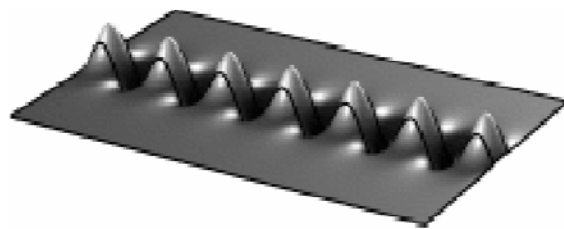
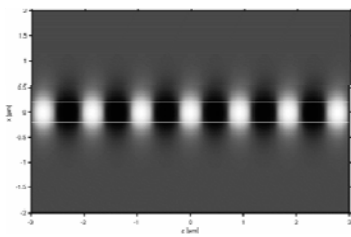


FIBRAS OPTICAS

INTRODUCCIÓN

Introducción



Introducción

Métodos de análisis

Óptica Geométrica → “Teoría de Rayos”

Válida sólo si longitud de onda \ll Dimensiones de la estructura de guiado.

Uso de las leyes de la Reflexión y Refracción + Óptica de Fresnel

Concepto de onda plana.

Óptica Física (ondulatoria) → Ecuaciones de Maxwell +
Condiciones de contorno.

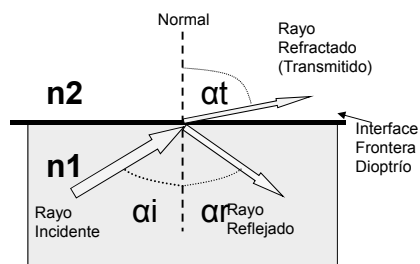
Válida para toda estructura, condiciones de propagación y frecuencia.

Concepto de Modo de Propagación.

Óptica Cuántica

Introducción

Leyes de Snell



1ª Ley: Reflexión

-Rayo incidente y reflejado están
en el mismo plano

$$\alpha_i = \alpha_r$$

2ª Ley: Refracción

-Rayo incidente y refractado
están en el mismo plano

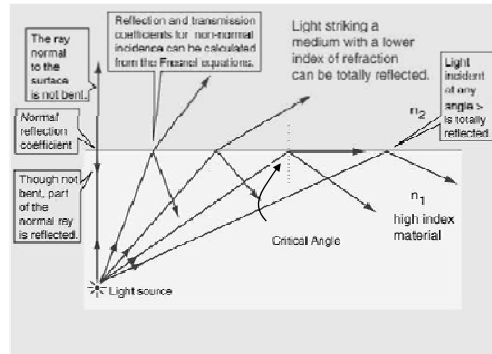
$$n_1 \cdot \text{sen}(\alpha_i) = n_2 \cdot \text{sen}(\alpha_t)$$

Introducción

Ángulo Crítico

$$n_1 \cdot \text{sen}(\alpha_{ic}) = n_2 \cdot \text{sen}(\pi / 2)$$

$$\alpha_{ic} = \text{sen}^{-1} \left(\frac{n_2}{n_1} \right)$$



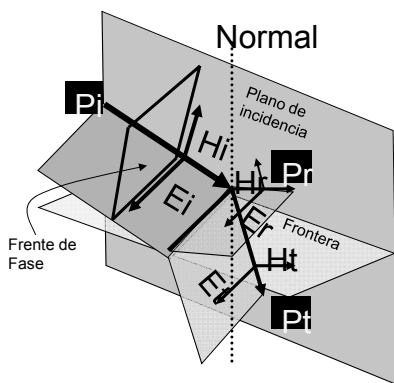
Ejemplos numéricos:

Interfase Aire / AsGa: $\alpha_c = \text{sen}^{-1} \frac{n_2}{n_1} = \text{sen}^{-1} \frac{0,3}{1} = 17,45^\circ$

Interfase Aire / Si: $\alpha_c = \text{sen}^{-1} \frac{n_2}{n_1} = \text{sen}^{-1} \frac{0,7}{1} = 44,42^\circ$

Introducción

Polarización TE



$$\mathfrak{R}_{TE} = \frac{n_1 \cdot \cos \alpha_i - \sqrt{n_2^2 - n_1^2 \cdot \text{sen}^2 \alpha_i}}{n_1 \cdot \cos \alpha_i + \sqrt{n_2^2 - n_1^2 \cdot \text{sen}^2 \alpha_i}}$$

$$= \frac{n_1 \cdot \cos \alpha_i - n_2 \cdot \cos \alpha_t}{n_1 \cdot \cos \alpha_i + n_2 \cdot \cos \alpha_t}$$

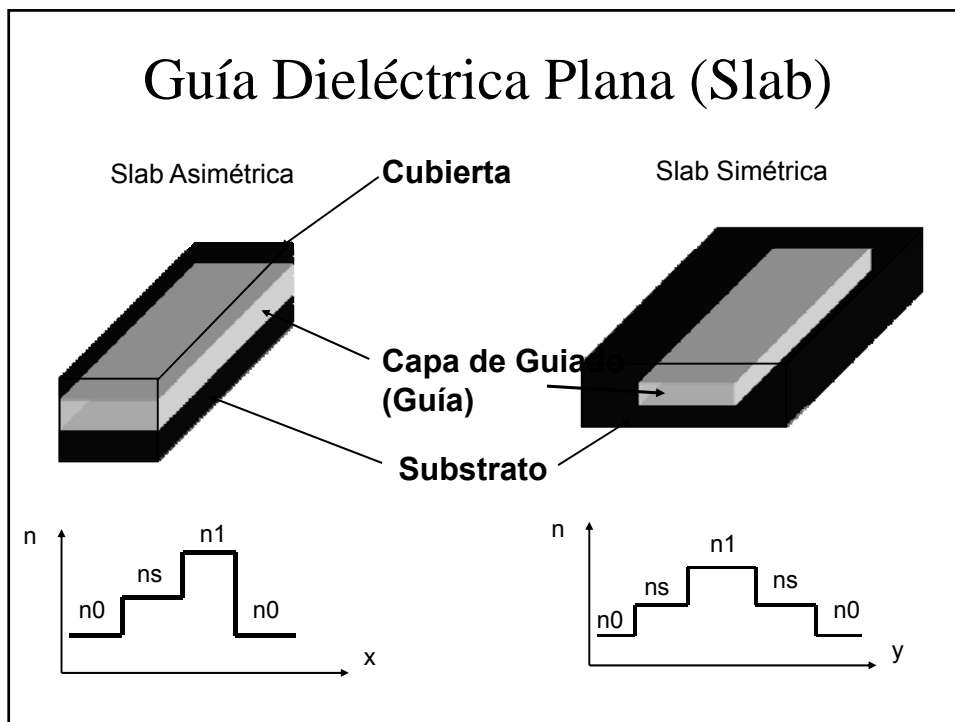
- Para ángulos de incidencia superiores al crítico (Reflexión Total):

$$\mathfrak{R}_{TE} = 1 \cdot e^{j\phi_{TE}}$$

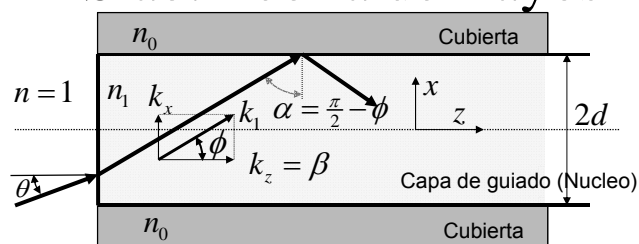
-La onda reflejada tiene la misma amplitud que la incidente pero ha sufrido un cierto desfase:

$$\phi_{TE} = 2 \arctan \frac{\sqrt{n_1^2 \cdot \text{sen}^2 \alpha_i - n_2^2}}{n_1 \cdot \cos \alpha_i} \quad \phi_{TM} = 2 \arctan \frac{\sqrt{\text{sen}^2 \alpha_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cdot \cos \alpha_i}$$

Guía Dieléctrica Plana (Slab)



