# Aerodynamics: Boundary Layer Theory – Multiple Choice Test

## 2018/19 Q2

1. Under which circumstances may the volume rate-of-change entail viscous shear stresses?
   (a) Never for incompressible flow.
   (b) Always under the Stokes hypothesis.
   (c) Always.
   (d) Never.
   (e) None of the other options.
2. What is true about the viscous blockage?
   (a) All options are correct.
   (b) It corresponds to a massflow reduction in the near-wall region due to the effects of viscosity.
   (c) It corresponds to a momentum flux reduction in the near-wall region due to the effects of viscosity.
   (d) It follows from the no-slip condition at the wall due to viscous effects.
   (e) It may be quantified by the displacement and momentum thicknesses of the boundary layer.
3. Which of the following factors may be held responsible for both advancing turbulent transition and inducing early separation of a boundary layer?
   (a) An adverse pressure gradient.
   (b) Wall roughness.
   (c) High preturbulence levels in the outer flow.
   (d) High Reynolds number.
   (e) None of the other options.
4. Laminar boundary layers, as opposed to turbulent,...
   (a) have lower friction but separate earlier.
   (b) have lower friction and separate later.
   (c) have higher friction and separate earlier.
   (d) have higher friction but separate later.
   (e) None of the other options.
5. Which of the following properties of the Navier-Stokes equations do the Euler equations retain?
   (a) None of the other options.
   (b) The no-slip condition can be applied on all solid walls.
   (c) They allow computation of friction drag.
   (d) They allow computation of form drag of closed bodies.
   (e) They can anticipate the occurrence of turbulence and wakes.
6. Which of the following terms of the streamwise momentum equation drops under the boundary layer hypothesis?
   (a) Streamwise diffusion.
   (b) Wall-normal diffusion.
   (c) Streamwise advection.
   (d) Wall-normal advection.
   (e) Streamwise pressure gradient.
7. The boundary layer (BL) integral equations...
   (a) result from integrating the BL local equations in the wall-normal coordinate over the BL thickness.
   (b) result from integrating the BL local equations along the streamwise coordinate.
   (c) result in as many unknowns as there are equations.
   (d) differ for laminar and turbulent BLs.
   (e) None of the other options.
8. How is the D'Alembert's paradox removed and the form drag obtained in an inviscid flow – boundary layer coupling calculation?
   (a) By solving the inviscid problem for a second time over a body enlarged by the displacement thickness and extended with the wake.
   (b) By directly solving the boundary layer equations.
   (c) The paradox remains no matter what.
   (d) By solving the inviscid problem over the original body.
   (e) The paradox was never there in the first place.
9. Which of the following inviscid potential flows admit self-similar solutions for the two-dimensional incompressible laminar boundary layer equations?
   (a) Wedge/corner flows.
   (b) Source and sink flows.
   (c) Irrotational vortex flow.
   (d) Doublet flow.
   (e) None of the other options.
10. For the laminar boundary layer developing in the vicinity of a stagnation point...
    (a) momentum thickness is locally finite and constant.
    (b) displacement thickness grows linearly.
    (c) wall shear stress is locally finite and constant.
    (d) the form factor is larger than for the Blasius solution.
    (e) None of the other options.
11. Which of the following statements does NOT describe turbulent flows?
    (a) Low energy dissipation.
    (b) Intrinsic three-dimensionality.
    (c) Intrinsically time-dependence.
    (d) Deterministic chaos.
    (e) Enhanced mixing capabilities.
12. Which of the following methods for solving turbulent flows simulates the large turbulent structures but filters out (and then models) the smaller scales?
    (a) Large Eddy Simulation (LES)
    (b) Reynolds Averaged Navier-Sokes (RANS).
    (c) Direct Navier-Stokes (DNS).
    (d) Direct Numerical Simulation (DNS).
    (e) None of the others.

13. What is the actual origin of the Reynolds stresses ($-\rho\overline{u_i'u_j'}$) in the RANS equations?
    (a) Advective momentum transport due to turbulent fluctuations.
    (b) Diffusive momentum transport due to enhanced fluid viscosity.
    (c) Turbulent pressure fluctuations.
    (d) Mass conservation in the presence of turbulent fluctuations.
    (e) None of the other options.
14. How do viscous $\mu\frac{\partial u}{\partial y}$ and Reynolds $-\rho\overline{u'v'}$ shear stresses compare within an incompressible, statistically two-dimensional, turbulent boundary layer (TBL)?
    (a) $\mu\frac{\partial u}{\partial y}$ prevails in the immediate vicinity of the wall.
    (b) $-\rho\overline{u'v'}$ prevails over the full TBL thickness.
    (c) $\mu\frac{\partial u}{\partial y}$ prevails over the full TBL thickness.
    (d) $-\rho\overline{u'v'}$ prevails in the immediate vicinity of the wall.
    (e) They compete in size over the full TBL thickness.
15. What is the sign of the streamwise-wall-normal velocity fluctuation correlations ($\overline{u'v'}$) within a turbulent boundary layer?
    (a) $\overline{u'v'} < 0$.
    (b) $\overline{u'v'} > 0$.
    (c) $\overline{u'v'} = 0$.
    (d) $\overline{u'v'} < 0$ close to the wall and $\overline{u'v'} > 0$ far from it.
    (e) $\overline{u'v'} > 0$ close to the wall and $\overline{u'v'} < 0$ far from it.
16. How can lift be estimated for an airfoil immersed in inviscid incompressible flow?
    (a) By introducing a circulation to enforce the Kutta condition.
    (b) In no way because potential flow is irrotational.
    (c) Enforcing wall impermeability, lift naturally arises.
    (d) By enforcing the no-slip condition on the walls to the potential flow equations.
    (e) None of the other options.
17. Which statement is true about the lifting problem within the frame of linear potential theory?
    (a) The perturbation potential is anti-symmetric in the vertical coordinate.
    (b) The vertical perturbation velocity is anti-symmetric in the vertical coordinate.
    (c) The horizontal perturbation velocity is symmetric in the vertical coordinate.
    (d) Both the vertical and horizontal perturbation velocities are symmetric in the vertical coordinate.
    (e) None of the other options are correct.
18. A problem that aims at computing the pressure distribution given the airfoil shape is known as...
    (a) a direct problem.
    (b) an inverse problem.
    (c) a symmetric problem.
    (d) an anti-symmetric problem.
    (e) None of the other options are correct.
19. On a finite wing producing lift...
    (a) None of the other options are correct.
    (b) Wingtip vortices roll from suction to pressure side.
    (c) The lift distribution is homogeneous along the span.
    (d) There may still be no lift-induced drag.
    (e) Streamlines are deflected towards the fuselage on the pressure side and towards wingtip on the suction side.
20. Prandtl's lifting-line theory for finite wings...
    (a) All options are correct.
    (b) leads to an integro-differential equation for the circulation distribution along the span.
    (c) shows that minimum lift-induced drag is obtained for an elliptical distribution of circulation along the span.
    (d) is based on assessing the span-dependent effective angle of attack that results from wingtip vortices.
    (e) provides both wing global lift and lift-induced drag coefficients.

## 2018/19 Q1

21. Into which terms may the total rate-of-change of a small fluid volume be split, provided nonlinear effects are neglected?
    (a) Translation, rotation, volume change, shear deformation.
    (b) Translation, rotation, volume change.
    (c) Translation, rotation, shear deformation.
    (d) Rotation, volume change, shear deformation.
    (e) Volume change, shear deformation.
22. Why is the drag on a sphere smaller when the separating boundary layer is turbulent rather than laminar?
    (a) Because wake drag dominates and is smaller.
    (b) Because friction drag dominates and is smaller.
    (c) Because both wake and friction drag are smaller.
    (d) Because wake drag is zero.
    (e) The assertion is plain wrong. The opposite is true.
23. What is true about the streamwise evolution of a boundary layer (possibly involving a turbulent transition) developing on a flat plate?
    (a) Its momentum thickness grows monotonically.
    (b) Its friction coefficient decreases monotonically.
    (c) Its momentum thickness decreases monotonically.
    (d) Its friction coefficient increases monotonically.
    (e) None of the other options.
24. Which statement is correct under the boundary layer hypothesis?
    (a) None of the other options.
    (b) Streamwise diffusion outweighs streamwise advection.
    (c) Streamwise diffusion outweighs wall-normal diffusion.
    (d) Wall-normal pressure gradients are large.
    (e) Fluid particles slip at the wall.

25. Which of the following interpretations of the displacement thickness is plain FALSE?
    (a) It quantifies the friction drag coefficient to be expected.
    (b) It quantifies the deviation of streamlines outside of the boundary layer.
    (c) It quantifies the massflow blockage due to viscous effects.
    (d) It quantifies the amount of wall-normal momentum that must appear for mass preservation within the boundary layer.
    (e) All statements are TRUE.
26. Which of the following boundary layers is at a higher risk of separation, all other parameters unaltered?
    (a) One that has a larger thickness.
    (b) One that has a larger outer velocity.
    (c) One that is subject to a stronger negative (favorable) pressure gradient.
    (d) One that is turbulent.
    (e) One that is subject to a milder deceleration of the outer flow.
27. What are the usual assumptions in order to devise an integral method for both laminar and turbulent boundary layers?
    (a) $H$ and $\frac{C_f}{2}Re_\theta$ taken from flat plate values.
    (b) $H$ and $\frac{C_f}{2}Re_\theta$ taken from stagnation point values.
    (c) $H$ and $\frac{C_f}{2}Re_\theta$ taken from separation profile values.
    (d) $H$ taken from flat plate and $\frac{C_f}{2}Re_\theta$ from stagnation point values.
    (e) $H$ taken from stagnation point and $\frac{C_f}{2}Re_\theta$ from separation profile values.
28. Which term of the Navier-Stokes equations is responsible for the appearance of the Reynolds stresses in the RANS equations?
    (a) The advection term.
    (b) The pressure term.
    (c) The time-derivative term.
    (d) The viscous diffusion term.
    (e) The volume forces term.
29. Which of the following statements is true about the Reynolds stresses inside a turbulent, statistically two-dimensional, incompressible boundary layer?
    (a) $\overline{u'v'} < 0$.
    (b) $\overline{v'^2} > \overline{u'^2}$.
    (c) $\overline{w'^2} = 0$.
    (d) $\overline{u'v'} \neq 0$ at the wall.
    (e) $\overline{u'^2}, \overline{v'^2}, \overline{w'^2} \neq 0$ at the wall.
30. To which of the following regions in a turbulent boundary layer does the near wall hypothesis (constant shear $\partial\tau/\partial y \simeq 0$, linear mixing length $l \simeq \chi y^+$) apply?
    (a) All options are correct.
    (b) To the inner region.
    (c) To the viscous sublayer.
    (d) To the buffer/overlap/blending region.
    (e) To the log-law layer.

## 2017/18 Q2

31. What does viscosity depend on for Newtonian fluids such as water or air?
    (a) Temperature.
    (b) Shear deformation rate.
    (c) Both temperature and shear deformation rate.
    (d) Neither temperature nor shear deformation rate.
    (e) Reynolds number.
32. Which of the following conditions entails separation of a two-dimensional flow?
    (a) Zero wall friction.
    (b) Zero wall velocity.
    (c) Infinite wall-normal gradient of streamwise velocity.
    (d) Zero boundary layer momentum thickness.
    (e) Zero boundary layer form factor.
33. Which momentum transport mechanisms are mainly competing within a laminar boundary layer?
    (a) Streamwise advection and wall-normal diffusion.
    (b) Streamwise diffusion and wall-normal advection.
    (c) Streamwise diffusion and streamwise advection.
    (d) Wall-normal advection and wall-normal diffusion.
    (e) Wall-normal and streamwise diffusion.
34. How are the integral boundary layer equations obtained?
    (a) By integrating the local equations over the boundary layer thickness.
    (b) By integrating the local equations along the streamwise coordinate.
    (c) By searching for self-similar solutions of the local equations.
    (d) By applying the boundary layer hypothesis to the Navier-Stokes equations in their integral form.
    (e) By adding whole wheat to the fluid.
35. How does the momentum thickness evolve for a laminar boundary layer in the close vicinity of a stagnation point?
    (a) It is finite and constant.
    (b) It remains of negligible size.
    (c) It debuts with a finite value and grows linearly.
    (d) It debuts at zero and grows linearly.
    (e) It debuts at infinity and decreases hyperbolically.
36. What can be said about the friction coefficient $C_f$ when comparing growing laminar and turbulent boundary layers (BLs) developing under the same outer velocity conditions?
    (a) It is higher and decreases slowlier for turbulent BL.
    (b) It is higher and decreases slowlier for laminar BL.
    (c) It is higher but decreases faster for turbulent BL.
    (d) It is higher but decreases faster for laminar BL.
    (e) None of the others.
37. Theoretical-empirical turbulent friction laws result from the matching of the velocity profile in what regions/layers of the turbulent boundary layer universal profile?
    (a) Inner and outer regions.
    (b) Viscous sublayer and buffer region.
    (c) Buffer region and log-law layer.
    (d) Viscous sublayer and log-law layer.
    (e) Buffer region and inner region.

