

HINTS: (ii) $x^2 + x + 1 = (x + 1/2)^2 + 3/4$; (v) $x^3 - 2x^2 - 9x + 18 = (x - 2)(x - 3)(x + 3)$; (vi) $x^4 - 2x^2 + 1 = (x - 1)^2(x + 1)^2$.

Problem 9.3 Obtain the following primitives doing an appropriate change of variable:

$$\begin{array}{lll}
 \text{(i)} \int x^2 \sqrt{x-1} dx; & \text{(ix)} \int \frac{dx}{(2+x)\sqrt{1+x}}; & \text{(xvii)} \int \sqrt{\sqrt{x} + 1} dx; \\
 \text{(ii)} \int x^2 \sin \sqrt{x^3} dx; & \text{(x)} \int \frac{dx}{1 + \sqrt[3]{1-x}}; & \text{(xviii)} \int \frac{\sqrt{x+2}}{1 + \sqrt{x+2}} dx; \\
 \text{(iii)} \int \cos(\log x) dx; & \text{(xi)} \int \frac{e^{4x}}{e^{2x} + 2e^x + 2} dx; & \text{(xix)} \int \sqrt{2 + e^x} dx; \\
 \text{(iv)} \int \sin(\log x) dx; & \text{(xii)} \int \frac{dx}{\sqrt{e^{2x} - 1}}; & \text{(xx)} \int \frac{\sin x + 3 \cos x}{\sin x + 2 \cos x} dx; \\
 \text{(v)} \int \cos^2(\log x) dx; & \text{(xiii)} \int \sqrt{e^x - 1} dx; & \text{(xxi)} \int \frac{\sin x + 3 \cos x}{\sin x \cos x + 2 \sin x} dx; \\
 \text{(vi)} \int \frac{\sqrt{x} + 1}{x + 3} dx; & \text{(xiv)} \int \frac{\sin^2 x \cos^5 x}{\tan^3 x} dx; & \text{(xxii)} \int \frac{\sqrt{1 + \sqrt[3]{x}}}{\sqrt[3]{x}} dx; \\
 \text{(vii)} \int \frac{(x+1)^3}{\sqrt{1 - (x+1)^2}} dx; & \text{(xv)} \int \frac{dx}{3 + \sqrt{2x+5}}; & \text{(xxiii)} \int \frac{dx}{(x+1)\sqrt[3]{x+2}}; \\
 \text{(viii)} \int \frac{x^3}{(1+x^2)^3} dx; & \text{(xvi)} \int \sqrt{\frac{x-1}{x+1}} dx; & \text{(xxiv)} \int \frac{dx}{e^x - 4e^{-x}} dx.
 \end{array}$$

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| (i) $\int \sin^2 x dx;$ | (vi) $\int \sin^2 x \cos^2 x dx;$ | (xi) $\int \cos^3 x \sin^2 x dx;$ |
| (ii) $\int \cos^2 x dx;$ | (vii) $\int \tan^2 x dx;$ | (xii) $\int \sec^6 x dx;$ |
| (iii) $\int \sin^4 x dx;$ | (viii) $\int \tan^4 x dx;$ | (xiii) $\int \sin^3 x \cos^2 x dx;$ |
| (iv) $\int \cos^4 x dx;$ | (ix) $\int \frac{dx}{\cos^4 x};$ | (xiv) $\int \tan^3 x dx;$ |
| (v) $\int \cos^6 x dx;$ | (x) $\int \sin^5 x dx;$ | (xv) $\int \tan^3 x \sec^4 x dx.$ |

HINTS: Identities to use: $2 \cos^2 x = 1 + \cos 2x$; $2 \sin^2 x = 1 - \cos 2x$; $\cos^2 x + \sin^2 x = 1$; $\sec^2 x = 1 + \tan^2 x$.

Problem 9.5 Integrate by parts to obtain the following primitives:

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| (i) $\int x \tan^2(2x) dx;$ | (v) $\int \tan^2(3x) \sec^3(3x) dx;$ | (ix) $\int (\log x)^3 dx;$ |
| (ii) $\int e^x \sin \pi x dx;$ | (vi) $\int e^{\sin x} \cos^3 x dx;$ | (x) $\int x(\log x)^2 dx;$ |
| (iii) $\int e^x \cos 2x dx;$ | (vii) $\int x^2 \log x dx;$ | (xi) $\int \frac{x \log x}{(1+x^2)^2} dx;$ |
| (iv) $\int \sec^3 x dx;$ | (viii) $\int x^m \log x dx;$ | (xii) $\int \arctan \sqrt[3]{x} dx.$ |

Problem 9.6 Obtain the following primitives by performing a trigonometric substitution:

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|---|--|---|
| (i) $\int \frac{x^2 + 1}{\sqrt{x^2 - 1}} dx;$ | (iii) $\int \frac{x^2}{(1 - x^2)^{3/2}} dx;$ | (v) $\int \frac{dx}{x^2 \sqrt{9 - x^2}}.$ |
| (ii) $\int \frac{x^2}{(x^2 + 1)^{5/2}} dx;$ | (iv) $\int \frac{dx}{x^2 \sqrt{1 - x^2}};$ | |

Problem 10.2

- (a) Prove that if f is odd then $\int_{-a}^a f(x) dx = 0$.
- (b) Prove that if f is even then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.
- (c) Calculate the integral $\int_6^{10} \sin(\sin((x-8)^3)) dx$.