Slab. Teoría de Rayos



Reflexión Total $n_1 \sin\left(\frac{\pi}{2} - \phi\right) \geq n_0 \quad \text{sen } \theta = n_1 \text{ sen } \phi$

Ángulo de incidencia máximo o apertura numérica (NA)

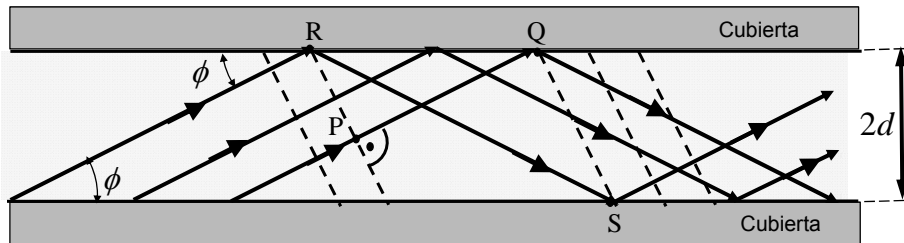
$$\theta_{\max} = \text{sen}^{-1} \sqrt{n_1^2 - n_0^2} \Rightarrow \theta_{\max} \approx \sqrt{n_1^2 - n_0^2}$$

Índice relativo de refracción $\Delta = \frac{n_1^2 - n_0^2}{2n_1^2} \approx \frac{n_1 - n_0}{n_1}$

$$NA = \theta_{\max} \approx n_1 \sqrt{2\Delta}$$

$$\left. \begin{array}{l} n_1 = 1.47 \\ n_0 = 1.455 \end{array} \right\} \Rightarrow \Delta = 1\%; NA = 0.21; \theta_{\max} = 12^\circ; \phi_{\max} = 8.3^\circ$$

Slab



ϕ no puede ser arbitrario. Se debe cumplir que todos los rayos tengan la misma fase en los frente de onda

$$d_{RQ} = \frac{2d}{\tan \phi} - 2d \tan \phi \quad d_{PQ} = d_{RQ} \cos \phi = 2d \left(\frac{1}{\sin \phi} - 2 \tan \phi \right)$$

$$d_{RS} = \frac{2d}{\sin \phi}$$

$$\text{Misma fase en frente de onda} \quad -k_1 d_{RS} + 2\Phi + k_1 d_{PQ} = -2m\pi$$

Φ Desfasaje por reflexión; m número entero

Slab

$$\Phi_{TE} = 2 \arctan \frac{\sqrt{n_1^2 \cdot \sin^2 \alpha_i - n_2^2}}{n_1 \cdot \cos \alpha_i} = 2 \arctan \frac{\sqrt{n_1^2 \cdot \cos^2 \phi - n_0^2}}{n_1 \cdot \sin \phi}$$

$$\Phi_{TE} = 2 \arctan \sqrt{\frac{2\Delta}{\sin^2 \phi} - 1}$$

$$\text{Condición de propagación} \quad \tan \left(k_1 d \sin \phi - \frac{m\pi}{2} \right) = \sqrt{\frac{2\Delta}{\sin^2 \phi} - 1}$$

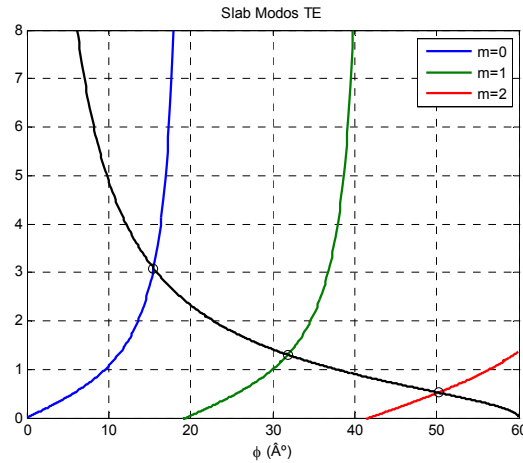
Solo hay un conjunto discreto de direcciones que permiten la propagación

A cada solución de la ecuación se le denomina modo y el correspondiente valor de β se le llama autovalor

Al modo que se propaga con el mínimo ángulo ϕ se le denomina modo fundamental y corresponde con $m = 0$

Slab

Solución gráfica. Modos TE $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



Solución numérica $\phi_1 = 15.4776^\circ$; $\phi_2 = 31.8463^\circ$; $\phi_3 = 50.2103^\circ$

Slab

$$\Phi_{\text{TM}} = 2 \arctan \frac{\sqrt{\sin^2 \alpha_i - (n_2/n_1)^2}}{(n_2/n_1)^2 \cdot \cos \alpha_i} = 2 \arctan \frac{\sqrt{\cos^2 \phi - (n_0/n_1)^2}}{(n_0/n_1)^2 \cdot \sin \phi}$$

$$\Phi_{\text{TM}} = 2 \arctan \left(\frac{n_1^2}{n_0^2} \sqrt{\frac{2\Delta}{\sin^2 \phi} - 1} \right)$$

$$\text{Condición de propagación } \tan \left(k_1 d \sin \phi - \frac{m\pi}{2} \right) = \frac{n_1^2}{n_0^2} \sqrt{\frac{2\Delta}{\sin^2 \phi} - 1}$$

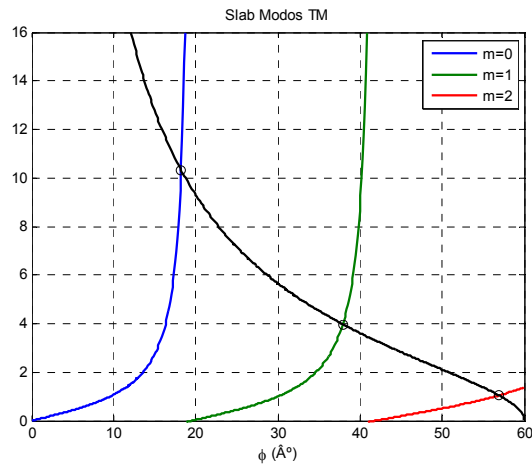
Solo hay un conjunto discreto de direcciones que permiten la propagación

A cada solución de la ecuación se le denomina modo y el correspondiente valor de β se le llama autovalor

Al modo que se propaga con el mínimo ángulo ϕ se le denomina modo fundamental y corresponde con $m = 0$

Slab

Solución gráfica. Modos $TM_{n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Solución numérica $\phi_1 = 18.2308^\circ$; $\phi_2 = 37.9051^\circ$; $\phi_3 = 56.9228^\circ$

Slab

- Sólo se propaga un conjunto discreto de trayectorias estables dentro de la guía slab.
- Cuanto mayor es el cociente entre d y λ menor es el ángulo de despegue respecto al eje longitudinal.
- Siempre existe un punto de corte, por tanto no existe situación de **corte** en el modo fundamental.
- Cada trayectoria permitida se identifica con un **MODO** de propagación.