## 2017/18 Q1

38. Which of the following phenomena can be ultimately ascribed to the action of viscosity?
    (a) All of them.
    (b) Lift.
    (c) Friction drag.
    (d) Wake drag.
    (e) Flow separation.
39. Which of the following actions might be beneficial in terms of drag coefficient reduction of a streamlined body (e.g. flat plate at $\alpha = 0$) with a laminar boundary layer naturally developing on its surface?
    (a) Preserving laminarity of the boundary layer all the way down to the trailing edge.
    (b) Artificially triggering turbulent transition in the boundary layer before it separates.
    (c) Reducing the Reynolds number.
    (d) Favouring surface rugosity.
    (e) None of the others.
40. Which of the following is NOT a consequence of the boundary layer hypothesis?
    (a) Negligible streamwise advection.
    (b) Wall-normal pressure gradient must cancel.
    (c) The local BL equations are parabolic.
    (d) Negligible streamwise diffusion.
    (e) Streamwise velocity outweighs wall-normal velocity.
41. When do boundary layers grow faster?
    (a) In adverse pressure gradient conditions.
    (b) In favourable pressure gradient conditions.
    (c) In conditions of accelerated outer flow.
    (d) In adverse (favourable) pressure gradient conditions if the boundary layer is laminar (turbulent).
    (e) In conditions of accelerated (decelerated) outer flow if the boundary layer is laminar (turbulent).
42. Upon averaging which term of the Navier-Stokes equations does the Reynolds stress tensor arise?
    (a) Advection term.
    (b) Diffusion term.
    (c) Time-derivative term.
    (d) Pressure gradient term.
    (e) None of the others.
43. What is true when comparing solution methods for laminar and turbulent boundary layers (BLs)?
    (a) The BL integral equations are formally the same.
    (b) The BL local equations are formally the same.
    (c) The Falkner-Skan self-similar solutions apply all the same, provided that the outer flow is a wedge flow.
    (d) Experimental input is needed for solving both laminar and turbulent BLs.
    (e) All of the others.
44. Which mechanism is predominantly responsible for shear in the log law layer of the inner region of a turbulent boundary layer?
    (a) Reynolds stresses $-\rho\overline{u'v'}$.
    (b) Viscous stresses $\mu\frac{\partial u}{\partial y}$.
    (c) Both Reynolds and viscous stresses are important.
    (d) Neither Reynolds nor viscous stresses are important.
    (e) None of the others.
45. Which of the following statements is true about the stresses that appear on a fluid volume in motion under the usual assumptions?
    (a) All options are correct.
    (b) Translation induces no stresses.
    (c) Solid body rotation induces no stresses.
    (d) Volume change induces no stresses, under the Stokes hypothesis.
    (e) Pure shear deformation does induce stresses.
46. Which of the following actions might be beneficial in terms of drag coefficient reduction of a bluff body with laminar boundary layers naturally developing on all surfaces?
    (a) Artificially triggering turbulent transition in the boundary layer before it separates.
    (b) Preserving laminarity of the boundary layer all the way down to the separation point.
    (c) Reducing the Reynolds number just below criticality to ensure a large wake.
    (d) Polishing all surfaces to remove any trace of rugosity.
    (e) None of the others.
47. Which of the following terms can be neglected under the boundary layer assumption?
    (a) Wall-normal advection and streamwise diffusion.
    (b) Streamwise advection and wall-normal diffusion.
    (c) Streamwise advection and streamwise diffusion.
    (d) Wall-normal advection and wall-normal diffusion.
    (e) Streamwise pressure gradient.
48. Which one of the following laminar boundary layers (BL) has the highest friction factor for equal local Reynolds number?
    (a) Laminar BL in the vicinity of a stagnation-point.
    (b) Separation Falkner-Skan BL.
    (c) Laminar BL on a flat plate at zero angle-of-attack.
    (d) An attached laminar BL with decelerated outer flow.
    (e) All laminar BL have the same friction factor.
49. What is the expected sign of the $\overline{u'v'}$ cross-correlation term of the Reynolds stresses in a statistically two-dimensional turbulent BL?
    (a) Negative.
    (b) Positive.
    (c) Null.
    (d) It might change sign depending on various factors.
    (e) It is a complex number.
50. Which of the following assertions is true about the integral method for solving the boundary layer developing along a streamlined surface?
    (a) The laminar and turbulent versions of the integral method are formally the same, although with different coefficients.
    (b) The integral method is exact for all external flows ranging from the stagnation point to the flat plate.
    (c) The integral method provides the complete velocity fields within the boundary layer.
    (d) The integral method is based on the mass conservation integral equation.
    (e) All options are correct.

## 2016/17 Q2

51. For a turbulent boundary layer, the velocity defect law...
    (a) applies to the outer region.
    (b) applies to the viscous sublayer of the inner region.
    (c) applies to the buffer layer of the inner region.
    (d) applies to the log law layer of the inner region.
    (e) applies to the full thickness of the boundary layer.

## 2016/17 Q1

52. Vorticity production occurs...
    (a) both in laminar and turbulent boundary layers.
    (b) only in laminar boundary layers.
    (c) only in turbulent boundary layers.
    (d) only in inviscid flow.
    (e) None of the others.

53. Drag of sufficiently fast ($Re > 10^4$) incompressible flow around bluff bodies is dominated by...
    (a) wake suction always.
    (b) friction always.
    (c) wake suction in the subcritical regime and friction in the supercritical regime.
    (d) friction in the subcritical regime and wake suction in the supercritical regime.
    (e) compressibility effects.

54. Which statement is correct about turbulent flow?
    (a) None of the others.
    (b) It is a stationary phenomenon.
    (c) It is a two-dimensional phenomenon.
    (d) Properties homogenisation across the fluid domain is milder than for laminar flow.
    (e) Energy dissipation is lower than for laminar flow.

55. Which of the following boundary layers (BL) has the highest form factor?
    (a) Separation Falkner-Skan BL.
    (b) Laminar BL in the vicinity of a stagnation-point.
    (c) Laminar BL on a flat plate at zero angle-of-attack.
    (d) Turbulent BL on a flat plate at zero angle-of-attack.
    (e) Accelerated flow Falkner-Skan BL.

56. What is the origin of Reynolds stresses in turbulent flows?
    (a) Turbulent velocity fluctuations.
    (b) Increased fluid viscosity.
    (c) Increased fluid density.
    (d) Turbulent pressure fluctuations.
    (e) None of the others.

57. How must the airfoil geometry be modified for the second inviscid flow calculation in an inviscid flow – boundary layer coupling computation?
    (a) Its thickness must be enlarged by the displacement thickness and the airfoil extended to include the wake.
    (b) Its thickness must be enlarged by the momentum thickness and the airfoil extended to include the wake.
    (c) Its thickness must be enlarged by both the momentum and displacement thicknesses and the airfoil extended to include the wake.
    (d) The airfoil must be merely extended to include the wake with no further modification to its thickness.
    (e) The geometry must be kept unaltered.

58. What is false about the near-wall hypothesis in determining the universal structure of a turbulent boundary layer?
    (a) It applies exclusively to the viscous sublayer.
    (b) The mixing length is assumed to increase with wall distance proportional to the von Kárman constant.
    (c) Total shear (viscous plus turbulent) is assumed constant.
    (d) It does not apply to the outer region.
    (e) None of the others (All options are true).

## 2015/16 Q2

59. At increasingly high bulk flow Reynolds number, turbulent boundary layers tend to...
    (a) be thinner.
    (b) be thicker.
    (c) produce a higher friction drag coefficient.
    (d) separate earlier.
    (e) None of the others.

60. When compared with inviscid flow, the viscous blockage of viscous flows entails...
    (a) both a near wall massflow and momentum reduction.
    (b) a near wall massflow reduction.
    (c) a near wall momentum reduction.
    (d) higher resistance to flow separation.
    (e) lower skin friction.

61. Which of the following is not a property of turbulent flow?
    (a) Lower energy dissipation than laminar flow.
    (b) Nonstationarity.
    (c) Three dimensionality.
    (d) Efficient mixing.
    (e) All are turbulent flow properties.

62. Which of the following is not a necessary condition for the Falkner-Skan self-similar boundary layer solutions to exist?
    (a) Flat plate-type external velocity distribution.
    (b) Wedge-flow-type external velocity distribution.
    (c) Laminar flow.
    (d) Incompressible flow.
    (e) All are necessary conditions.

63. How many additional unknowns arise from performing Reynolds averaging on the incompressible Navier-Stokes equations?
    (a) Six.
    (b) Four.
    (c) Three.
    (d) One.
    (e) None of the others.

64. Which of the following airfoil aerodynamic forces can only be obtained after preforming the second inviscid flow calculation in an inviscid flow – boundary layer coupling computation?
    (a) Form drag.
    (b) Friction drag.
    (c) Lift.
    (d) Lift-induced drag.
    (e) All forces can be computed, if only approximately.

65. How is the mixing length ($l$) assumed to behave within the inner region of the turbulent boundary layer under the near wall hypothesis?
   (a) Linearly.
   (b) As an hyperbolic tangent.
   (c) Exponentially.
   (d) Logarithmically.
   (e) Constant.

## 2015/16 Q1

66. The boundary layer hypothesis only applies...
   (a) at sufficiently high bulk-flow Reynolds numbers.
   (b) to inviscid flows.
   (c) to incompressible flows.
   (d) to isothermal flows.
   (e) to 2D flows.
67. Viscosity is responsible for...
   (a) All options are correct.
   (b) momentum diffusion.
   (c) turbulent energy dissipation.
   (d) friction drag.
   (e) shear stresses.
68. Which of the following has an enhancing effect on boundary layer separation?
   (a) A large boundary layer thickness.
   (b) A negative (favourable) pressure gradient.
   (c) High outer streamwise velocities.
   (d) Boundary layer turbulent transition.
   (e) None of the other options.
69. What is true about an ideal laminar boundary layer developing on a flat plate at zero angle of attack immersed in an incompressible flow?
   (a) The velocity profile is self-similar.
   (b) The form factor is constant at $H = 1.4$.
   (c) The wall friction coefficient is constant.
   (d) The momentum thickness is constant.
   (e) The displacement thickness is constant.
70. RANS turbulent viscosity models...
   (a) assimilate advection momentum transport due to turbulent fluctuations to an apparent turbulent diffusion.
   (b) assimilate turbulent diffusion due to turbulent fluctuations to an apparent advection momentum transport.
   (c) individually model all six Reynolds stresses.
   (d) resolve the large turbulent eddies directly.
   (e) None of the others.
71. Which of the following is not a step of an inviscid flow – boundary layer coupling computation of an airfoil aerodynamic performances?
   (a) Solve the viscous problem on the infinite domain around the original airfoil.
   (b) Solve the inviscid problem around the original airfoil.
   (c) Solve the boundary layer equations for a previously computed outer velocity distribution.
   (d) Solve the inviscid problem around an airfoil made thicker by adding the displacement thickness.
   (e) All other options are necessary steps.
72. Which of the following is not a name for a layer of the inner region of the turbulent boundary layer?
   (a) Velocity deffect law layer.
   (b) Viscous sublayer.
   (c) Buffer region.
   (d) Log law region.
   (e) All options belong in the inner region.

## 2014/15 Q2

73. The rate-of-change of a fluid volume can be decomposed, up to first order, in...
   (a) translation, rotation, isotropic volume change and shear deformation.
   (b) translation and rotation.
   (c) rotation, isotropic volume change and shear deformation.
   (d) isotropic volume change and shear deformation.
   (e) translation and shear deformation.
74. The viscosity of a Newtonian fluid...
   (a) depends on temperature.
   (b) is constant.
   (c) depends on shearing rate.
   (d) depends on temperature and shearing rate.
   (e) is null.
75. How do friction and resistance to separation compare for laminar and turbulent boundary layers?
   (a) Laminar has less friction while turbulent resists separation better.
   (b) Laminar resists separation better while turbulent has less friction .
   (c) Laminar has less friction and resists separation better.
   (d) Turbulent has less friction and resists separation better.
   (e) Both laminar and turbulent have comparable properties.
76. What is the major difference from Navier-Stokes to Euler equations?
   (a) Reduction of the order of space derivatives.
   (b) Reduction of the order of time derivatives.
   (c) The disappearance of the pressure term.
   (d) The disappearance of the advection term.
   (e) None of the others.
77. Which additional momentum transport mechanism is characteristic of a RANS averaged turbulent boundary layer when compared to a laminar boundary layer?
   (a) Wall-normal turbulent diffusion.
   (b) Streamwise turbulent diffusion.
   (c) Streamwise viscous diffusion.
   (d) Wall-normal viscous diffusion.
   (e) None of the others.

78. What can be generally said about the form factor $H$ of a boundary layer?
   (a) A higher value denotes a boundary layer that is prone to earlier separation.
   (b) In turbulent boundary layers typical of aeronautical applications at Reynolds numbers below $10^7$ it can be assumed that $H \approx 2.591$.
   (c) It is defined as the ratio of momentum to displacement boundary layer thicknesses $H = \theta/\delta_1$.
   (d) For a laminar boundary layer developing on any surface it takes a constant value $H = 2.591$.
   (e) All other options are incorrect.

79. Which of the following methods for solving turbulent flows attempts at completely modelling all turbulent scales?
   (a) Reynolds-Averaged Navier-Stokes (RANS).
   (b) Direct Navier-Stokes (DNS).
   (c) Large Eddy Simulation (LES).
   (d) All of them.
   (e) None of them.

80. In which regime, regarding boundary layer transition, will a blunt body experience a lower drag coefficient?
   (a) Supercritical.
   (b) Subcritical.
   (c) Compressible.
   (d) Supersonic.
   (e) Hypersonic.

81. What makes the separation of a laminar boundary layer on an airfoil upper surface only slightly dependent on the Reynolds number?
   (a) The effect of the viscous blockage on the outer inviscid flow.
   (b) Actually it is completely independent.
   (c) Actually it is highly dependent.
   (d) The modification of the apparent viscosity due to turbulent fluctuations.
   (e) The constant friction coefficient along the airfoil chord that is characteristic of laminar boundary layers.