Problem 10.4 Calculate $F(x) = \int_{-1}^x f(t) dt$, with $-1 \leq x \leq 1$, for the following functions:

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| (i) $f(x) = x e^{- x };$ | (v) $f(x) = \begin{cases} 1, & -1 \leq x \leq 0, \\ x+1, & 0 < x \leq 1; \end{cases}$ |
| (ii) $f(x) = x - 1/2 ;$ | (vi) $f(x) = \begin{cases} 1+x, & -1 \leq x \leq -\frac{1}{2}, \\ \frac{1}{x}, & -\frac{1}{2} < x < 1 \end{cases}$ |
| (iii) $f(x) = \begin{cases} -1, & -1 \leq x < 0, \\ 1, & 0 \leq x \leq 1; \end{cases}$ | |

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$$f(x) = \begin{cases} \frac{x}{x^2}, & x < 0, \end{cases}$$

Problems

Problem 11.1 Calculate the area delimited by the following curves:

- (i) $y = x^2, y = (x-2)^2, y = (2-x)/6$;
 (ii) $x^2 + y^2 = 1, x^2 + y^2 = 2x$;
 (iii) $y = \frac{1-x}{1+x}, y = \frac{2-x}{1+x}, y = 0, y = 1$;
 (iv) one loop of the curve $y^2 = (x-a)(x-b)^2$, with $a < b$.

Problem 11.2 Determine the area between the curve $f(x) = \frac{x(x^2-1)}{(x^2+1)^{3/2}}$ and the X axis.

Problem 11.3 Calculate the area delimited by the following curves:

- (i) $r = a\theta$ (Archimedes's spiral), $0 \leq \theta \leq 2\pi$, and the segment $\{(x, 0) : 0 \leq x \leq 2\pi a\}$;
 (ii) a petal of the three-petal rose $r = a \cos 3\theta, -\pi/6 \leq \theta \leq \pi/6$;
 (iii) half a *lemniscata* $r = a\sqrt{\cos 2\theta}, -\pi/4 \leq \theta \leq \pi/4$.

Problem 11.4 Let A the plane figure limited by the curves $y = x^2$ and $y = \sqrt{x}$. Determine:

- (a) the area of A;

Problem 12.1 Discuss the convergence of the following improper integrals:

- (i) $\int_0^{\infty} e^{-t} t^{p-1} dt$; (iv) $\int_{-\infty}^{\infty} e^{-x} dx$; (vii) $\int_1^2 \frac{\log t + t - 1}{(t-1)^{3/2}} dt$;
 (ii) $\int_0^a e^{1/x} x^p dx$; (v) $\int_1^{\infty} \frac{dx}{x^\beta \sqrt{1+x^2}}$; (viii) $\int_{-\infty}^{\infty} \frac{\cos ax}{k^2 + x^2} dx$, with $k \neq 0$;
 (iii) $\int_{-\infty}^{\infty} e^{-x^2} dx$; (vi) $\int_0^1 \log x dx$; (ix) $\int_1^{\infty} \left(\frac{1}{\sqrt{x}} - \arctan \frac{1}{\sqrt{x}} \right) dx$.

Problem 12.2 Prove that the following integrals converge for the specified range of parameters:

- (i) $\int_0^1 x^p (1-x)^q dx$, for $p, q > -1$; (iii) $\int_0^{\pi/2} \log \left(\frac{1+s \cos x}{1-s \cos x} \right) \frac{dx}{\cos x}$, for $|s| \leq 1$;
 (ii) $\int_0^{\infty} \log \left(1 + \frac{a^2}{x^2} \right) dx$, for $a \in \mathbb{R}$; (iv) $\int_0^1 \frac{x^p - 1}{\log x} dx$, for $p > -1$.

Problem 12.3 Consider the improper integral of the first kind

$$\int_0^{\infty} \left(\frac{1}{\sqrt{1+x^2}} - \frac{\alpha}{x+1} \right) dx.$$

- (a) Find the only value of α for which this integral converges.
 (b) Calculate the integral for that value of α .



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