Slab. Ecuación de onda

Modos TE

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_{ci}^2\right) h_z^i = 0 \Rightarrow \begin{cases} \left(\frac{\partial^2}{\partial x^2} + k_{ci}^2\right) h_z^i = 0 \\ \frac{\partial}{\partial y} = 0 \end{cases} \Rightarrow \begin{cases} u^2 = k_{c1}^2 = k^2 n_1^2 - \beta^2 \\ w^2 = -k_{c0}^2 = \beta^2 - k^2 n_0^2 \end{cases}$$

Solución general

Núcleo		$h_z(x) = \begin{cases} A_{no} \cos ux & \text{Modo Impar} \\ A_{ne} \text{sen } ux & \text{Modo Par} \end{cases}$	
Cubierta		$h_z(x) = \begin{cases} A_{co} e^{-wx} & x > d \\ A_{co} e^{wx} & x < -d \end{cases} \Rightarrow \text{Modo Impar}$ $\begin{cases} A_{ce} e^{-wx} & x > d \\ -A_{ce} e^{wx} & x < -d \end{cases} \Rightarrow \text{Modo Par}$	

Slab

Para que las soluciones representen ondas físicas reales:

$$\left. \begin{aligned} u^2 > 0 &\Rightarrow k n_1 > \beta \\ w^2 > 0 &\Rightarrow \beta > k n_0 \end{aligned} \right\} \Rightarrow k n_0 < \beta < k n_1$$

Modo TE impar. Condiciones de contorno

$$h_z(d^-) = h_z(d^+); \quad h_z(-d^-) = h_z(-d^+)$$

$$A_{no} \cos(ud) = A_{co} e^{-wd}$$

$$e_y(d^-) = e_y(d^+); \quad e_y(-d^-) = e_y(-d^+)$$

$$\frac{j\omega\mu}{k_{c1}^2} \frac{\partial h_z}{\partial x} \Big|_{x=d^-} = \frac{j\omega\mu}{k_{c0}^2} \frac{\partial h_z}{\partial x} \Big|_{x=d^+}; \quad \frac{j\omega\mu}{k_{c1}^2} \frac{\partial h_z}{\partial x} \Big|_{x=-d^-} = \frac{j\omega\mu}{k_{c0}^2} \frac{\partial h_z}{\partial x} \Big|_{x=-d^+}$$

$$\frac{-u A_{no} \text{sen}(ud)}{u^2} = \frac{-w A_{co} e^{-wd}}{-w^2} \Rightarrow w A_{no} \text{sen}(ud) = -u A_{co} e^{-wd}$$

Ecuación de dispersión

$$w \tan(ud) = -u$$

Slab

Modo TE par. Condiciones de contorno

$$h_z(d^-) = h_z(d^+); \quad h_z(-d^-) = h_z(-d^+)$$

$$A_{ne} \operatorname{sen}(u d) = A_{ce} e^{-w d}$$

$$e_y(d^-) = e_y(d^+); \quad e_y(-d^-) = e_y(-d^+)$$

$$\frac{j\omega\mu}{k_{c1}^2} \frac{\partial h_z}{\partial x} \Big|_{x=d^-} = \frac{j\omega\mu}{k_{c0}^2} \frac{\partial h_z}{\partial x} \Big|_{x=d^+}; \quad \frac{j\omega\mu}{k_{c1}^2} \frac{\partial h_z}{\partial x} \Big|_{x=-d^-} = \frac{j\omega\mu}{k_{c0}^2} \frac{\partial h_z}{\partial x} \Big|_{x=-d^+}$$

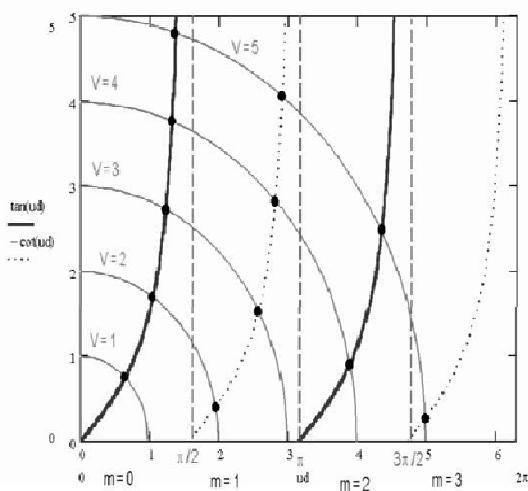
$$\frac{u A_{ne} \cos(u d)}{u^2} = \frac{-w A_{ce} e^{-w d}}{-w^2} \Rightarrow w A_{ne} \cos(u d) = u A_{ce} e^{-w d}$$

Ecuación de dispersión

$$u \tan(u d) = w$$

Slab

Se define la **FRECUENCIA NORMALIZADA** de la guía (o Anchura Normalizada) **V**:



$$V \triangleq \sqrt{(u d)^2 + (w d)^2} =$$

$$(u^2 + w^2)^{1/2} d =$$

$$(k^2 n_1^2 - \beta^2 + \beta^2 - k^2 n_2^2)^{1/2} d =$$

$$k (n_1^2 - n_2^2)^{1/2} d$$

Slab

Campos transversales. Modos TE

$$h_x = \frac{-j\beta}{k_c^2} \frac{\partial h_z}{\partial x} \quad e_x = \frac{-j\omega\mu}{k_c^2} \frac{\partial h_z}{\partial y}$$

$$h_y = \frac{-j\beta}{k_c^2} \frac{\partial H_z}{\partial y} \quad e_y = \frac{j\omega\mu}{k_c^2} \frac{\partial h_z}{\partial x}$$

Slab $h_y = 0 \quad e_x = 0$

Modo Impar

$$h_z = \begin{cases} A_{no} \cos ux & |x| < d \\ A_{co} e^{-wx} & x > d \\ A_{co} e^{wx} & x < -d \end{cases} \quad \begin{matrix} \Rightarrow \\ \Rightarrow \end{matrix} \quad h_z = \begin{cases} A_{no} \cos ux & |x| < d \\ A_{no} \cos ud e^{-w(x-d)} & x > d \\ A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases}$$

$$A_{co} = A_{no} \cos(ud) e^{wd}$$

Slab

Campos transversales. Modo TE impar

$$e_y = \begin{cases} -\frac{j\omega\mu}{u} A_{no} \operatorname{sen} ux & |x| < d \\ \frac{j\omega\mu}{w} A_{no} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\omega\mu}{-w} A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases} \quad \begin{matrix} \Rightarrow \\ \Rightarrow \end{matrix} \quad -u \frac{\cos(ud)}{\operatorname{sen}(ud)} = w$$

$$e_y = \begin{cases} -\frac{j\omega\mu}{u} A_{no} \operatorname{sen} ux & |x| < d \\ -\frac{j\omega\mu}{u} A_{no} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ \frac{j\omega\mu}{u} A_{no} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Campos transversales. Modo TE impar

$$h_x = \begin{cases} \frac{j\beta}{u} A_{no} \operatorname{sen} ux & |x| < d \\ \frac{-j\beta}{w} A_{no} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\beta}{w} A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases}$$

$-u \frac{\cos(ud)}{\operatorname{sen}(ud)} = w$

$$h_x = \begin{cases} \frac{j\beta}{u} A_{no} \operatorname{sen} ux & |x| < d \\ \frac{j\beta}{u} A_{no} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ -\frac{j\beta}{u} A_{no} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Modo Par

$$h_z = \begin{cases} A_{ne} \operatorname{sen} ux & |x| < d \\ A_{ce} e^{-wx} & x > d \\ -A_{ce} e^{wx} & x < -d \end{cases} \Rightarrow h_z = \begin{cases} A_{ne} \operatorname{sen} ux & |x| < d \\ A_{ne} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ -A_{ne} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

$$A_{ce} = A_{ne} \operatorname{sen}(ud) e^{wd}$$

$$e_y = \begin{cases} \frac{j\omega\mu}{u} A_{ne} \cos ux & |x| < d \\ \frac{j\omega\mu}{u} A_{ne} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\omega\mu}{u} A_{ne} \cos ud e^{w(x+d)} & x < -d \end{cases} \quad h_x = \begin{cases} \frac{j\beta}{u} A_{ne} \cos ux & |x| < d \\ \frac{j\beta}{u} A_{ne} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\beta}{u} A_{ne} \cos ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Capa de guiado: $-d < x < d$

$$E_y(x) = \begin{cases} E_e \cdot \cos(u_m \cdot x); m = 0, 2, \dots \\ E_o \cdot \text{sen}(u_m \cdot x); m = 1, 3, \dots \end{cases} \Big|_{|x| \leq d}$$

Capas de confinamiento: $x < -d$; $x > d$

$$E_y(x) = E_s \cdot e^{-|w_m \cdot x|}; m = 0, 1, 2, 3, \dots; |x| \geq d$$

Número de Modos propagados:

Si $V < \pi/2 \rightarrow$ sólo un punto de corte \rightarrow sólo se propaga el modo TE_0

Si $\pi/2 < V < \pi \rightarrow$ se propagan los modos: TE_0 (par), y TE_1 (impar)

