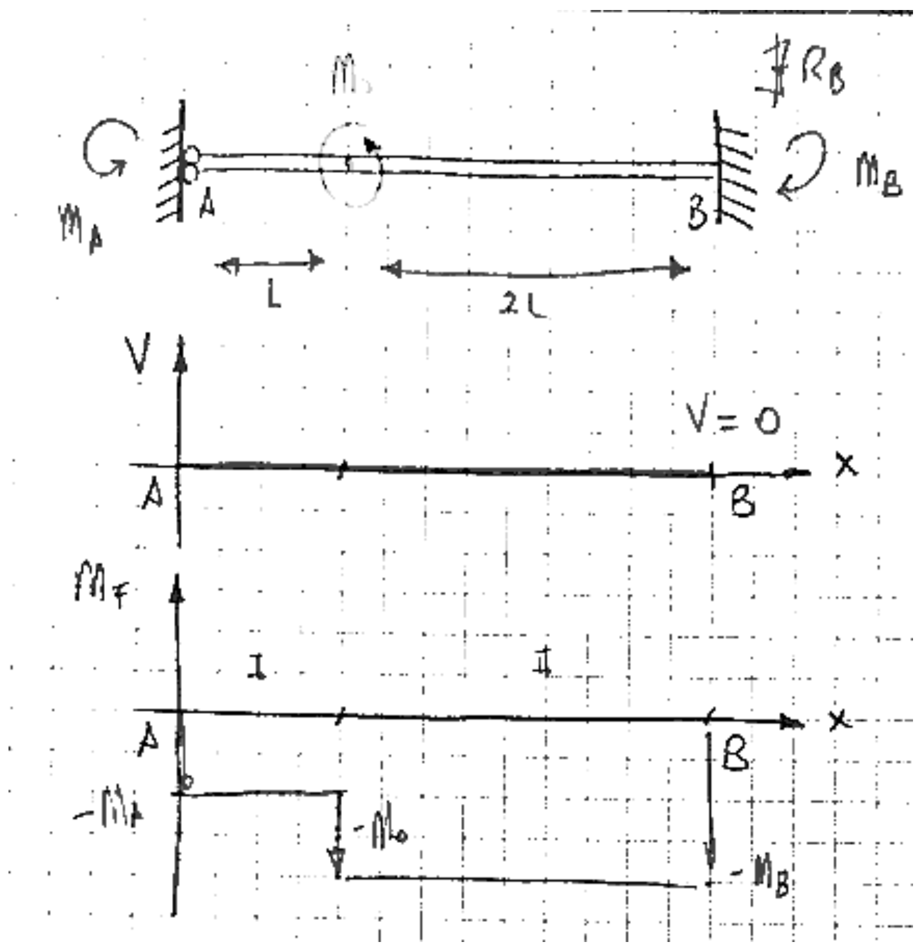


ⓐ $r = 1$

ⓑ $\nexists\, R_A, R_B \leftrightarrow \nexists\, f_y^{ext} \rightarrow V = 0 \quad \forall x$

$$\sum M_A^{ext} = 0 \rightarrow +M_A + M_0 - M_B = 0 \rightarrow M_B = M_A + M_0$$

$M_I(0) = -M_A$ $\quad$ M_A $\quad$ $M_F = -M_A$

$$M_I(x) = cte = -M_A$$

$x = L$ $\quad \left.\begin{array}{l} \Delta M = -M_0 \\ \quad \shortparallel \\ M_f - \underbrace{M_i}_{-M_A} \end{array}\right\} M_f = M_{II}(L) = -M_0 - M_A = -M_B$

$$M_{II}(x) = cte = -M_B = -M_0 - M_A$$

→ Método I: Plástico

(T1) $EI_z V_I'' = -M_A \xrightarrow{\int} EI_z V_I' = -M_A X + C_1 \xrightarrow{\int} EI_z V_I = -M_A \frac{X^2}{2} + C_1 X + D_1$