82. Regarding the structure of the turbulent boundary layer, the near wall hypothesis that states that shear stress is approximately constant holds...
   (a) across the whole of the inner region.
   (b) only in the viscous sublayer.
   (c) only in the viscous sublayer and the buffer region.
   (d) across the whole of the turbulent boundary layer.
   (e) only within the log-law layer.

## 2014/15 Q1

83. Which of the following components of the rate-of-change of a Newtonian fluid volume entails viscous forces?
   (a) Shear deformation
   (b) Translation
   (c) Rotation
   (d) Volume change, under the Stokes hypothesis
   (e) None does.

84. Which of the following is a necessary condition for boundary layer separation in 2D?
   (a) An adverse pressure gradient.
   (b) A favourable pressure gradient.
   (c) The boundary layer must be laminar.
   (d) The boundary layer must be turbulent.
   (e) None of the others.

85. How does the friction coefficent evolve along an airfoil with boundary layer transition midway down the chord?
   (a) None of the others.
   (b) It decreases monotonically.
   (c) It increases monotonically.
   (d) It increases except upon transition where it undergoes a sudden drop.
   (e) It remains fairly constant.

86. What is a major drawback of making the Reynolds number tend to infinity in the Navier-Stokes equations?
   (a) The no-slip condition at walls is lost.
   (b) All forms of drag are lost.
   (c) Computations result in zero lift.
   (d) The flow becomes turbulent.
   (e) None of the others.

87. Which two momentum transport mechanisms compete inside a laminar boundary layer?
   (a) Streamwise advection vs wall-normal viscous diffusion.
   (b) Wall-normal viscous diffusion vs wall-normal advection.
   (c) Wall-normal vs streamwise viscous diffusion.
   (d) Streamwise viscous diffusion vs wall-normal advection.
   (e) Wall-normal vs strteamwise advection.

88. The boundary layer displacement thickness...
   (a) all options are correct.
   (b) can be interpreted, for a flat plate, as the deflection of streamlines outside the boundary layer.
   (c) corrects for the massflow reduction due to the viscous blockage.
   (d) is larger than the momentum thickness.
   (e) grows faster with adverse pressure gradients than in accelerated external flow.

89. Which of the following methods for solving turbulent flows resolves all turbulent scales (no turbulence modelling)?
   (a) Direct Navier-Stokes (DNS).
   (b) Reynolds-Averaged Navier-Stokes (RANS).
   (c) Large Eddy Simulation (LES).
   (d) All of them.
   (e) None of them.

90. Which of the following is not a property of turbulent flow when compared to laminar?
   (a) Lower friction and lower energy dissipation.
   (b) Intrinsically 3D.
   (c) Intrinsically time-dependent.
   (d) Better resistance to boundary layer separation.
   (e) Better mixing capabilities.

91. To solve the boundary layer on an airfoil, the integral method...
    (a) can be made exact for the flat plate.
    (b) is exact for any decelerated flow.
    (c) is more accurate than solving the local boundary layer equations.
    (d) does not require the external velocity distribution to be applied.
    (e) works for laminar but not for turbulent flows.

92. The universal turbulent boundary layer profile produces a turbulent friction law by matching...
    (a) the log-law layer and the velocity defect law.
    (b) the viscous sublayer and the buffer region.
    (c) the viscous sublayer and the log-law layer.
    (d) the buffer-region and the velocity defect law.
    (e) the log-law layer and the buffer region.

## 2013/14 Q2

93. Which type of drag dominates in the case of a thin airfoil at very high attack angle, close to stall?
    (a) Form (wake) drag.
    (b) Friction drag.
    (c) Wave drag.
    (d) Interference drag.
    (e) None of the others.

94. A rectangular flat plate at zero angle of attack...
    (a) has less drag if it is oriented longitudinally.
    (b) has less drag if it is oriented transversally.
    (c) has the same drag whatever the orientation.
    (d) has no drag.
    (e) the optimal orientation is different for a turbulent and a laminar boundary layer.

95. A separation criterion must take into account...
    (a) pressure gradients and the laminar/turbulent nature of the boundary layer.
    (b) pressure gradients but not the laminar/turbulent nature of the boundary layer.
    (c) the laminar/turbulent nature of the boundary layer but not pressure gradients.
    (d) neither the laminar/turbulent nature of the boundary layer nor pressure gradients.
    (e) the court costs, marital assets and kids custody.

96. Which of the following components of the rate-of-change of a fluid volume does not entail stresses on its boundary?
    (a) Neither translation nor rotation.
    (b) Neither translation nor rate-of-shear.
    (c) Neither rotation nor rate-of-shear.
    (d) Neither rate-of-shear nor volume-change.
    (e) None of the others is correct.

97. Which of the following is not a property of turbulent flow?
    (a) Stationarity.
    (b) Efficient mixing.
    (c) Properties homogeneisation.
    (d) Increased wall friction.
    (e) Better resistence to boundary layer separation.

98. What happens with the skin friction coefficient as a turbulent boundary layer grows along a flat plate?
    (a) It decreases.
    (b) It increases.
    (c) It remains constant.
    (d) It increases for a while, then it decreases.
    (e) It depends on outer flow velocity.

99. Which of the following momentum transport mechanisms dominates in the nearwall region of a boundary layer?
    (a) Viscous diffusion.
    (b) Streamwise advection.
    (c) Turbulent diffusion.
    (d) Spanwise advection.
    (e) wall-normal advection.

100. Which one of the following boundary layer thicknesses is smaller in normal circumstances?
    (a) Momentum thickness.
    (b) Displacement thickness.
    (c) Total boundary layer thickness.
    (d) They are all the same.
    (e) It depends on many factors.

101. In the mixing length model for the Reynolds stresses in a 2D turbulent boundary layer, the mixing length is defined as...
    (a) the wall-normal distance over which a displaced fluid particle can retain its longitudinal momentum.
    (b) the longitudinal distance over which a displaced fluid particle can retain its longitudinal momentum.
    (c) the wall-normal distance over which a displaced fluid particle can retain its wall-normal momentum.
    (d) the longitudinal distance over which a displaced fluid particle can retain its wall-normal momentum.
    (e) None of the others is correct.

102. Which of the following assertions is true about the structure of a turbulent boundary layer?
    (a) The inner region consists of a viscous sublayer, a buffer layer and a log-law layer.
    (b) The outer region consists of a viscous sublayer, a buffer layer and a log-law layer.
    (c) The log-law layer consists of an inner region and an outer region.
    (d) The buffer layer consists of an inner region and an outer region.
    (e) The viscous sublayer consists of two inner regions and a bunch of buffer layers.

## 2013/14 Q1

103. Which of the following is a consequence of viscosity?
    (a) All of them.
    (b) Friction drag.
    (c) Near-wall massflow reduction.
    (d) Near-wall momentum reduction.
    (e) Lift production.

104. Why does a turbulent boundary layer resist separation better than a laminar one?
   (a) Due to its higher momentum near the wall.
   (b) Due to its higher thickness.
   (c) Due to its instability.
   (d) Due to its negative (favorable) pressure gradients.
   (e) Laminar resists better.

105. Which of the following is a consequence of assuming the Boundary Layer hypothesis?
   (a) Streamwise diffusion is negligible.
   (b) Streamwise pressure gradients are negligible.
   (c) Streamwise advection is negligible.
   (d) Wall-normal advection is negligible.
   (e) Wall-normal diffusion is negligible.

106. Which of the following boundary layer characteristic properties is a measure of the near-wall massflow blockage?
   (a) the displacement thickness.
   (b) the momentum thickness.
   (c) the skin friction coefficient.
   (d) the form factor.
   (e) the local Reynolds number.

107. Which of the following wedge/corner flows represent locally an adverse (positive) pressure gradient over an airfoil?
   (a) $\beta < 0$.
   (b) $k < 0$.
   (c) $m > 0$.
   (d) $\alpha > 0$.
   (e) All conditions are equally representative.

108. Which of the following statements is true about laminar and turbulent boundary layers?
   (a) Laminar BLs grow more slowly while the friction coefficient reduces faster along the chord.
   (b) Laminar BLs grow faster and the friction coefficient reduces faster along the chord.
   (c) Laminar BLs grow more slowly and the friction coefficient reduces more slowly along the chord.
   (d) Laminar BLs grow faster while the friction coefficient reduces more slowly along the chord.
   (e) Laminar and turbulent BLs are equivalent in terms of growth and friction coefficient.

109. When using the approximate integral method to solve a transitional boundary layer computation, which property is assumed preserved across the transition point?
   (a) Momentum thickness.
   (b) Displacement thickness.
   (c) Form factor.
   (d) Boundary layer thickness.
   (e) Skin friction coefficent.

110. Which turbulent structures (eddies) are responsible for viscous dissipation?
   (a) The small structures.
   (b) The large structures.
   (c) The intermediate structures.
   (d) All turbulent structures are equally dissipative.
   (e) Turbulence dissipation is not viscous.

111. The compatibility of which two turbulent boundary layer regions allows for the obtention of a turbulent friction law?
   (a) Log law region and outer region.
   (b) Viscous sublayer and outer region.
   (c) Viscous sublayer and log law region.
   (d) Buffer layer and log law region.
   (e) Log law region and buffer layer.

## 2012/13 Q2

112. Which of the following components of the rate-of-change of an infinitesimal fluid domain entails the appearance of viscous stresses?
   (a) Rate of shear.
   (b) Rate of volume change.
   (c) Rotation velocity.
   (d) Translation velocity.
   (e) None of the others.

113. Which of the following factors enhance turbulent transition in boundary layers?
   (a) High velocities.
   (b) Low outer-flow pre-turbulence levels.
   (c) Wall suction.
   (d) Negative (favorable) pressure gradients.
   (e) High viscosity.

114. Which of the following is a consequence of assuming the Boundary Layer hypothesis?
   (a) There is no wall-normal pressure gradient.
   (b) There is no streamwise pressure gradient.
   (c) There is no streamwise advection.
   (d) There is no wall-normal advection.
   (e) There is no wall-normal viscous diffusion.

115. The momentum thickness $\theta$ can be interpreted as...
   (a) being proportional to the friction drag coefficient.
   (b) the deflection of exterior streamlines.
   (c) the consequence of the massflow increase in the proximity of the wall induced by viscosity.
   (d) the distance from the wall at which the boundary layer velocity matches the outer flow velocity.
   (e) None of the others.

116. Which of the following conditions indicates separation for a 2D boundary layer?
   (a) Zero wall friction.
   (b) An inflection point in the streamwise velocity profile.
   (c) No transverse velocity gradient at the outer edge of the boundary layer.
   (d) Fulfillment of a certain transition criterion.
   (e) All given conditions are equivalently indicative.

117. Which of the following statements is true about laminar and turbulent boundary layers?

(a) Laminar BLs induce less friction but separate earlier in the presence of adverse pressure gradients.
(b) Turbulent BLs induce less friction but separate earlier in the presence of adverse pressure gradients.
(c) Laminar BLs induce less friction and separate later in the presence of adverse pressure gradients.
(d) Turbulent BLs induce less friction and separate later in the presence of adverse pressure gradients.
(e) Laminar and turbulent BLs are equivalent in terms of friction and separation.

118. When solving a coupled Inviscid Flow / Boundary Layer computation, how must the airfoil be modified for the final inviscid flow computation that yields the form drag?

(a) It must be enlarged by the displacement thickness.
(b) It must be enlarged by the momentum thickness.
(c) It must be enlarged by the form factor.
(d) It must be enlarged by the boundary layer thickness.
(e) It must be left as is.

119. Which term of the Navier Stokes equations is responsible for the appearance of the Reynolds stresses when averaging to obtain the RANS equations?

(a) The advection term.
(b) The diffusion term.
(c) The pressure term.
(d) The volume forces term.
(e) The transient term.

120. Which distinct properties characterise the log-law region in a turbulent boundary layer?

(a) Advection of average momentum is negligible and turbulent diffusion outweighs viscous diffusion.
(b) Advection of average momentum is negligible and viscous diffusion outweighs turbulent diffusion.
(c) Advection of average momentum outweighs both viscous and turbulent diffusion.
(d) Advection of average momentum is negligible and viscous and turbulent diffusion are comparable.
(e) Advection of average momentum and turbulent diffusion outweigh viscous diffusion.

121. Which statement is FALSE about turbulence?

(a) Turbulent flow has poor mixing capabilities.
(b) Turbulent flow is intrinsically 3D.
(c) Turbulent flow is intrinsically time-dependent.
(d) Navier Stokes are not applicable for turbulent flows.
(e) Turbulent flow is considered deterministic chaos.

## 2012/13 Q1

122. Which of the following statements is true about viscosity?

(a) Friction drag is a consequence of viscosity.
(b) Lift has nothing to do with viscosity.
(c) All forms of drag are a consequence of viscosity.
(d) Fluid flows may undergo spontaneous separation in inviscid calculations.
(e) None of the others.

123. Which of the following factors enhance turbulent transition in boundary layers?

(a) High outer-flow preturbulent levels.
(b) Boundary layer suction.
(c) Low velocities.
(d) Negative (favorable) pressure gradients.
(e) High viscosity.

124. What is the dominant mechanism for momentum transport inside a laminar boundary layer?

(a) Both advection and viscous diffusion.
(b) Advection.
(c) Turbulent diffusion.
(d) Viscous diffusion.
(e) Buoyancy.