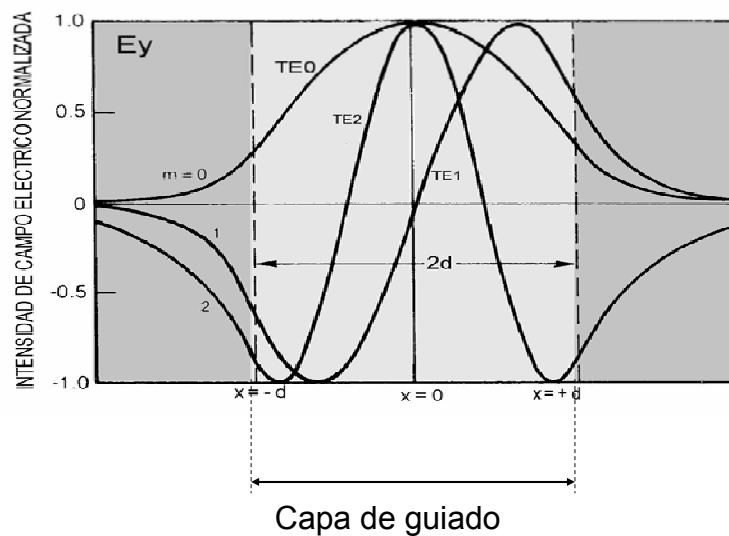
Si $\pi < V < 3\pi/2 \rightarrow$ se propagan los modos: TE_0 , TE_1 y TE_2

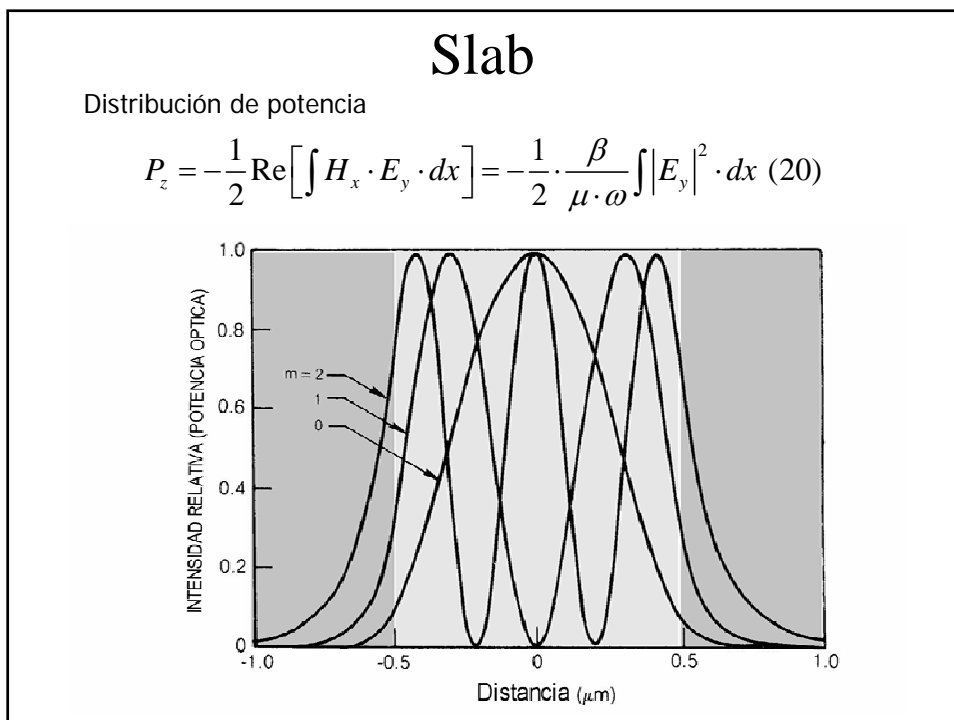
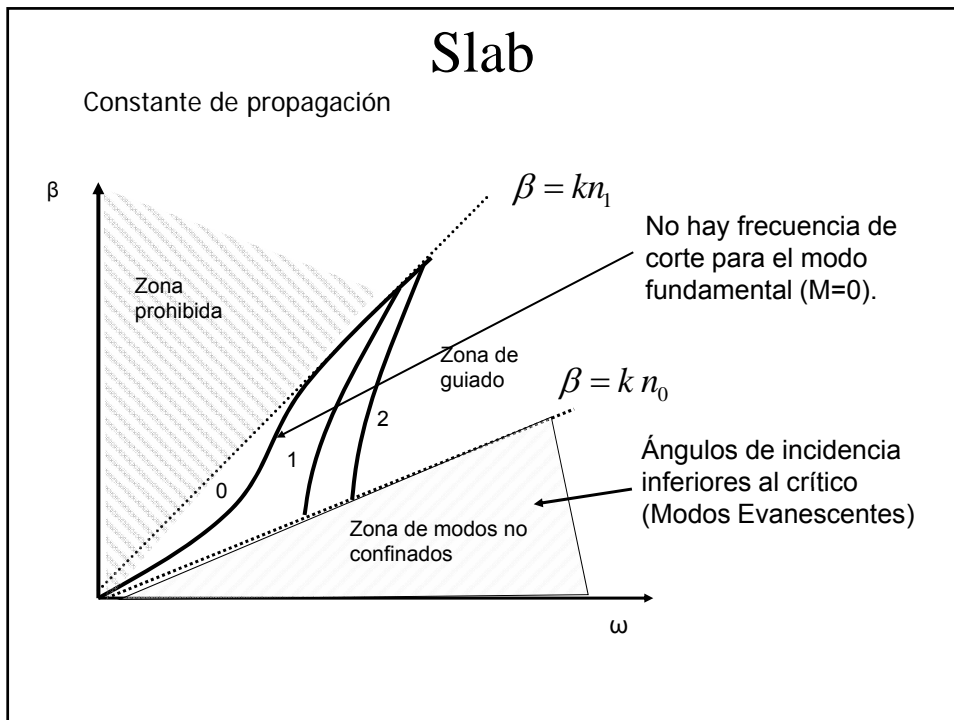
En general: Número de modos TE que soporta el slab es :

$$M = E \uparrow \left[\frac{2 \cdot V}{\pi} \right]$$

Slab

Distribución de campo eléctrico





Slab

Factor de confinamiento

$$\Gamma \triangleq \frac{I_1}{I_1 + I_0} = \frac{I_{in}}{I_{in} + I_{out}} = \frac{\int_{-d}^d |E_y(x)|^2 dx}{\int_{-\infty}^{\infty} |E_y(x)|^2 dx}$$

Para los modos TE

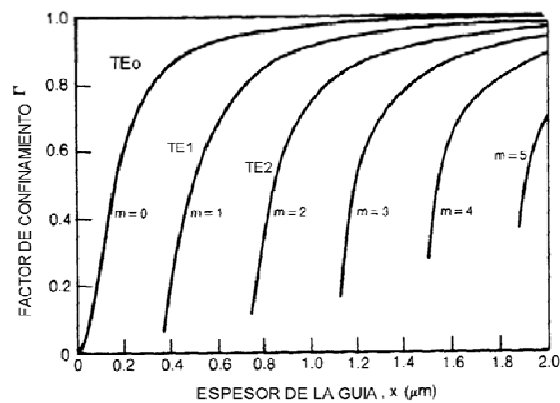
$$\Gamma = \frac{1}{1 + \frac{w_m \cdot \left[d + \left(\frac{1}{u_m} \right) \cdot \text{sen}(u_m \cdot d) \cdot \cos(u_m \cdot d) \right]}{\cos^2(u_m \cdot d)}}$$