(T2) $EI_z V_{II}'' = -M_0 - M_A \rightarrow EI_z V_{II}' = -(M_0 + M_A)X + C_2 \rightarrow EI_z V_{II}$

$= -(M_0 + M_A)\frac{X^2}{2} + C_2 X + D_2$

5 CC

✓ $\theta_A = 0 = V_I'(0) \rightarrow \boxed{C_1 = 0}$

✓ $\theta_B = 0 = V_{II}'(3L) \rightarrow$ $\boxed{C_2 = (M_0 + M_A)3L}$

$V_I(L) = V_{II}(L)$

$V_I'(L) = V_{II}'(L)$

✓ $V_{II}(3L) = 0 \rightarrow$

$\rightarrow -M_A \frac{L^2}{2} + D_1 = -(M_0 + M_A)\frac{L^2}{2} + (M_0 + M_A)3L^2 + -(M_0 + M_A)\frac{9L^2}{2}$

$D_1 = (M_0 + M_A)L^2 \underbrace{\left[-\frac{1}{2} + 3 - \frac{9}{2}\right]}_{\frac{-1+6-9}{2} = -2} + M_A \frac{L^2}{2} = -M_0 2L^2 + M_A L^2 \underbrace{\left[\frac{1}{2} - 2\right]}_{-\frac{3}{2}}$

• $V_{II}(3L) = 0 \rightarrow -(M_0 + M_A)\frac{9L^2}{2} + (M_0 + M_A)(3L)^2 + D_2$

$\boxed{D_2 = -(M_0 + M_A)\frac{9L^2}{2}}$

$\boxed{D_1 = -2M_0 L^2 - \frac{3M_A L^2}{2}}$

• $V_I'(L) = V_{II}'(L) \rightarrow -M_A \cancel{L} = -(M_0 + M_A)\cancel{L} + (M_0 + M_A)3\cancel{L}$

$-\cancel{M_A} + \cancel{M_A} - 3M_A = -M_0 + 3M_0 = 2M_0$

$\boxed{M_A = -\frac{2M_0}{3}}$

→ Método II Menabrea

$\boxed{\frac{\partial U}{\partial M_A}=0}$ como $U=U_1+U_2 \to \frac{\partial U_1}{\partial M_A}+\frac{\partial U_2}{\partial M_A}=0$

$$\frac{\partial U_1}{\partial M_A} = \int_0^L \frac{M_I(x)}{EI_z}\,\frac{\partial M_I}{\partial M_A}\,dx = \int_0^L \frac{+M_A}{EI_z}\,dx = \frac{M_A}{EI_z}L$$

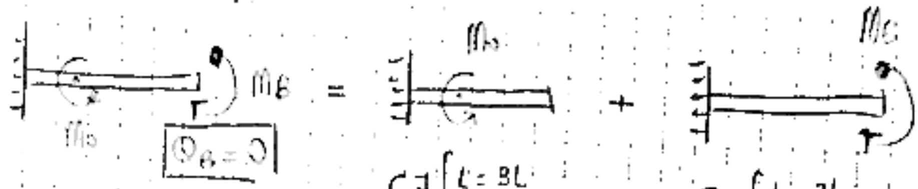
$$\frac{\partial U_2}{\partial M_A} = \int_L^{3L} \frac{M_{II}(x)}{EI_z}\,\overset{-1}{\underset{-1}{\frac{\partial M_{II}}{\partial M_A}}}\,dx = \frac{(M_A+M_0)}{EI_z}(\underbrace{3L-L}_{2L})$$

$$\frac{M_A L}{\cancel{EI_z}} + \frac{(M_A+M_0)\,2L}{\cancel{EI_z}} = 0 \to 3M_A + 2M_0 = 0$$

$\frac{3-2}{3} = \frac{M_0}{3}$ $\boxed{M_A = -\frac{2}{3}M_0}$

E.E. → $M_B = M_A + M_0 = \left(1-\frac{2}{3}\right)M_0 \to \boxed{M_B = \frac{M_0}{3}}$

→ Método III: Superposición.