125. The displacement thickness $\delta_1$ can be interpreted as...

(a) the distance by which exterior streamlines are deflected.
(b) being proportional to the friction drag coefficient.
(c) the consequence of the momentum defect (loss) induced by viscosity due to the blockage caused by the no-slip boundary condition at the wall.
(d) the distance from the wall at which the boundary layer velocity matches the outer flow velocity.
(e) None of the others.

126. Which of the following is NOT a step in a coupled Inviscid Flow / Boundary Layer computation?

(a) Solve the Navier-Stokes equations in the full domain.
(b) Solve the inviscid problem around the original body.
(c) Solve the boundary layer equations for an outer flow resulting from a previous step.
(d) Solve the inviscid problem around the original body enlarged by the displacement thickness and extended with a wake.
(e) Compute friction drag from the boundary layer computation and lift and form drag from the improved inviscid solution.

127. For a stagnation point, the Falkner-Skan self-similar solutions result in $\frac{Re_\theta}{\sqrt{Re_x}} = 0.2923$. If the inviscid calculation around a given airfoil produces at the stagnation point $u_e = kx$, how thick is the boundary layer at this point?

(a) $\theta = 0.2923\sqrt{\nu/k}$.
(b) $\theta = 0.2923\sqrt{k/\nu}$.
(c) $\theta = 0$.
(d) $\theta = 0.2923 Re_x^{-1/2}$.
(e) None of the others.

128. Which of the following statements is true about the Reynolds Averaged Navier Stokes (RANS) equations?

(a) The mass conservation equation for the average velocity fields remains the same.
(b) The momentum conservation equation for the average fields remains the same.
(c) The equations for the average fields constitute a closed set (there are as many equations as there are unknowns).
(d) The effect of the fluctuations on the mean fields appear in the form of 9 Reynolds stresses.
(e) None of the others.

129. Of which distinct regions, and in which order starting from the wall, consists a turbulent boundary layer?

(a) The internal region (viscous sublayer, buffer region, log-law region) and the outer region.

(b) The internal region (buffer region, viscous sublayer, log-law region) and the outer region.

(c) The internal region (log-law region, viscous sublayer, buffer region) and the outer region.

(d) The internal region (viscous sublayer, buffer region) and the outer or log-law region.

(e) None of the others.

# Aerodynamics

An airfoil of chord $l = 1.49\,\text{m}$ is cruising at speed $U_\infty = 60\,\text{m/s}$ and altitude $h = 2000\,\text{m}$ under ISA conditions. The velocity distribution on the upper and lower surfaces can be approximated by

$$u_e(x) = \left[u_{le}^2 - \left(u_{le}^2 - u_{te}^2\right)\left(\frac{x}{l}\right)\right]^{\frac{1}{2}} \qquad 0 \le x \le l$$

with $u_{le}^{us} = 75\,\text{m/s}$, $u_{le}^{ls} = 60\,\text{m/s}$ and $u_{te}^{us} = u_{te}^{ls} = 48\,\text{m/s}$, the corresponding upper ($us$) and lower ($ls$) surface leading ($le$) and trailing ($te$) edge velocities, respectively.

1. (1p) Calculate the temperature ($T_\infty$), pressure ($p_\infty$), density ($\rho_\infty$) and kinematic viscosity ($\nu_\infty = \mu_\infty/\rho_\infty$) at cruising altitude, assuming ISA conditions. Estimate the Reynolds ($Re_l = lU_\infty/\nu_\infty$) and Mach ($M$) numbers.
2. (1p) Nondimensionalise the velocity distributions on the upper ($\bar{u}^{us}(\bar{x})$) and lower ($\bar{u}^{ls}(\bar{x})$) surfaces using $U_\infty$ and $l$ as velocity and length scales, such that $\bar{u} = u_e/U_\infty$ and $\bar{x} = x/l$. What are the values of $\bar{u}_{le}^{us}$, $\bar{u}_{le}^{ls}$, $\bar{u}_{te}^{us}$, $\bar{u}_{te}^{ls}$?
3. (1p) Find the pressure coefficient distributions on upper ($C_p^{us}(\bar{x})$) and lower ($C_p^{ls}(\bar{x})$) surfaces and compute the airfoil lift coefficient ($C_L$).
4. (1p) Compute the pitching moment coefficient with respect to the leading edge ($C_{m_0}$) and the location of the centre of pressure ($\bar{x}_{\text{CP}}$).

The boundary layers developing on the upper and lower surfaces can be assumed as starting with negligible momentum thickness ($\theta_{\text{SP}} = \theta(0) \simeq 0$) at the leading edge and considered turbulent from outset. The turbulent version of the separation criterion applies:

$$-\frac{\theta}{u_e}\frac{du_e}{dx} > C_t = 0.0035$$

5. (2p) Nondimensionalise both the separation criterion and the integral method.
6. (2p) Use the nondimensional integral method to find the non-dimensional trailing edge momentum thickness on the upper ($\bar{\theta}_{te}^{us}$) and lower ($\bar{\theta}_{te}^{ls}$) surfaces.
7. (1p) Use the Squire & Young formula for non-symmetric velocity distributions to estimate the drag coefficient ($C_D$).
8. (1p) Use the turbulent version of the separation criterion to show that the boundary layer remains attached all the way down to the trailing edge on both the upper and lower surfaces.

## Solutions

1. Using the ISA relations, we have

$$\begin{aligned}
T_\infty &= T_0\left(1+\frac{T_h}{T_0}h\right) = 275.15\,\mathrm{K}\\
p_\infty &= p_0\left(\frac{T}{T_0}\right)^{-\frac{g}{RT_h}} = 79494.4\,\mathrm{Pa}\\
\rho_\infty &= \rho_0\left(\frac{T}{T_0}\right)^{-\left(\frac{g}{RT_h}+1\right)} = 1.00648\,\mathrm{kg/m^3}\\
\nu_\infty &= \frac{\mu_\infty}{\rho_\infty} = \frac{\mu_{\mathrm{ref}}}{\rho}\frac{T_{\mathrm{ref}}+C}{T+C}\left(\frac{T}{T_{\mathrm{ref}}}\right)^{3/2} = 1.70977\times 10^{-5}\,\mathrm{m^2/s}
\end{aligned}$$

The Reynolds and Mach numbers are

$$Re_l = \frac{U_\infty l}{\nu_\infty} = 5.229\times 10^6 \qquad M = \frac{U_\infty}{\sqrt{\gamma RT}} = 0.1804$$

2. Nondimensionalisation yields

$$\bar{u}(\bar{x}) \equiv \frac{u_e(x)}{U_\infty} = \left[\bar{u}_{le}^2 - \left(\bar{u}_{le}^2 - \bar{u}_{te}^2\right)\bar{x}\right]^{\frac{1}{2}} \qquad 0\le \bar{x}\le 1$$

with $\bar{u}_{le}^{us} = u_{le}^{us}/U_\infty = 1.25$, $\bar{u}_{le}^{ls} = u_{le}^{ls}/U_\infty = 1$ and $\bar{u}_{te}^{us} = \bar{u}_{te}^{ls} = 0.8$.

3. The pressure coefficient distributions result from the Bernoulli equation as

$$p_\infty + \rho_\infty\frac{U_\infty^2}{2} = p_e + \rho_e\frac{u_e^2}{2} \quad\longrightarrow\quad C_p \equiv \frac{p_e-p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = 1-\bar{u}^2$$

assuming incompressible flow ($\rho_e = \rho_\infty$). The lift coefficient is obtained as

$$\begin{aligned}
C_L &= \frac{L}{\frac{1}{2}\rho_\infty U_\infty^2 l} = \frac{-1}{\frac{1}{2}\rho_\infty U_\infty^2 l}\int_{\mathrm{airfoil}} p\hat{n}\cdot\hat{y}\,ds = \frac{-1}{\frac{1}{2}\rho_\infty U_\infty^2 l}\int_{\mathrm{airfoil}} (p-p_\infty)\,\hat{n}\cdot\hat{y}\,ds =\\
&= -\frac{1}{l}\int_0^l \left(C_p^{us}-C_p^{ls}\right)dx = -\int_0^1\left(C_p^{us}-C_p^{ls}\right)d\bar{x} = \int_0^1 \bar{u}_{us}^2-\bar{u}_{ls}^2\,d\bar{x} =\\
&= \int_0^1 \left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right] - \left[\left((\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right)-\left((\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right)\right]\bar{x}\,d\bar{x} =\\
&= \left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right] - \frac{1}{2}\left[\left((\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right)-\left((\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right)\right] =\\
&= \frac{\left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right]+\left[(\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right]}{2} = 0.28125
\end{aligned}$$

4. The moment coefficient can be computed as

$$\begin{aligned}
C_{m_0} &= \frac{M_0}{\frac{1}{2}\rho_\infty U_\infty^2 l^2} = \frac{-1}{\frac{1}{2}\rho_\infty U_\infty^2 l^2}\int_{\mathrm{airfoil}} p\,(\vec{r}\times\hat{n})\cdot\hat{z}\,ds = \frac{-1}{\frac{1}{2}\rho_\infty U_\infty^2 l}\int_{\mathrm{airfoil}} (p-p_\infty)\,(\vec{r}\times\hat{n})\cdot\hat{z}\,ds \simeq\\
&\simeq -\frac{1}{l^2}\int_0^l x\,\left(C_p^{us}-C_p^{ls}\right)dx = -\int_0^1 \bar{x}\,\left(C_p^{us}-C_p^{ls}\right)d\bar{x} = \int_0^1 \bar{x}\,\left[\bar{u}_{us}^2-\bar{u}_{ls}^2\right]d\bar{x} =\\
&= \int_0^1 \left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right]\bar{x} - \left[\left((\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right)-\left((\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right)\right]\bar{x}^2\,d\bar{x} =\\
&= \frac{1}{2}\left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right] - \frac{1}{3}\left[\left((\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right)-\left((\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right)\right] =\\
&= \frac{\left[(\bar{u}_{le}^{us})^2-(\bar{u}_{le}^{ls})^2\right]+\left[(\bar{u}_{te}^{us})^2-(\bar{u}_{te}^{ls})^2\right]}{6} = 0.09375
\end{aligned}$$

The centre of pressure is determined from

$$\vec{M}_0 + (\vec{r}_0 - \vec{r}_{\mathrm{CP}}) \times \vec{F} = \vec{0} \quad \longrightarrow \quad M_0 + L\,(x_0 - x_{\mathrm{CP}}) = 0$$

$$\longrightarrow \quad C_{m_0} + C_L\,(\bar{x}_0 - \bar{x}_{\mathrm{CP}}) = 0 \quad \longrightarrow \quad \bar{x}_{\mathrm{CP}} = \cancel{\bar{x}_0} + \frac{C_{m_0}}{C_L} = \frac{1}{3}$$

5. Nondimensionalising the separation criterion yields

$$-\frac{\theta}{u_e}\frac{du_e}{dx} = -\frac{L}{U_\infty}\frac{\bar{\theta}}{\bar{u}}\frac{U_\infty}{L}\frac{d\bar{u}}{d\bar{x}} = -\frac{\bar{\theta}}{\bar{u}}\frac{d\bar{u}}{d\bar{x}} > C_s$$

Taking the expression for $\bar{u}(\bar{x})$, the streamwise derivative is

$$\frac{d\bar{u}}{d\bar{x}} = -\frac{1}{2}\frac{\bar{u}_{le}^2 - \bar{u}_{te}^2}{\left[\bar{u}_{le}^2 - (\bar{u}_{le}^2 - \bar{u}_{te}^2)\,\bar{x}\right]^{\frac{1}{2}}} = -\frac{1}{2}\frac{\bar{u}_{le}^2 - \bar{u}_{te}^2}{\bar{u}}$$

All in all, the separation criterion predicts separation for

$$\frac{\bar{u}_{le}^2 - \bar{u}_{te}^2}{2}\frac{\bar{\theta}}{\bar{u}^2} > C_s$$

Nondimensionalising the integral method with $L$ and $U_\infty$ provides

$$\bar{\theta}^{m_0+1}\bar{u}^{(m_0+1)(H+2)} = \bar{\theta}_0^{\,m_0+1}\bar{u}_0^{\,(m_0+1)(H+2)} + \frac{(m_0+1)b}{Re_l^{m_0}}\int_{\bar{x}_0}^{1} \bar{u}^{(m_0+1)(H+2)-m_0}\,d\bar{x}$$

which, after substituting the values for its turbulent version ($H = 1.4$, $m_0 = 0.2$ and $b = 0.0086$) and assuming negligible boundary layer thickness at $\bar{x}_0 = 0$, results in

$$\bar{\theta}^{1.2}\bar{u}^{4.08} = \cancel{\bar{\theta}_0^{\,1.2}\bar{u}_0^{\,4.08}} + \frac{0.01032}{Re_l^{0.2}}\int_0^1 \bar{u}^{3.88}\,d\bar{x}$$

6. The trailing edge momentum thickness is therefore given by

$$\begin{aligned}\bar{\theta}_{te}^{1.2} &= \frac{0.0132}{Re_l^{0.2}u_{te}^{4.08}}\int_0^1 \left[\bar{u}_{le}^2 - \left(\bar{u}_{le}^2 - \bar{u}_{te}^2\right)\bar{x}\right]^{1.94} d\bar{x} = \\ &= \frac{0.0035102}{Re_l^{0.2}u_{te}^{4.08}}\frac{\bar{u}_{le}^{5.88} - \bar{u}_{te}^{5.88}}{\bar{u}_{le}^2 - \bar{u}_{te}^2} = \frac{0.0035102}{Re_l^{0.2}u_{te}^{0.2}}\frac{(\bar{u}_{le}/\bar{u}_{te})^{5.88} - 1}{(\bar{u}_{le}/\bar{u}_{te})^2 - 1}\end{aligned}$$