Para los modos TE₀

$$\Gamma \approx \frac{2 \cdot V^2}{1 + 2 \cdot V^2}$$

Slab

Factor de confinamiento



Datos de la guía: $n_1 = 3,590$; $n_2 = 3,385$; $\lambda = 0,9 \mu m$

Slab. Ecuación de onda

Modos TM

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_{ci}^2\right) e_z^i = 0 \Rightarrow \begin{cases} \left(\frac{\partial^2}{\partial x^2} + k_{ci}^2\right) e_z^i = 0 \\ \frac{\partial}{\partial y} = 0 \end{cases} \Rightarrow \begin{cases} k_{ci}^2 = k^2 n_i^2 - \beta^2 \\ u^2 = k_{c1}^2 = k^2 n_1^2 - \beta^2 \\ w^2 = -k_{c0}^2 = \beta^2 - k^2 n_0^2 \end{cases}$$

Solución general

Núcleo		$e_z(x) = \begin{cases} A_{no} \cos ux & \text{Modo Impar} \\ A_{ne} \sen ux & \text{Modo Par} \end{cases}$	
Cubierta		$e_z(x) = \begin{cases} A_{co} e^{-wx} & x > d \\ A_{co} e^{wx} & x < -d \end{cases} \Rightarrow \text{Modo Impar}$	
		$\begin{cases} A_{ce} e^{-wx} & x > d \\ -A_{ce} e^{wx} & x < -d \end{cases} \Rightarrow \text{Modo Par}$	

Slab

Para que las soluciones representen ondas físicas reales:

$$\left. \begin{aligned} u^2 > 0 &\Rightarrow k n_1 > \beta \\ w^2 > 0 &\Rightarrow \beta > k n_0 \end{aligned} \right\} \Rightarrow k n_0 < \beta < k n_1$$

Modo TM impar. Condiciones de contorno

$$e_z(d^-) = e_z(d^+); \quad e_z(-d^-) = e_z(-d^+)$$

$$A_{no} \cos(ud) = A_{co} e^{-wd}$$

$$h_y(d^-) = h_y(d^+); \quad h_y(-d^-) = h_y(-d^+)$$

$$\frac{j\omega\epsilon_1}{k_{c1}^2} \frac{\partial e_z}{\partial x} \Big|_{x=d^-} = \frac{j\omega\epsilon_0}{k_{c0}^2} \frac{\partial e_z}{\partial x} \Big|_{x=d^+}; \quad \frac{j\omega\epsilon_1}{k_{c1}^2} \frac{\partial e_z}{\partial x} \Big|_{x=-d^-} = \frac{j\omega\epsilon_0}{k_{c0}^2} \frac{\partial e_z}{\partial x} \Big|_{x=-d^+}$$

$$\frac{-u\epsilon_1 A_{no} \sen(ud)}{u^2} = \frac{-w\epsilon_0 A_{co} e^{-wd}}{-w^2} \Rightarrow w\epsilon_1 A_{no} \sen(ud) = -u\epsilon_0 A_{co} e^{-wd}$$

Ecuación de dispersión

$$w\epsilon_1 \tan(ud) = -u\epsilon_0$$

Slab

Modo TM par. Condiciones de contorno

$$e_z(d^-) = e_z(d^+); \quad e_z(-d^-) = e_z(-d^+)$$

$$A_{ne} \operatorname{sen}(u d) = A_{ce} e^{-w d}$$

$$h_y(d^-) = h_y(d^+); \quad h_y(-d^-) = h_y(-d^+)$$

$$\frac{j\omega\varepsilon_1}{k_{c1}^2} \frac{\partial e_z}{\partial x} \Big|_{x=d^-} = \frac{j\omega\varepsilon_0}{k_{c0}^2} \frac{\partial e_z}{\partial x} \Big|_{x=d^+}; \quad \frac{j\omega\varepsilon_1}{k_{c1}^2} \frac{\partial e_z}{\partial x} \Big|_{x=-d^-} = \frac{j\omega\varepsilon_0}{k_{c0}^2} \frac{\partial e_z}{\partial x} \Big|_{x=-d^+}$$

$$\frac{u\varepsilon_1 A_{ne} \cos(u d)}{u^2} = \frac{-w\varepsilon_0 A_{ce} e^{-w d}}{-w^2} \Rightarrow w\varepsilon_1 A_{ne} \cos(u d) = u\varepsilon_0 A_{ce} e^{-w d}$$

Ecuación de dispersión

$$u\varepsilon_0 \tan(u d) = w\varepsilon_1$$

Slab

Campos transversales. Modos TM

$$e_x = \frac{-j\beta}{k_c^2} \frac{\partial e_z}{\partial x} \quad h_x = \frac{j\omega\varepsilon}{k_c^2} \frac{\partial e_z}{\partial y}$$

$$e_y = \frac{-j\beta}{k_c^2} \frac{\partial e_z}{\partial y} \quad h_y = \frac{-j\omega\varepsilon}{k_c^2} \frac{\partial e_z}{\partial x}$$

Slab $e_y = 0 \quad h_x = 0$

Modo Impar

$$e_z = \begin{cases} A_{no} \cos ux & |x| < d \\ A_{co} e^{-wx} & x > d \\ A_{co} e^{wx} & x < -d \end{cases} \Rightarrow e_z = \begin{cases} A_{no} \cos ux & |x| < d \\ A_{no} \cos ud e^{-w(x-d)} & x > d \\ A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases}$$

$$A_{co} = A_{no} \cos(ud) e^{wd}$$

Slab

Campos transversales. Modo TM impar

$$h_y = \begin{cases} \frac{j\omega\varepsilon_1}{u} A_{no} \operatorname{sen} ux & |x| < d \\ -\frac{j\omega\varepsilon_0}{w} A_{no} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\omega\varepsilon_0}{w} A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases} \quad -\frac{u \cos(ud)}{\varepsilon_1 \operatorname{sen}(ud)} = \frac{w}{\varepsilon_0}$$

$$h_y = \begin{cases} \frac{j\omega\varepsilon_1}{u} A_{no} \operatorname{sen} ux & |x| < d \\ \frac{j\omega\varepsilon_1}{u} A_{no} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ -\frac{j\omega\varepsilon_1}{u} A_{no} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Campos transversales. Modo TM impar

$$e_x = \begin{cases} \frac{j\beta}{u} A_{no} \operatorname{sen} ux & |x| < d \\ -\frac{j\beta}{w} A_{no} \cos ud e^{-w(x-d)} & x > d \\ \frac{j\beta}{w} A_{no} \cos ud e^{w(x+d)} & x < -d \end{cases} \quad -\frac{u \cos(ud)}{\varepsilon_1 \operatorname{sen}(ud)} = \frac{w}{\varepsilon_0}$$

$$e_x = \begin{cases} \frac{j\beta}{u} A_{no} \operatorname{sen} ux & |x| < d \\ \frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} A_{no} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ -\frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} A_{no} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Modo Par

$$e_z = \begin{cases} A_{ne} \operatorname{sen} ux & |x| < d \\ A_{ce} e^{-wx} & x > d \\ -A_{ce} e^{wx} & x < -d \end{cases} \Rightarrow e_z = \begin{cases} A_{ne} \operatorname{sen} ux & |x| < d \\ A_{ne} \operatorname{sen} ud e^{-w(x-d)} & x > d \\ -A_{ne} \operatorname{sen} ud e^{w(x+d)} & x < -d \end{cases}$$

$$A_{ce} = A_{ne} \operatorname{sen}(ud) e^{wd}$$

$$h_y = \begin{cases} -\frac{j\omega\varepsilon_1}{u} A_{ne} \cos ux & |x| < d \\ -\frac{j\omega\varepsilon_1}{u} A_{ne} \cos ud e^{-w(x-d)} & x > d \\ -\frac{j\omega\varepsilon_1}{u} A_{ne} \cos ud e^{w(x+d)} & x < -d \end{cases} e_x = \begin{cases} -\frac{j\beta}{u} A_{ne} \cos ux & |x| < d \\ -\frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} A_{ne} \cos ud e^{-w(x-d)} & x > d \\ -\frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} A_{ne} \cos ud e^{w(x+d)} & x < -d \end{cases}$$