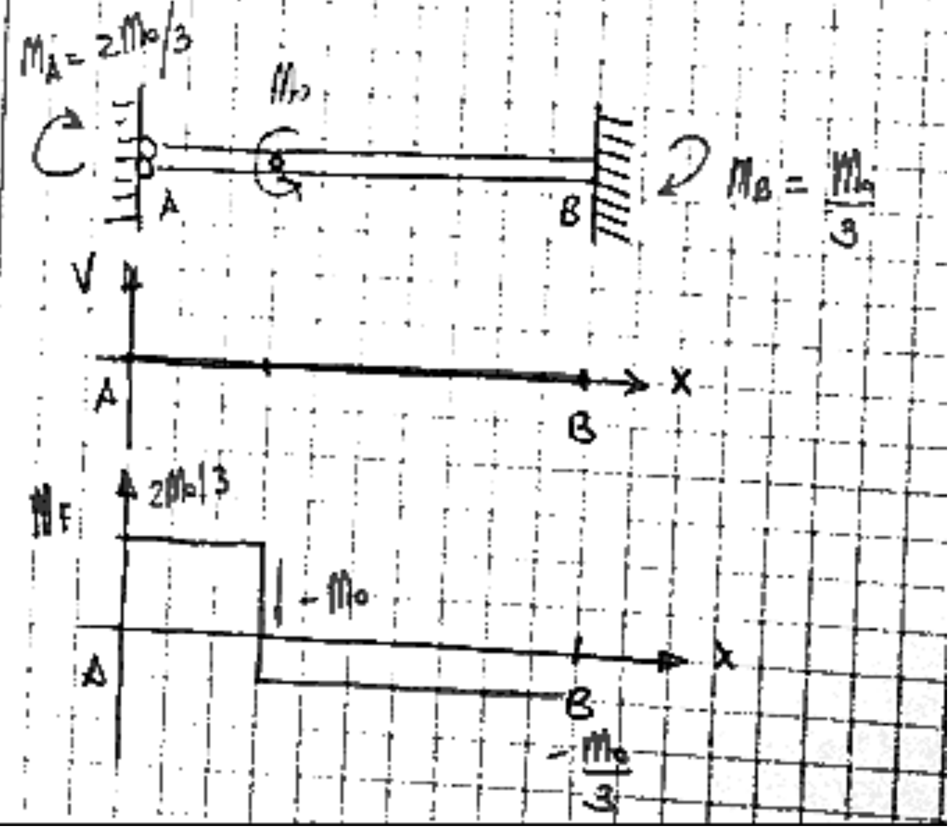


C7 $\begin{cases} L = 3L \\ a = L \\ M_0 = -M_0 \end{cases}$ C6 $\begin{cases} L = 3L \\ M_0 = +M_B \end{cases}$

$\theta_B^{M_0} = \frac{M_0 L}{EI_z}$ $\theta_B^{M_B} = -\frac{M_B\,3L}{EI_z}$

$$\theta_B = 0 = \frac{M_0 L}{EI_z} - \frac{M_B\,3L}{EI_z} \to \boxed{M_B = \frac{M_0}{3}}$$ ∎

- Diagramas



$$V_I = -\overset{-\frac{2}{3}M_0}{M_A}\frac{x^2}{2} - 2M_0 L^2 - \frac{3\overset{-\frac{2}{3}M_0}{M_A}}{2}L^2 = \frac{2}{3}M_0\frac{x^2}{2}\overbrace{-2M_0 L^2 + M_0 L^2}^{-M_0 L^2}$$

$$V_I' = \frac{2}{3}M_0 X$$

$$\delta_A = -M_0 L^2 = V_{mx}$$

$$\theta_{mx} = V_I'(L) = \frac{2}{3}M_0 L$$