The upper and lower trailing edge momentum thicknesses are then

$$\bar{\theta}_{te}^{us} = 4.375 \times 10^{-3} \qquad \bar{\theta}_{te}^{ls} = 2.633 \times 10^{-3}$$

7. The Squire & Young formula yields

$$C_D = 2\left[\bar{\theta}_{te}^{us}(\bar{u}_{te}^{us})^{3.2} + \bar{\theta}_{te}^{us}(\bar{u}_{te}^{ls})^{3.2}\right] = 6.86 \times 10^{-3}$$

8. At the trailing edge, the streamwise outer velocity gradient is

$$\left(\frac{d\bar{u}}{d\bar{x}}\right)_{\bar{x}_{te}=1} = -\frac{\bar{u}_{te}}{2}\left[\left(\frac{\bar{u}_{le}}{\bar{u}_{te}}\right)^2 - 1\right]$$

Evaluating the separation criteria at the trailing edge results in

$$C = -\frac{\bar{\theta}_{te}}{\bar{u}_{te}}\left(\frac{d\bar{u}}{d\bar{x}}\right)_{\bar{x}_{te}} = \frac{\bar{\theta}_{te}}{2}\left[\left(\frac{\bar{u}_{le}}{\bar{u}_{te}}\right)^2 - 1\right]$$

with $C_{us} = 3.15 \times 10^{-3}$ and $C_{ls} = 7.40 \times 10^{-4}$, both safely below $C_s = 3.5 \times 10^{-3}$

# Aerodynamics: Boundary layer Problem

Consider the following approximation to the Blasius solution for the incompressible laminar boundary layer developing on a smooth flat plate:

$$\frac{u}{u_e} = \begin{cases} \dfrac{y}{\delta} & 0 \leq y \leq \delta \\ 1 & y > \delta \end{cases}$$

where $u = u(x, y)$ is the velocity profile, $\delta = \delta(x)$ is the boundary layer thickness and $u_e$ is the constant outer flow velocity. The density of the fluid is $\rho$ and its kinematic viscosity $\nu$.

1. (1p) Enumerate which of the properties of a laminar boundary layer developing on a flat plate this approximation satisfies exactly.
2. (1p) Express the displacement thickness $\delta_1$ as a function of the boundary layer thickness $\delta$.
3. (2p) Express the momentum thickness $\theta$ as a function of the boundary layer thickness $\delta$. Which value does the form factor $H$ take?
4. (2p) Express the friction coefficent $C_f$ as a function of the boundary layer thickness $\delta$.
5. (2p) Use the momentum integral equation to find out how the boundary layer thickness $\delta$ evolves as a function of the local Reynolds number $Re_x = u_e x/\nu$.
6. (2p) Recast $(\delta_1/x)$, $(\theta/x)$ and $C_f$ as functions of $Re_x$. Comment on how accurate (in %) these results are with respect to Blasius boundary layer values?

# Solutions

1. $u(0) = 0$ (no-slip at wall) and $u(\delta) = u_e$ (recovery of outer velocity at boundary).

2. $\delta_1 = \dfrac{1}{2}\,\delta$

3. $\theta = \dfrac{1}{6}\,\delta$ and $H = \dfrac{\delta_1}{\theta} = \dfrac{5}{2} = 3$

4. $C_f = \dfrac{2\nu}{\delta u_e}$

5. $\dfrac{\delta}{x} = \dfrac{\sqrt{12}}{Re_x^{1/2}} = \dfrac{3.4641}{Re_x^{1/2}}$

6. $\dfrac{\delta_1}{x} = \dfrac{1.7321}{Re_x^{1/2}}$, $\dfrac{\theta}{x} = \dfrac{0.5774}{Re_x^{1/2}}$, $\dfrac{C_f}{2} = \dfrac{0.2887}{Re_x^{1/2}}$ and $H = 3$, which are 0.65%, −13.1%, −13.1% and 15.8% accurate, respectively, when compared to the exact Blasius boundary layer solution for a smooth flat plate.

**Aerodynamics & Flight Mechanics** **EETAC**

**Q2**, 08/06/2018 **Final Exam**

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Aerodynamics: Boundary layer Problem

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Consider the laminar boundary layer that develops on the surface of a circular cylinder of radius $R$, immersed in an incompressible inviscid flow with upstream velocity $U_\infty$ (Fig. 1).

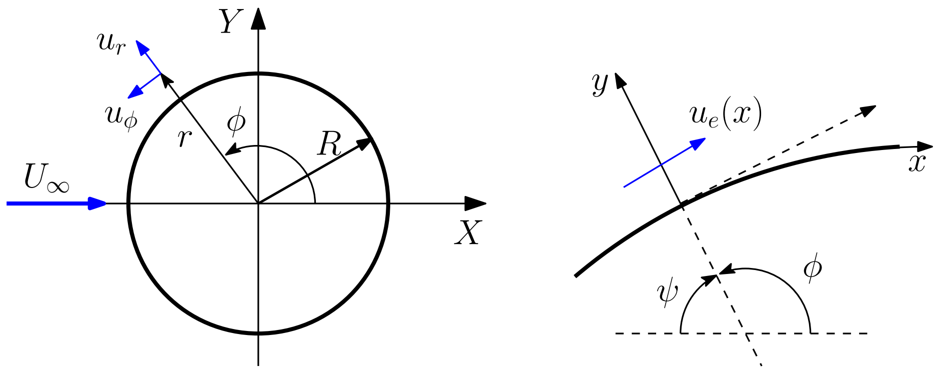


Figure 1: Potential flow around the cylinder: global and local coordinates.

The velocity field in polar coordinates $(r, \theta)$ is given by the doublet solution to the potential flow equations:

$$\begin{aligned} u_r(r,\phi) &= U_\infty \left(1 - \frac{R^2}{r^2}\right) \cos\phi \\ u_\phi(r,\phi) &= -U_\infty \left(1 + \frac{R^2}{r^2}\right) \sin\phi \end{aligned}$$

A local reference system with coordinates $(x, y)$ is defined along the cylinder surface for the analysis of the boundary layer. For ease of treatment, the complementary angle is defined as $\psi \equiv \pi - \phi$ and the origin for the local curvilinear reference system is placed the stagnation point $(r, \psi) = (R, 0)$. The Reynolds number is defined as $Re_D = U_\infty D/\nu$, with $D$ the cylinder diameter and $\nu$ the kinematic viscosity of the fluid.

1. (2p) Express the velocity field in the modified polar coordinates $[u_r, u_\psi](r, \psi)$. Draw a sketch.
2. (2p) Express the local coordinate $x(\psi)$ as a function of the angular coordinate $\psi$. Find the outer velocity distribution $u_e(\psi)$.
3. (2p) What is the boundary layer nondimensional momentum thickness $\bar{\theta}_0 \equiv \theta(0)/R$ at the stagnation point? Express it in terms of $Re_D$.
4. (2p) Formulate the integral method for the nondimensional momentum thickness distribution $\bar{\theta}(x) \equiv \theta(\psi)/R$ along the cylinder surface.
5. (2p) Find the non-closed-form expression for the separation point $\psi_s$ given the laminar separation criterion

$$-\frac{\theta^2}{\nu}\frac{du_e}{dx} > 0.06815$$

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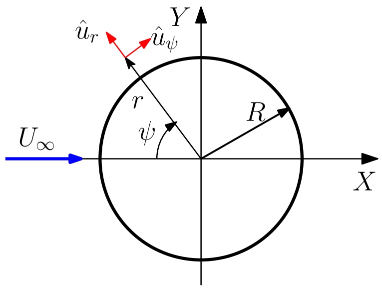
## Solutions

1. The velocity field in the new coordinates is simply expressed as




$$\begin{aligned}\hat{u}_r(r,\psi) &= u_r(r,\pi-\psi) = -U_\infty\left(1-\frac{R^2}{r^2}\right)\cos\psi \\ \hat{u}_\psi(r,\phi) &= -u_\phi(r,\pi-\psi) = U_\infty\left(1+\frac{R^2}{r^2}\right)\sin\psi\end{aligned}$$

2. The local coordinate and the velocity distribution are

$$\begin{aligned}x(\psi) &= R\psi \\ u_e(\psi) &= \hat{u}_\psi(R,\phi) = 2U_\infty\sin\psi\end{aligned}$$

3. At a stagnation point, potential flow around a $m=\beta=1$, $\alpha=\pi/2$ corner indicates that $u_e(x)=kx$. In our case we have

$$k = \left(\frac{du_e}{dx}\right)_{x=0} = \left(\frac{du_e}{d\psi}\right)_{\psi=0}\left(\frac{d\psi}{dx}\right)_{x=0} = (2U_\infty\cos\psi)_{\psi=0}\frac{1}{R} = \frac{2U_\infty}{R}.$$

The Falkner-Skan solution for the self-similar laminar boundary layer solution around the stagnation point is

$$\theta_{SP} = 0.2923\sqrt{\nu/k} = 0.2923\sqrt{\frac{\nu R}{2U_\infty}} \qquad \longrightarrow \qquad \bar{\theta}_0 = \frac{\theta_{SP}}{R} = \frac{0.2923}{\sqrt{Re_D}}.$$

4. The laminar version for the approximate integral method uses $m_0=1$, $H=2.591$ and $b=0.2205$:

$$\theta(x)^2 u_e(x)^{9.182} = \theta_0^2 u_0^{9.182} + 0.441\nu\int_0^\psi u_e(x)^{8.182}\,dx.$$

Nondimensionalisation with $R$ and $U_\infty$ yields

$$\bar{\theta}(\psi)^2(\sin\psi)^{9.182} = \cancel{\frac{0.2923^2}{Re_D}(\sin\psi_0)^{9.182}} + \frac{0.441}{Re_D}\int_0^\psi(\sin\psi)^{8.182}\,d\psi.$$

5. The laminar separation criterion may be nondimensionalised thus

$$-\frac{\theta^2}{\nu}\frac{du_e}{dx} = -\frac{\theta^2}{\nu}\frac{du_e}{d\psi}\frac{d\psi}{dx} = -\frac{\theta^2}{\nu}\frac{2U_\infty\cos\psi}{R} = -Re_D\bar{\theta}^2\cos\psi > 0.06815.$$

Substitution of the expression for the nondimensional momentum thickness results in

$$\frac{\cos\psi_s}{(\sin\psi_s)^{9.182}}\int_0^{\psi_s}(\sin\psi)^{9.182}\,d\psi = 0.1545,$$

independent of $Re_D$. Solving this equation numerically yields $\psi_s = 1.777\,\text{rad} = 101.8°$.

## Aerodynamics: Boundary layer Problem

The area of a 2D air duct (fig. 2) decreases exponentially as $A(x) = A_0 \mathrm{e}^{-kx}$, with $A(0) = A_0$ the outlet area and $k > 0$ the area decrease rate factor. For simplicity, the duct will be considered as extending upstream to infinity ($x = -\infty$), and the airflow through it incompressible and 1D (except within boundary layers). The boundary layers developing on the upper and lower walls are assumed to start with zero thickness at $x = -\infty$ and to be turbulent right from outset. The air has density $\rho$ and kinematic viscosity $\nu$.

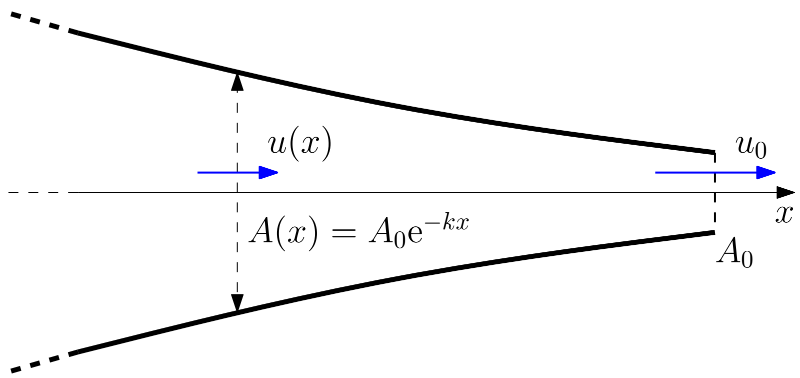



Figure 2: 2D air duct.

1. (2p) If the outlet velocity is $u(0) = u_0$, how does the velocity $u(x)$ evolve along the duct?
2. (3p) Use the integral method to find the momentum thickness $\theta(x)$ distribution along the duct walls.
3. (3p) Use the integral momentum equation to produce the friction coefficient $C_f(x)$ distribution.
4. (2p) Calculate the friction drag per span unit $D$ on the duct.

Leave all results as a function of $u_0$, $k$, $\rho$, $\nu$ and $x$, whenever appropriate.