Slab

Modo TM Par. Energía

$$\vec{E} = \begin{cases} A_{ne} \left(\operatorname{sen} ux \hat{z} - \frac{j\beta}{u} \cos ux \hat{x} \right) e^{-j\beta z} & |x| < d \\ A_{ne} e^{-w(x-d)} \left(\operatorname{sen} ud \hat{z} - \frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} \cos ud \hat{x} \right) e^{-j\beta z} & x > d \\ -A_{ne} e^{w(x+d)} \left(\operatorname{sen} ud \hat{z} - \frac{j\beta}{u} \frac{\varepsilon_1}{\varepsilon_0} \cos ud \hat{x} \right) e^{-j\beta z} & x < -d \end{cases}$$

$$\vec{H}^* = \begin{cases} \frac{j\omega\varepsilon_1}{u} A_{ne}^* \cos ux e^{j\beta z} \hat{y} & |x| < d \\ \frac{j\omega\varepsilon_1}{u} A_{ne}^* \cos ud e^{-w(x-d)} e^{j\beta z} \hat{y} & x > d \\ \frac{j\omega\varepsilon_1}{u} A_{ne}^* \cos ud e^{w(x+d)} e^{j\beta z} \hat{y} & x < -d \end{cases}$$

Slab

Modo TM Par. Energía

$$\frac{1}{2} \operatorname{Re}[\vec{E} \times \vec{H}^*] = \begin{cases} \hat{z} \frac{|A_{ne}|^2}{2} \frac{\beta \omega \varepsilon_1}{u^2} \cos^2 ux & |x| < d \\ \hat{z} \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud e^{-2w(x-d)} & x > d \\ \hat{z} \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud e^{2w(x+d)} & x < -d \end{cases}$$

$$P_T = \int_{-\infty}^{\infty} \frac{1}{2} \operatorname{Re}[\vec{E} \times \vec{H}^*] \cdot \hat{z} dx = I_1 + I_2 + I_3 \Rightarrow \begin{cases} I_1 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega \varepsilon_1}{u^2} \int_{-d}^d \cos^2 ux dx \\ I_2 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud \int_d^{\infty} e^{-2w(x-d)} dx \\ I_3 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud \int_{-\infty}^{-d} e^{2w(x+d)} dx \end{cases}$$

Slab

Modo TM Par. Energía

$$I_1 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega \varepsilon_1}{u^2} \left(d + \frac{\operatorname{sen} 2ud}{2u} \right)$$

$$I_2 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud \frac{1}{2w}$$

$$I_3 = I_2 = \frac{|A_{ne}|^2}{2} \frac{\beta \omega}{u^2} \frac{\varepsilon_1^2}{\varepsilon_0} \cos^2 ud \frac{1}{2w}$$

$$P_T = \frac{|A_{ne}|^2}{2} \frac{\beta \omega \varepsilon_1}{u^2} \left(d + \frac{\operatorname{sen} 2ud}{2u} + \frac{\varepsilon_1}{w \varepsilon_0} \cos^2 ud \right) \operatorname{Wm}^{-1}$$

Slab

Modo TM Par. Energía

$$I_1 = \frac{|A_{ne}|^2}{2} \frac{\beta\omega\epsilon_1}{u^2} \left(d + \frac{\text{sen } 2ud}{2u} \right)$$

$$I_2 = \frac{|A_{ne}|^2}{2} \frac{\beta\omega}{u^2} \frac{\epsilon_1^2}{\epsilon_0} \cos^2 ud \frac{1}{2w}$$

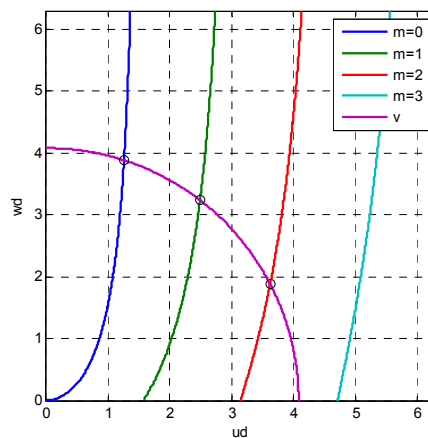
$$I_3 = I_2 = \frac{|A_{ne}|^2}{2} \frac{\beta\omega}{u^2} \frac{\epsilon_1^2}{\epsilon_0} \cos^2 ud \frac{1}{2w}$$

$$P_T = \frac{|A_{ne}|^2}{2} \frac{\beta\omega\epsilon_1}{u^2} \left(d + \frac{\text{sen } 2ud}{2u} + \frac{\epsilon_1}{w\epsilon_0} \cos^2 ud \right) \text{Wm}^{-1}$$

Slab

Solución gráfica. Modos TE $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{GHz}$; $d = 25 \text{mm}$

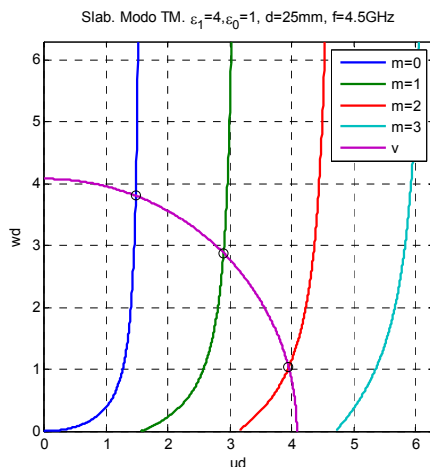
Slab. Modo TE. $\epsilon_1=4, \epsilon_0=1, d=25\text{mm}, f=4.5\text{GHz}$



Solución numérica $\phi_1 = 15.4776^\circ$; $\phi_2 = 31.8463^\circ$; $\phi_3 = 50.2103^\circ$

Slab

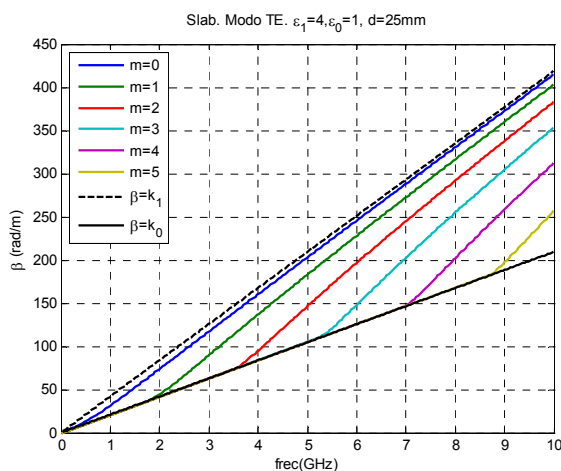
Solución grafica. Modos $TM_{n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Solución numérica $\phi_1 = 18.2308^\circ; \phi_2 = 37.9051^\circ; \phi_3 = 56.9228^\circ$

Slab

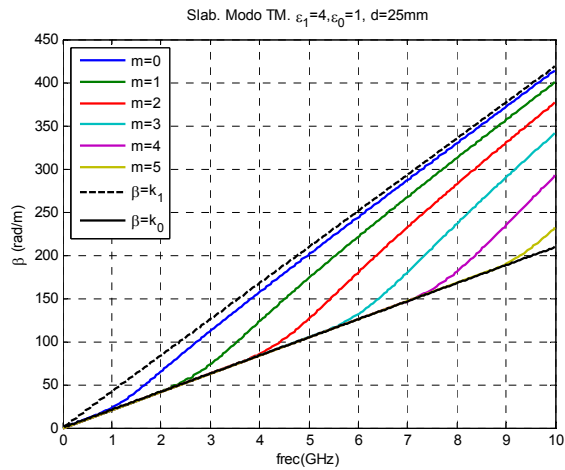
Modos TE $n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Constante de propagación