# Solutions

1. Mass conservation for incompressible flow along a 1D duct yields

$$\rho_0 A_0 u_0 = \rho(x)A(x)u(x) = \rho_0 A_0 \mathrm{e}^{-kx}u(x) \qquad \longrightarrow \qquad u(x) = u_0\mathrm{e}^{kx}.$$

2. Taking the turbulent version of the integral method ($m_0 = 0.2$, $H = 1.4$ and $b = 0.0086$), we have

$$\begin{aligned}\theta(x)^{1.2}u(x)^{4.08} &= \cancel{\theta_{-\infty}^{1.2}u_{-\infty}^{4.08}} + 0.01032\nu^{0.2}\int_{-\infty}^{x} u(x)^{3.88}\,dx = \\ &= 0.01032\nu^{0.2}u_0^{3.88}\int_{-\infty}^{x}\mathrm{e}^{3.88kx}\,dx = 2.66\cdot 10^{-3}\frac{\nu^{0.2}u_0^{3.88}}{k}\mathrm{e}^{3.88kx},\end{aligned}$$

which results in

$$\theta(x) = 7.146\cdot 10^{-3}\frac{1}{k}\left(\frac{\nu k}{u_0}\right)^{1/6}\mathrm{e}^{-kx/6}.$$

3. Substituting the expressions for $u(x)$ and $\theta(x)$ into the momentum integral equation yields

$$\begin{aligned}\frac{C_f}{2}(x) &= \frac{d\theta}{dx} + \theta\frac{H+2}{u}\frac{du}{dx} = 7.146\cdot 10^{-3}\left(\frac{\nu k}{u_0}\right)^{1/6}\mathrm{e}^{-kx/6}\left(-\frac{1}{6} + (H+2)\right) = \\ &= 0.0231\left(\frac{\nu k}{u_0}\right)^{1/6}\mathrm{e}^{-kx/6}\end{aligned}$$

4. Friction drag can be obtained by integration on both walls (upper and lower) of the wall shear stress.

$$\begin{aligned}D &= 2\int_{-\infty}^{x}\tau_w\,dx = 2\int_{-\infty}^{x}\rho u(x)^2\frac{C_f}{2}(x)\,dx = 0.04621\rho u_0^2\left(\frac{\nu k}{u_0}\right)^{1/6}\int_{-\infty}^{x}\mathrm{e}^{11kx/6}\,dx = \\ &= 0.02521\frac{\rho u_0^2}{k}\left(\frac{\nu k}{u_0}\right)^{1/6}.\end{aligned}$$

Aerodynamics: Boundary layer Problem

The slanted rear of a hatchback car can be modelled as a backward-facing ramp with deflection angle $\alpha$ (fig. 3). The flow past the deflection can be treated as the two-dimensional inviscid potential flow around an ideal corner or wedge with constant $k$ (defined as the velocity at $x = 1$ on a streamline going through the origin), and the boundary layer developing on the ramp can be assumed laminar, self-similar (Falkner-Skan type) and starting at the corner (e.g. because a boundary layer trap sucks the boundary layer that has evolved on the horizontal surface). Take $\nu$ as the kinematic viscosity of the fluid.

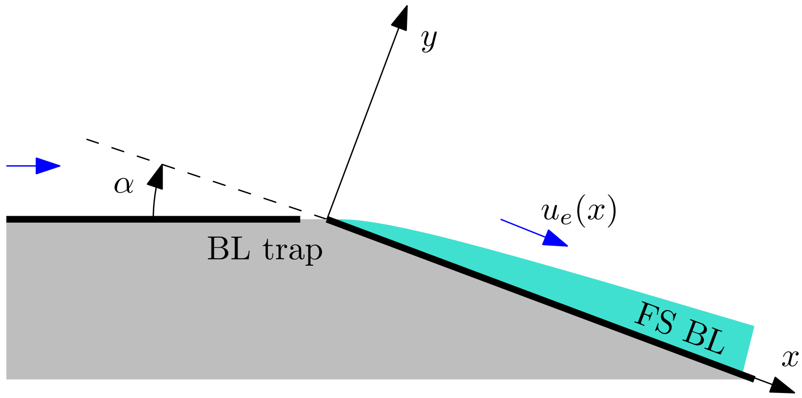



Figure 3: Backward-facing ramp with deflection angle $\alpha$.

1. (2p) What is the maximum acceptable deflection angle ($\alpha$) for the boundary layer to remain attached past the corner?
2. (2p) What is the corresponding outer flow velocity ($u_e(x)$)?
3. (2p) How does the friction coefficient ($C_f(x)$) evolve along the ramp?
4. (2p) How do the displacement ($\delta_1(x)$) and momentum ($\theta(x)$) thicknesses evolve along the ramp?
5. (2p) What is the form factor ($H$)?

Leave all results as a function of $k$, $\nu$ and $x$, whenever appropriate.

## Solutions

1. In the idealisation of a two-dimensional inviscid potential flow around a corner, the velocity distribution along the streamlines going through the origin is $u_e(x) = kx$. The self-similar boundary layer that develops with this outer flow is of Falkner-Skan type and the maximum acceptable deflection angle for the boundary layer to remain attached is that corresponding to the separation boundary layer, which has $m = -0.09043$ and $\beta = -0.19884$ (see the table for Falkner-Skan boundary layer solutions). The corresponding deflection angle is

$$\alpha = \frac{\pi m}{m+1} = \frac{\pi}{2}\beta = -0.31234\,\mathrm{rad} = -17.90^\circ$$

2. The outer flow velocity that results in the separation Falkner-Skan solution is

$$u_e(x) = kx^m = kx^{-0.09043}$$

3. The friction coefficient of the separation Falkner-Skan boundary layer solution is precisely that whcih defines separation, namely

$$C_f = 0$$

4. The displacement thickness can be read from the Falkner-Skan tables again:

$$\frac{Re_{\delta_1}}{\sqrt{Re_x}} = \frac{Re_{\delta_1}}{Re_x}\sqrt{Re_x} = \frac{\delta_1}{x}\sqrt{Re_x} = 3.49779,$$

so that

$$\delta_1(x) = 3.49779\frac{x}{\sqrt{Re_x}} = 3.49779\frac{x}{\sqrt{\frac{u_e(x)x}{\nu}}} = 3.49779\frac{x}{\sqrt{\frac{kx^m x}{\nu}}} = \sqrt{\frac{\nu}{k}}x^{\frac{1-m}{2}} = 3.49779\sqrt{\frac{\nu}{k}}x^{0.5452}$$

The momentum thickness can be found analogously as

$$\frac{Re_\theta}{\sqrt{Re_x}} = \frac{Re_\theta}{Re_x}\sqrt{Re_x} = \frac{\theta}{x}\sqrt{Re_x} = 0.86811,$$

so that

$$\theta(x) = 0.86811\frac{x}{\sqrt{Re_x}} = 0.86811\frac{x}{\sqrt{\frac{u_e(x)x}{\nu}}} = 0.86811\frac{x}{\sqrt{\frac{kx^m x}{\nu}}} = \sqrt{\frac{\nu}{k}}x^{\frac{1-m}{2}} = 0.86811\sqrt{\frac{\nu}{k}}x^{0.5452}$$

5. The form factor can be obtained by dividing the expressions for $\delta_1$ and $\theta$ or simply by reading the value for the separation boundary layer from the table:

$$H = \frac{\delta_1}{\theta} = 4.02923.$$

**Aerodynamics & Flight Mechanics** **EETAC**

**Q1**, 16/01/2017 **Final Exam**

## Aerodynamics: Boundary layer Problem

A low subsonic closed-loop wind tunnel is to be fitted with a flow straightener in the settling chamber, immediately upstream from the contraction, to reduce swirl and feed a homogeneous inlet velocity profile into the test section. The honeycomb (fig. 4a) consists of a grid of $N$ square section ducts (fig. 4b) of side $a = 5\,\text{mm}$ and length $L = 4\,\text{cm}$, completely filling the cross-section of the wind tunnel at the installation location in the plenum. The total area is $A = 0.5\,\text{m}^2$ and the average incoming axial velocity is $U = 10\,\text{m/s}$ at full power.

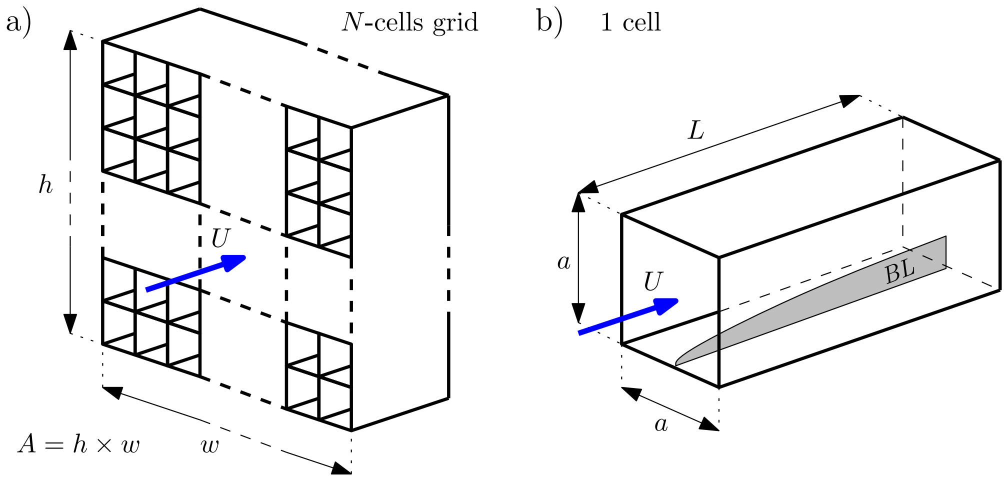



Figure 4: a) Flow straightener (Honeycomb) consisting of $N$ individual cells. b) Detail of one cell: a duct of square cross-section. The boundary layer (BL) develops on all four internal walls.

The boundary layers developing on the interior walls of each duct, to be taken as one-sided flat plates, are incompressible and can be considered two-dimensional and fully laminar. Both the blockage due to plates thickness and three-dimensional effects deriving from finite spanwise size shall be neglected. Assume sea level ISA air density ($\rho = 1.225\,\text{kg/m}^2$) and kinematic viscosity ($\nu = 1.4531 \times 10^{-5}\,\text{m}^2/\text{s}$).

1. (2p) Assess the boundary layer displacement thickness on the walls at the exit of a cell ($\delta_{1_{te}}$).
2. (1p) Comment on viscous blockage and the validity of the flat plate boundary layer assumption.

Neglect further the effects of viscous blockage.

3. (2p) Find the boundary layer momentum thickness on the walls at the exit of a cell ($\theta_{te}$).
4. (3p) Assess the drag experienced by each individual cell ($D_1$) and by the whole grid ($D$).
5. (2p) What is the additional power that will be required to drive the flow through the wind tunnel once the flow straightener has been installed?

# Solutions

1. For a laminar boundary layer on a flat plate, we have

$$\frac{\delta_1}{x} = \frac{1.721}{Re_x^{1/2}} = \frac{1.721}{Re_L^{1/2}}\left(\frac{x}{L}\right)^{-1/2} \longrightarrow \frac{\delta_{1_{te}}}{L} = \frac{1.721}{Re_L^{1/2}} \longrightarrow \boxed{\delta_{1_{te}} = 4.15\times10^{-4}\,\mathrm{m}},$$

where $Re_L = UL/\nu = 2.75\times10^4$. This is a low $Re_L$, and transition does not occur despite the high turbulence levels in the return circuit due to turns and, above all, the swirl and turbulence introduced by the driving fan.

2. The cross-sectional area of every cell is reduced from a geometric $S = a^2 = 25\,\mathrm{mm}^2$ to an effective $S_{\text{eff}} = (a - 2\delta_{1_{te}})^2 = 17.4\,\mathrm{mm}^2$ due to viscous blockage. The outer flow velocity will accordingly increase across the cell, so that the boundary layer developing on its walls will not be the assumed open-air flat plate boundary layer but rather the boundary layer for an accelerated external flow. The combined effect of the thinner resulting boundary layer and higher outer velocities will result in increased friction. Neglecting viscous blockage will therefore underestimate real friction.

3. The momentum thickness is

$$\frac{\theta}{x} = \frac{0.664}{Re_x^{1/2}} = \frac{0.664}{Re_L^{1/2}}\left(\frac{x}{L}\right)^{-1/2} \longrightarrow \frac{\theta_{te}}{L} = \frac{0.664}{Re_L^{1/2}} = \frac{1}{H}\frac{\delta_{1_{te}}}{L} \longrightarrow \boxed{\theta_{te} = 1.60\times10^{-4}\,\mathrm{m}}$$

where $H = 2.591$ is the form factor of the flat plate laminar boundary layer.

4. For a one-sided flat plate, $C_D^{\text{fp}} = 2\theta_{te}/L$. A cell consisting of a duct with four internal walls of span $a$ (or,equivalently, one folded wall of span $4a$) has a drag

$$D_1 = \frac{1}{2}\rho S U^2 C_D^{\text{fp}} = \frac{0.664\rho L(4a)U^2}{Re_L^{1/2}} = \frac{2.656\rho L a U^2}{Re_L^{1/2}} = \boxed{3.92\times10^{-4}\,\mathrm{N}}$$

A rather awkward alternative would have been to integrate the wall shear stress $\tau_w$ along the chord of the wall.

To cover an area $A$ with cells of $a^2$ cross-section each, a total of $N = A/a^2 = 2\times10^4$ cells is required. the total drag is simply found as

$$D = ND_1 = \boxed{7.84\,\mathrm{N}}$$

5. The additional power due to friction drag in the honeycomb is simply estimated as

$$P = DU = \boxed{78.44\,\mathrm{W}}$$

**Aerodynamics & Flight Mechanics** **EETAC**

**Q2**, 02/06/2016 **Final Exam**

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## Aerodynamics: Boundary layer Problem

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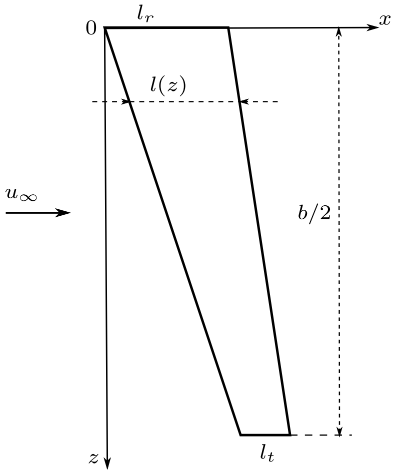
Consider the incompressible flow around the Honda Jet during a takeoff run at speed $u_\infty = 100\,\mathrm{m/s}$. The airplane is characterised by a wingspan $b = 12.15\,\mathrm{m}$, an aspect ratio $\lambda = 8.5$ and a taper $t = l_t/l_r = 0.38$.