Slab

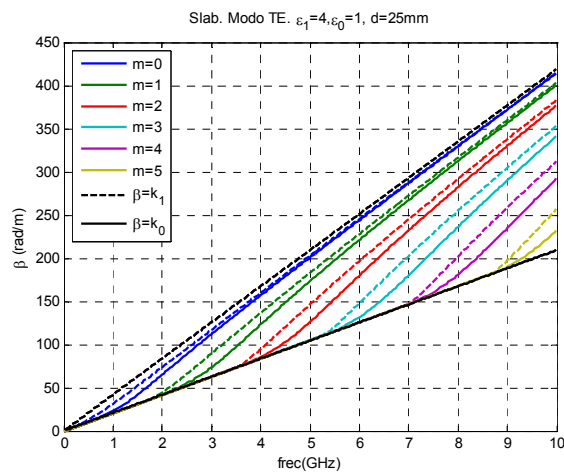
Modos TE $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



Constante de propagación

Slab

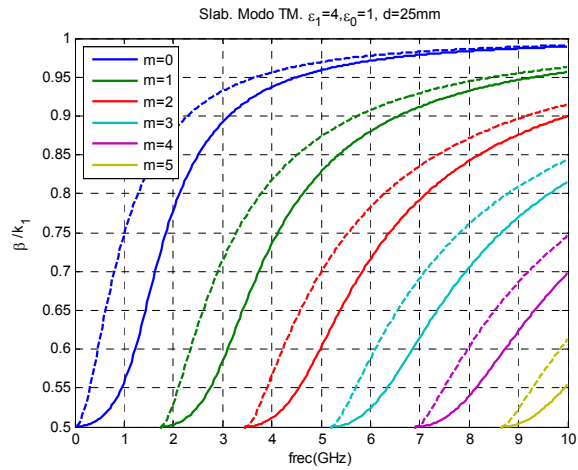
Modos TE y TM $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



Constante de propagación

Slab

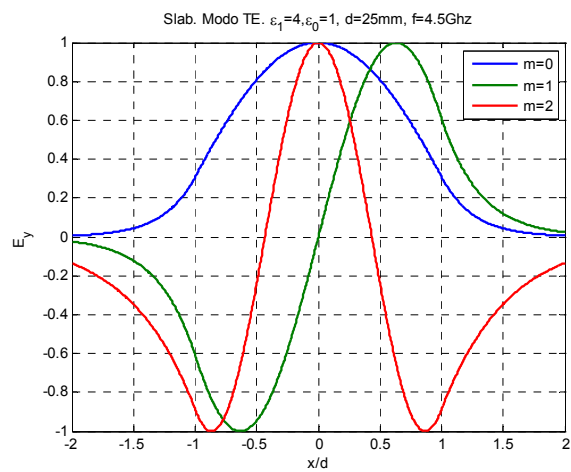
Modos TE y TM $n_1 = 2$; $n_0 = 1$; $f = 4.5$ GHz; $d = 25$ mm



Constante de propagación

Slab

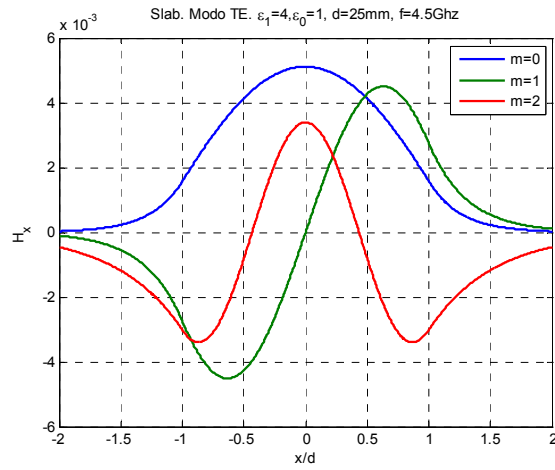
Modos TE $n_1 = 2$; $n_0 = 1$; $f = 4.5$ GHz; $d = 25$ mm



Campo E_y

Slab

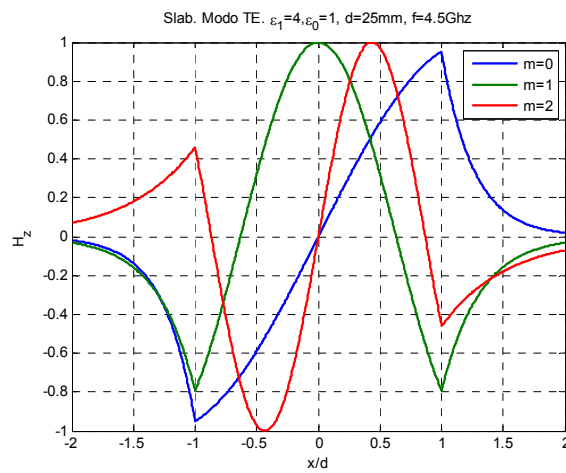
Modos TE $n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Campo H_x

Slab

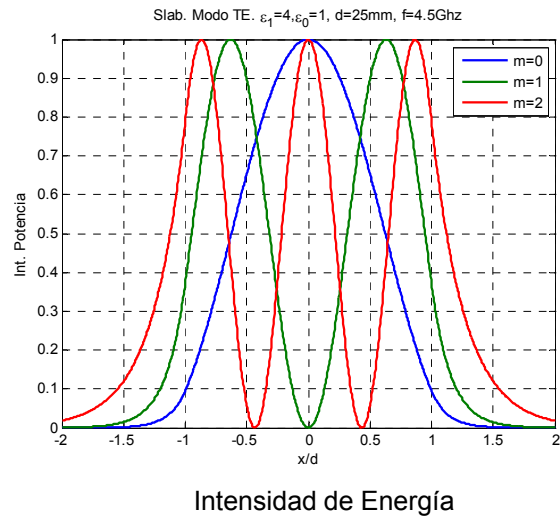
Modos TE $n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Campo H_z

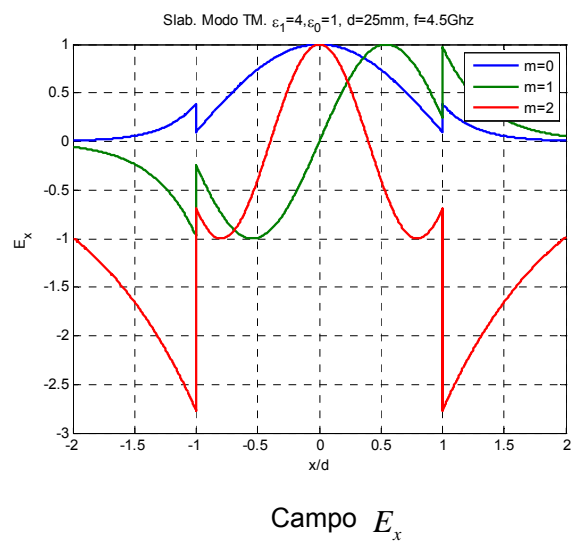
Slab

Modos TE $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



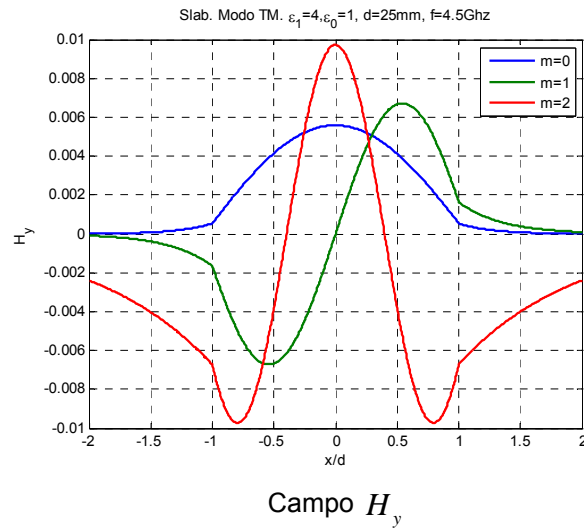
Slab

Modos TM $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



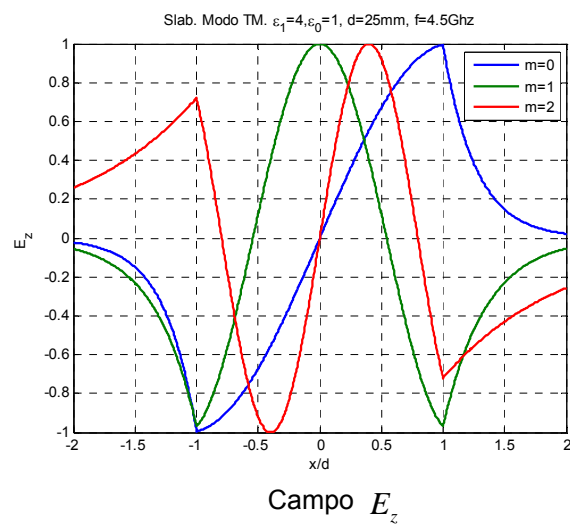
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Modos TM $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