For the sake of simplicity, neglect both the presence of the fuselage and wing twist and incidence and assume that the wing can be approximated by a flat plate at zero angle of attack as shown in the figure.




Neglect further any source of transverse flow such as sweep or wingtip lift-induced vortices, such that the assumption of purely streamwise flow can be made (there is no spanwise flow). Take the kinematic viscosity of air at sea level as $\nu = 1.4531 \cdot 10^{-5}\,\mathrm{m^2/s}$.

1. (2p) Find the projected wing surface $S$, the mean chord $\bar{l}$ and the Reynolds number based on half the span $Re_{b/2}$.

2. (2p) Compute the root and tip chords and express the nondimensional chord $\hat{l}(\hat{z}) = l(z)/(b/2)$ as a function of the nondimensional spanwise coordinate $\hat{z} = z/(b/2)$.

3. (2p) Express the friction drag coefficient $C_D$ of both wings combined, based on the total projected surface, as an integral along the nondimensional spanwise direction $\hat{z}$ of the nondimensional trailing edge momentum thickness $\hat{\theta}_{te}(\hat{z}) = \theta_{te}(z)/(b/2)$.

4. (2p) Assuming a fully turbulent boundary layer over the whole wing surface, express the nondimensional trailing edge momentum thickness $\hat{\theta}_{te}(\hat{z})$ as a function of the nondimensional spanwise coordinate $\hat{z}$.

5. (2p) Compute the Honda Jet wing friction drag coefficient $C_D$.

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# Solutions

1.
$$S = \frac{b^2}{\lambda} = 17.367\,\mathrm{m}^2 \qquad \bar{l} = \frac{S}{b} = \frac{b}{\lambda} = 1.429\,\mathrm{m} \qquad Re_{b/2} = \frac{b\,u_\infty}{2\,\nu} = 4.1807 \cdot 10^7$$

2.
$$l_t = \frac{2\,b\,t}{\lambda\,(t+1)} = 0.787\,\mathrm{m} \qquad l_r = \frac{2\,b}{\lambda\,(t+1)} = 2.072\,\mathrm{m}$$
$$\hat{l}(\hat{z}) = \frac{l(z)}{b/2} = \frac{4}{\lambda\,(t+1)}\left[(t-1)\,\frac{\hat{z}}{b} + 1\right] = -0.21142\,\hat{z} + 0.34101$$

3.
$$C_D = \frac{8\,\lambda}{b^2}\int_0^{b/2} \theta_{te}(z)\,dz = 2\,\lambda\int_0^1 \hat{\theta}_{te}(\hat{z})\,d\hat{z} = 17\int_0^1 \hat{\theta}_{te}(\hat{z})\,d\hat{z}$$

4.
$$\begin{aligned}\hat{\theta}_{te}(\hat{z}) &= \frac{\theta_{TE}(z)}{b/2} = \frac{0.0221}{Re_{l(z)}^{1/6}}\frac{l(z)}{b/2} = \frac{0.0221}{Re_{b/2}^{1/6}}\hat{l}(\hat{z})^{5/6} = \left(\frac{0.01031}{Re_{b/2}^{1/5}}\,\hat{l}(\hat{z})\right)^{5/6} = \\ &= \left(-6.518955 \times 10^{-5}\,\hat{z} + 1.051435 \times 10^{-4}\right)^{5/6}\end{aligned}$$

5.
$$C_D = 0.07654\left(\frac{\lambda}{Re_{b/2}}\right)^{1/6}\frac{t^{11/6} - 1}{(t-1)(t+1)^{5/6}} = 6.0102 \cdot 10^{-3}$$

**Aerodynamics & Flight Mechanics** **EETAC**

**Q1**, 14/01/2016 **Final Exam**

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## Aerodynamics: Boundary layer Problem

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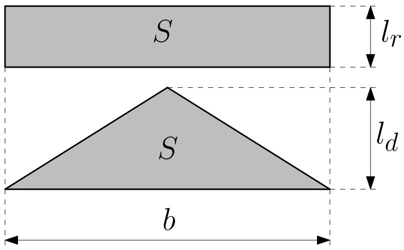
Consider the incompressible flow around two wings flying at speed $u_\infty$ in an atmosphere of density $\rho$ and kinematic viscosity $\nu$. Both wings are characterised by the same wingspan $b$ and projected wing area $S$, and only differ in their planform shape: one is rectangular while the other is a triangular delta wing (see the figure below).




Assume that both wings can be approximated by flat plates, of the corresponding wing planform shape, at zero angle of attack. Neglect further any source of transverse flow such as sweep or wingtip lift-induced vortices, such that the assumption of purely streamwise flow can be made (there is no spanwise flow). The boundary layer can be considered fully laminar.

1. (2p) Express the skin friction coefficient ($C_f$) for a generic flat plate of chord $l$, as a function of $x/l$ and $Re_l \equiv \frac{u_\infty l}{\nu}$.

2. (2p) Integrate the wall shear stress along the chord to find the drag per unit span $D/b$ as a function of $l$ and $Re_l$.

3. (1p) Use the previous result to calculate the friction drag coefficent for the rectangular wing $C_D^{\text{rect}}$.

4. (2p) Compute the momentum thickness $\theta$ at the trailing edge and show its relation with the drag coefficient.

5. (1p) Express $l = l(z)$ for the delta wing in terms of $\bar{l} = S/b$.

6. (2p) Use the result from section 2 to calculate the friction drag coefficient for the delta wing $C_D^{\text{delta}}$. Beware that $l = l(z)$ now, and that integration along the span is required.

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## Solutions

1. The friction coefficient can be looked up at the table for flat plate boundary layer characteristic properties as

$$C_f = \frac{k_1}{Re_x^{s_0}} = \frac{k_1}{Re_l^{s_0}} \left(\frac{x}{l}\right)^{-s_0}$$

2. The drag per spanwise unit results from simple integration along the chord of the wall shear stress with $u_e = u_\infty$ for a flat plate.

$$D/b = \int_0^l \tau_w \, dx = \int_0^l \frac{1}{2}\rho u_e^2 C_f \, dx = \frac{1}{2}\rho u_\infty^2 \frac{k_1}{Re_l^{s_0}} \int_0^l \left(\frac{x}{l}\right)^{-s_0} dx = \frac{1}{2}\rho u_\infty^2 \frac{k_1 l}{Re_l^{s_0}\,(1-s_0)}$$

3. Using $l \equiv l_r$, the drag coefficient is readily computed as

$$C_D^{\text{rect}} = \frac{2(D/b)b}{\frac{1}{2}\rho u_\infty^2 S} = \frac{2k_1}{Re_{l_r}^{s_0}\,(1-s_0)},$$

where $S = \bar{l}b$ and the 2 factor in the numerator comes from the fact that the wing is double-sided.

4. Using again the tables, we find

$$\frac{\theta}{l} = \frac{\theta}{x}\frac{x}{l} = \frac{k_1'}{Re_l^{s_0}}\frac{x}{l}$$

At the trailing edge, this is

$$\frac{\theta(l_r)}{l_r} = \frac{k_1'}{Re_{l_r}^{s_0}} = C_D^{\text{rect}} \frac{k_1'\,(1-s_0)}{2k_1} = C_D^{\text{rect}}/4,$$

exactly as seen in class for a double-sided flat plate.

5. The expression for the span-depending chord is a straight line of the form

$$l(z) = l_d \left(1 - \frac{z}{b/2}\right) \qquad \text{for } z \in [0, b/2],$$

the other half being symmetric. To relate $l_d$ with $l_r$ all that is needed is computing the average along the span:

$$l_r = S/b = \bar{l} = \frac{1}{b/2}\int_0^{b/2} l(z)\,dz = \frac{l_d}{b/2}\int_0^{b/2}\left(1 - \frac{z}{b/2}\right)dz = \frac{l_d}{2} \qquad \longrightarrow \qquad l/l_r = 2\left(1 - \frac{z}{b/2}\right)$$

6. To compute the drag coefficient, now an integration along the span is required

$$\begin{aligned} C_D^{\text{delta}} &= \frac{2}{\frac{1}{2}\rho u_\infty^2 S}\, 2\int_0^{b/2} (D/b)\,dz = \frac{4}{S}\frac{k_1}{(1-s_0)}\int_0^{b/2} \frac{l(z)}{Re_{l(z)}^{s_0}}\,dz = \\ &= \frac{4\,l_r}{S Re_{l_r}^{s_0}}\frac{k_1}{(1-s_0)}\int_0^{b/2} (l/l_r)^{1-s_0}\,dz = \frac{4}{b Re_{l_r}^{s_0}}\frac{k_1}{(1-s_0)}\int_0^{b/2}\left[2\left(1-\frac{z}{b/2}\right)\right]^{1-s_0} dz = \\ &= \frac{2k_1}{Re_{l_r}^{s_0}\,(1-s_0)}\frac{2^{1-s_0}}{2-s_0} = C_D^{\text{rect}}\frac{2^{1-s_0}}{2-s_0}, \end{aligned}$$

where the second 2 factor corresponds to the other half wing (symmetric), given that we have integrated only over half the span.

**Aerodynamics & Flight Mechanics** **EETAC**
**Q2**, 05/06/2015 **Final Exam**

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Aerodynamics: Boundary layer Problem

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Experimental results on a flat plate boundary layer show that the effects of free-stream preturbulence levels ($Tu$) on the natural laminar-turbulent transition can be modelled by the following law:

$$Re_{\theta_T} = \frac{u_e \theta_T}{\nu} = 201 - 206\,\log(16.8\,Tu),$$

where log is the natural logarithm, $u_e = U_\infty$ is the free stream velocity and $\theta_T$ is the boundary layer momentum thickness at transition. Remember that the drag coefficient for a flat plate with transition at the nondimensional coordinate $\bar{x}_T = x_T/L$, with $L$ the chord, is:

$$\left(\frac{C_D}{2}\right)^{6/5} = \frac{0.61191}{Re_L^{3/5}}\,\bar{x}_T^{3/5} + \frac{0.01031}{Re_L^{1/5}}\,(1 - \bar{x}_T).$$

1. (3p) Using the boundary layer results for a flat plate, find a relation between $\bar{x}_T$ and $Re_{\theta_T}$, and estimate the transition point $\bar{x}_T$ for $Tu = 0.001, 0.002$ and $Re_L = 3 \cdot 10^6, 10^7$.
2. (2p) Assess the drag coefficient $C_D$ for the case $Tu = 0.001$ and $Re_L = 10^7$. [use $\bar{x}_T = 0.2$ from here on if you got stuck in the first question]

With the intention of reproducing the natural transition experimental results for $Tu = 0.001$ and $Re_L = 10^7$ but at the lower $Re_L = 3 \cdot 10^6$, the laminar boundary layer is tripped by a wire placed at a convenient position. The new criterion for immediate transition at the wire location, which overrides natural transition, is

$$Re_{d_w} = \frac{u_e d_w}{\nu} = 900,$$

where $d_w$ is the diameter of the wire.

3. (3p) Assess what the diameter of a wire must be ($d_w/\theta_T$) to trigger transition at $Re_L = 3 \cdot 10^6$ when placed at the $\bar{x}_T$ where transition occurred naturally for $Tu = 0.001$ and $Re_L = 10^7$.
4. (2p) What is the corresponding drag coefficient $C_D$? Does inducing transition at the same point result in the same drag coefficient as for natural transition?

---

## Solutions

1. The flat plate laminar boundary layer states that

$$\frac{\theta}{x} = \frac{Re_\theta}{Re_x} = \frac{0.66411}{Re_x^{1/2}} \qquad \longrightarrow \qquad Re_x = \frac{Re_\theta^2}{0.44104},$$

which at the transition point demands that

$$Re_{x_T} = Re_L \left(\frac{x_T}{L}\right) = \frac{Re_{\theta_T}^2}{0.44104} \qquad \longrightarrow \qquad \bar{x}_T = \frac{x_T}{L} = 2.26736\frac{Re_{\theta_T}^2}{Re_L}.$$

Substituting the values for $Tu$ and $Re_L$ provided, the transition point results in

| $\bar{x}_T$ | Tu | |
|---|---|---|
| $Re_L$ | 0.001 | 0.002 |
| $3 \cdot 10^6$ | 0.8219 | 0.6122 |
| $10^7$ | 0.2466 | 0.1866 |

As expected, the higher the $Re_L$ and $Tu$, the earlier the transition.