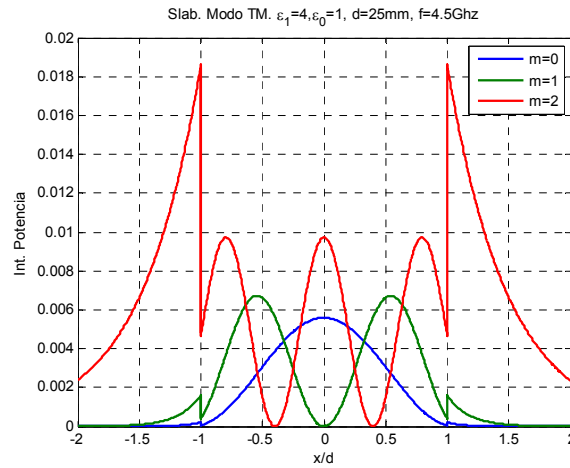
Slab

Modos TM $n_1 = 2$; $n_0 = 1$; $f = 4.5 \text{ GHz}$; $d = 25 \text{ mm}$



Slab

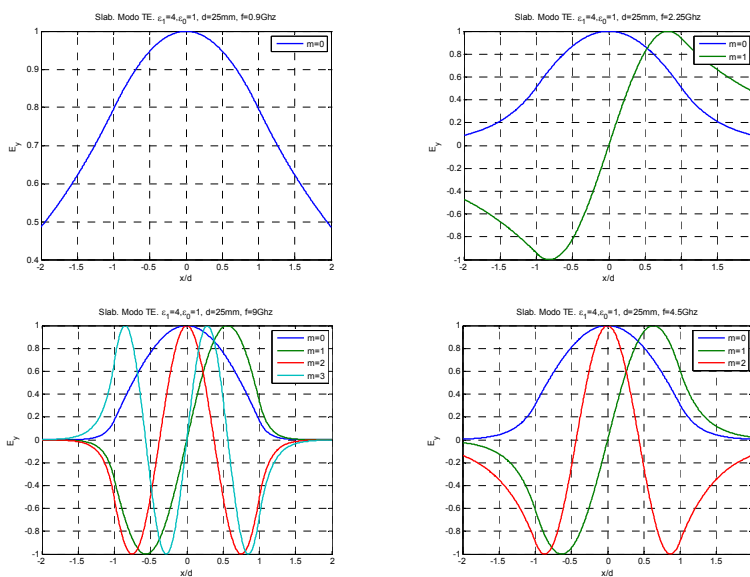
Modos TM $n_1 = 2; n_0 = 1; f = 4.5 \text{ GHz}; d = 25 \text{ mm}$



Intensidad de Energía

Slab

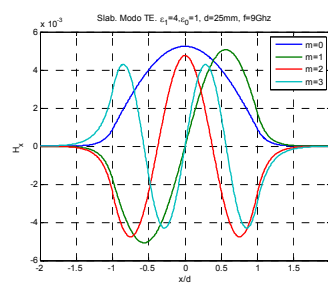
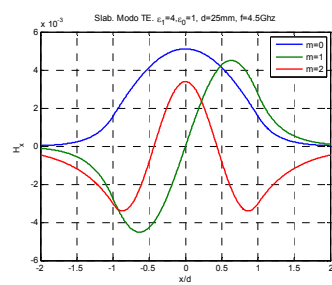
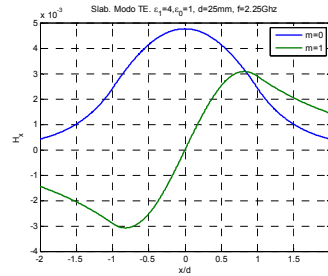
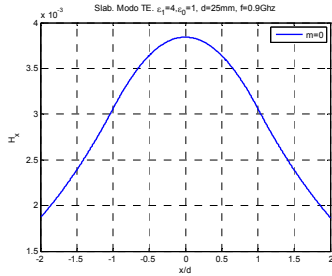
Modos TE. Campo E_y $n_1 = 2; n_0 = 1; d = 25 \text{ mm}$



Slab

Modos TE. Campo H_x

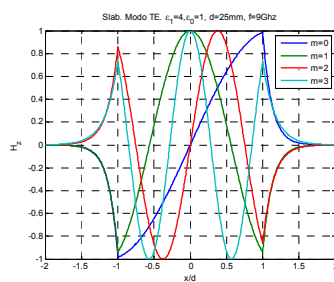
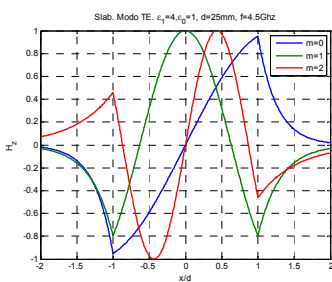
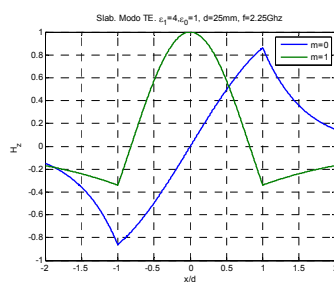
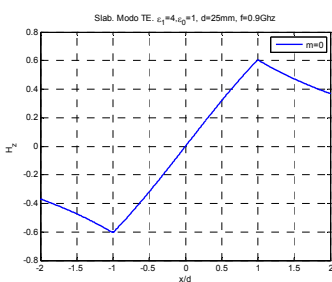
$n_1 = 2; n_0 = 1; d = 25 \text{ mm}$



Slab

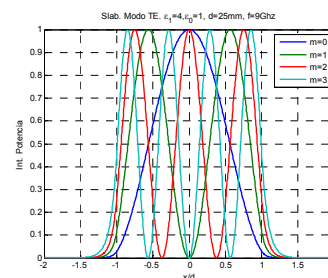
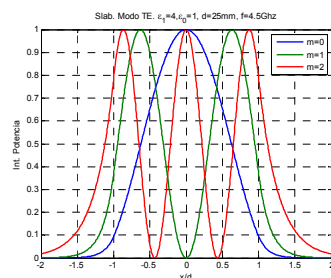
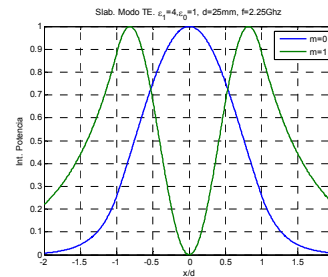
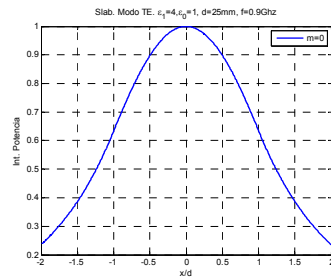
Modos TE. Campo H_z

$n_1 = 2; n_0 = 1; d = 25 \text{ mm}$



Slab

Modos TE. Intensidad de Potencia $n_1 = 2$; $n_0 = 1$; $d = 25\text{ mm}$



Slab

Conclusiones

Existen campos tanto en la capa de guiado como en las de confinamiento (aparente contradicción con la Teoría de Rayos).

El espesor de las capas de confinamiento debe ser suficientemente grande como para que los campos se extingan prácticamente en ellas.

La energía transportada ("intensidad de luz") es proporcional al cuadrado del E