2. Picking the case $(Re_L, Tu) = (10^7, 0.001)$ and substituting the corresponding $\bar{x}_T$ into the expression for the drag coefficient, we obtain

$$C_D = 2.485 \cdot 10^{-3}$$

3. Natural transition for $(Re_L, Tu) = (10^7, 0.001)$ occurs at $\bar{x}_T = 0.2466$, while for $(Re_L, Tu) = (3 \cdot 10^6, 0.001)$ it is retarded to $\bar{x}_T = 0.8219$. Now we will artificially advance transition at $Re_L = 3 \cdot 10^6$ back to $\bar{x}_T = 0.2466$ by placing a wire at that exact location. The diameter of the wire required is given by

$$\frac{d_w}{\theta_T} = \frac{Re_{d_w}}{Re_{\theta_T}} = \frac{900}{Re_{\theta_T}} = \frac{1355.2}{(Re_L \bar{x}_T)^{1/2}} = 1.5757,$$

or, in terms of the displacement thickness, $(d_w/\delta_{1_T}) = (d_w/\theta_T)/H = 0.6082$ ($H = 2.591$ is the laminar boundary layer form factor for the flat plate). The wire must protrude 1.5757 times the momentum thickness to trigger transition.

4. The corresponding drag coefficient, now with $Re_L = 3 \cdot 10^6$ is

$$C_D = 3.117 \cdot 10^{-3}.$$

Despite inducing transition at the same location, the drag coefficent is higher because the $Re_L$ is lower. To have the same drag coefficient as for natural transition we would have to place the wire further downstream. It is therefore impossible to reproduce both transition location and drag coefficient by tripping the boundary layer at a lower Reynolds number.

## Aerodynamics: Boundary layer Problem

In designing a 2D wind tunnel, the diffuser downstream from the test section (see Figure 5) has to reduce the air velocity by a factor $\bar{u}_o = u_o/u_i$. The height of the test section is $h_i$.
The diffuser, which can be considered quasi-one-dimensional, increases its height smoothly and linearly. Distances will be non-dimensionalised with $h_i$ and velocities with $u_i$, such that $\bar{h} = h/h_i$, $\bar{x} = x/h_i$, $\bar{l} = l/h_i$ and $\bar{u} = u/u_i$.

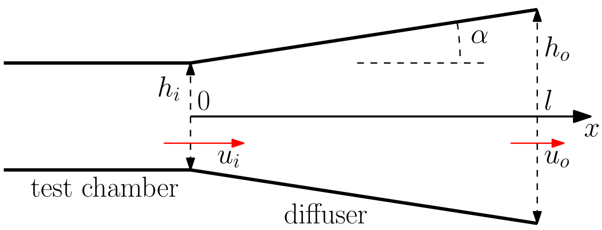


Figure 5: Sketch of the wind tunnel diffuser.

1. (1p) Express the evolution of the non-dimensional diffuser height $\bar{h}(\bar{x})$ in terms of geometric parameters.
2. (1p) Use mass conservation to express the evolution of the non-dimensional velocity $\bar{u}(\bar{x})$ in terms of $\bar{u}_o$ and $\bar{l}$.

A boundary layer trap at the entrance of the diffuser ensures that the boundary layer on the diffuser walls starts with negligible thickness and turbulent.

3. (2p) Non-dimensionalise the integral method.
4. (2p) Apply the non-dimensional integral method to express the momentum thickness at the exit of the diffuser $\bar{\theta}_o$ in terms of $Re_{h_i}$, $\bar{l}$ and $\bar{u}_o$.
5. (1p) Non-dimensionalise the following separation criterion for turbulent boundary layers:

$$-\frac{\theta}{u}\frac{du}{dx} = C = 0.0035$$

6. (2p) Use this criterion to determine the minimum length $\bar{l}$ of the diffuser to avoid separation.
7. (1p) Assuming $u_i = 30\,\mathrm{m/s}$, $h_i = 0.5\,\mathrm{m}$, $\bar{u}_o = 0.48$ and sea level ISA atmospheric conditions, calculate the length $l$ of the diffuser and the expansion semiangle $\alpha$.

# Solutions

1. Since the diffuser height grows linearly, we have:

$$\frac{h(x)-h_i}{x} = \frac{h_o-h_i}{l} \qquad \longrightarrow \qquad h(x) = h_i + (h_o-h_i)\frac{x}{l} \qquad \longrightarrow \qquad \bar{h}(\bar{x}) = 1 + (\bar{h}_o - 1)\frac{\bar{x}}{\bar{l}}$$

2. Mass conservation demands that $u_i h_i = u_o h_o = u(x)h(x)$. Hence,

$$\bar{u}_o\bar{h}_o = \bar{u}(\bar{x})\bar{h}(\bar{x}) = 1 \qquad \longrightarrow \qquad \bar{u}(\bar{x}) = \frac{\bar{u}_o}{\bar{u}_o + (1-\bar{u}_o)\dfrac{\bar{x}}{\bar{l}}}$$

3. Non-dimensionalising the integral method for a turbulent boundary layer with $m_0 = 0.2$, $H = 1.4$ and $b = 0.0086$, and assuming the boundary layer starts with negligible thickness, we obtain:

$$\bar{\theta}^{1.2}\bar{u}^{4.08} = \frac{0.01032}{Re_{h_i}^{0.2}}\int_0^{\bar{x}} \bar{u}^{3.88} d\bar{x}$$

4. Substituting $\bar{u}(x)$ into the above expression and integrating between 0 and $\bar{l}$, we get

$$\bar{\theta}_o^{1.2} = \frac{0.0035833\,\bar{l}}{Re_{h_i}^{0.2}\bar{u}_o^{3.08}}\,\frac{1-\bar{u}_o^{2.88}}{1-\bar{u}_o}$$

5. Straightforward non-dimensionalisation of the separation criterion yields

$$-\frac{\theta}{u}\frac{du}{dx} = -\frac{\bar{\theta}}{\bar{u}}\frac{d\bar{u}}{d\bar{x}} = C$$

6. Evaluating the above expression at the diffuser exit, we have

$$\left[-\frac{\bar{\theta}}{\bar{u}}\frac{d\bar{u}}{d\bar{x}}\right]_{\bar{l}} = \left[\bar{\theta}\frac{1-\bar{u}_o}{\bar{l}\left(\bar{u}_o + (1-\bar{u}_o)\dfrac{\bar{x}}{\bar{l}}\right)}\right]_{\bar{l}} = \frac{\bar{\theta}_o}{\bar{l}}\left(1-\bar{u}_o\right) = \frac{0.009162}{Re_{h_i}^{1/6}\bar{u}_o^{77/30}}\left(1-u_o^{2.88}\right)^{5/6}\left(\frac{1-\bar{u}_o}{\bar{l}}\right)^{1/6} = C$$

Isolating the diffuser length for separation at exit, we get

$$\bar{l} = \left(\frac{0.062403}{C}\right)^6 \frac{\left(1-\bar{u}_o\right)\left(1-u_o^{2.88}\right)^5}{Re_{h_i}\bar{u}_o^{77/5}}$$

7. Finally, substituting the numerical values, we obtain

$$\bar{l} = 6.931 \qquad \longrightarrow \qquad l = 3.47\,\mathrm{m},$$

which corresponds to an expansion semiangle

$$\alpha = \arctan\left(\frac{h_o-h_i}{2\,l}\right) = \arctan\left(\frac{1-\bar{u}_o}{2\,\bar{l}\,\bar{u}_o}\right) = 4.47^\circ$$

## Aerodynamics: Boundary layer Problem

With the aim of measuring wind velocity ($U_\infty$) in an airport runway, a system based on friction drag has been devised. The system, shown in Fig. 8, is based on a ground-level-mounted tile. The tile neither protrudes nor sinks, and is anchored to a very sensitive, high-precision underground balance that measures the tangential force exerted on it by wind friction. The tile is a flat plate

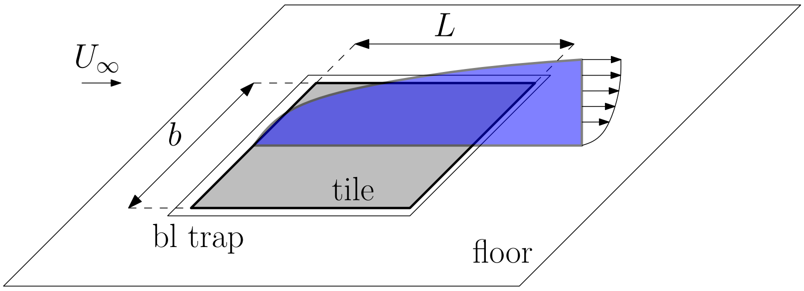


Figure 6: Friction-based anemometer.

of dimensions $L$ (chord) and $b$ (span) and a boundary layer trap in the ground just before the tile forces the boundary layer to start with no thickness and already turbulent at onset. The density $\rho$ and kinematic viscosity $\nu$ of air take the standard ISA sea level values (do not substitute with values until section 3).

1. (2p) Use the flat plate turbulent boundary expression for the friction coefficient ($C_f$) to express the wall shear stress ($\tau_w$) in an arbitrary position on the tile.
2. (4p) Integrate the wall shear stress on the tile to express friction ($D$) drag in terms of the tile dimensions ($L, b$), air properties ($\rho, \nu$) and a Reynolds number based on the tile chord and wind speed ($Re_L = U_\infty L/\nu$).
3. (2p) If $L = b = 1\,\mathrm{m}$ and the balance measures a friction drag $D = 0.5\,\mathrm{N}$, what should the wind speed reading be ($U_\infty$)?
4. (2p) Does the mantra "drag is proportional to airspeed squared" hold? Where's the catch?

## Solutions

1. The friction law of a turbulent boundary layer developing on a flat plate follows:
$$C_f = \frac{0.0368}{Re_x^{1/6}}$$
with $Re_x = u_e x/\nu = U_\infty x/\nu$. The wall shear stress on an arbitrary point on the flat plate can be written
$$\tau_w(x,z) = \tau_w(x) = \rho_e u_e(x)^2 \frac{C_f}{2}(x) = \rho\, U_\infty^2 \frac{C_f}{2}(x),$$
with the origin placed at the flat plate midspan leading edge, and where we have used that on a flat plate $u_e(x) = U_\infty$ and that there is no $z$-dependence.

2. Computing friction drag is just a matter of integrating wall shear stress over the surface:
$$D = \iint_S \tau_w\, dS = \int_{-b/2}^{b/2}\int_0^L \tau_w(x,z)\, dx\, dz = b\int_0^L \rho\, U_\infty^2 \frac{0.0184}{Re_x^{1/6}}\, dx = 0.0184\, b\, \rho\, U_\infty^2 \int_0^L Re_x^{-1/6}\, dx$$
To integrate, either we write $Re_x = U_\infty x/\nu$ or we can procede with a change of variables such that $dRe_x = U_\infty dx/\nu$:
$$D = 0.0184\, b\, \rho\, U_\infty \nu \int_0^{Re_L} Re_x^{-1/6}\, dRe_x = 0.0184\, b\, \rho\, U_\infty \nu \frac{6}{5}\left[Re_x^{5/6}\right]_0^{Re_L} = 0.02208\, b\, \rho\, U_\infty \nu\, Re_L^{5/6}$$
The $U_\infty$ dependence can be easily removed by combining it to form a $Re_L$:
$$D = 0.02208 \frac{b}{L}\, \rho\, \nu^2\, Re_L^{11/6}$$

3. Substituting $L = b = 1\,\mathrm{m}$, $\rho = 1.225\,\mathrm{kg/m^3}$, $\nu = 1.4604 \cdot 10^{-5}\,\mathrm{m^2/s}$ and $D = 0.5\,\mathrm{N}$, we can solve for $Re_L$:
$$Re_L = \left[\frac{D\, L}{0.02208\, b\, \rho\, \nu^2}\right]^{6/11} = 9.25 \cdot 10^5.$$
Finally, using the definition of $Re_L$, the wind velocity can be estimated:
$$U_\infty = \frac{Re_L\, \nu}{L} = 13.5\,\mathrm{m/s}.$$

4. If we substitute the $Re_L$ definition into the expression for drag we obtain:
$$D = 0.02208\, b\, L^{5/6}\, \nu^{1/6}\, \rho\, U_\infty^{11/6},$$
which clearly shows a $U_\infty^{11/6}$ dependence. Drag is therefore not proportional to airspeed squared. To understand this, lets build the $U_\infty^2$ dependence and see what is left behind:
$$D = 0.02208\, \rho\, U_\infty^2\, b\, L\, Re_L^{-1/6} = \frac{1}{2}\rho\, S\, U_\infty^2\, C_D,$$
where $S = b\, l$ is the area and
$$C_D = \frac{0.04416}{Re_L^{1/6}}$$
is the drag coefficient that clearly depends on $Re_L$, hence $U_\infty$. The airspeed squared dependence of drag is a simplification assuming a constant drag coefficient independent of airspeed, which is obviously not true but approximately acceptable over a reasonable $Re$-range.

As an aside, notice that, as repeateadly seen in class, $C_D = 2\dfrac{\theta(L)}{L}$.

**Aerodynamics & Flight Mechanics** **EETAC**

**Q1**, 21/01/2014 **Final Exam**

## Aerodynamics: Boundary layer Problem

A 2D airfoil is engaged in a horizontal sea level flight at a velocity $U_\infty = 80\,\mathrm{m/s}$. The chord of the airfoil is $l = 1.24\,\mathrm{m}$. The atmosphere is at rest (steady) and the air flow can be considered incompressible with density $\rho = 1.225\,\mathrm{kg/m^3}$. For the present flight conditions, the angle of attack is such that the pressure coefficient ($C_p$) can be considered as following a parabolic distribution along the chord on both the upper ($C_{p_u}$) and lower ($C_{p_l}$) surfaces (see figure 7 below).
The pressure coefficient distribution can be used, along with boundary layer theory, to estimate the friction drag on the airfoil. Consider the dynamic viscosity at current flight conditions to be $\mu = 1.78 \cdot 10^{-5}$ Pa s.

1. (2p) Use chord-averaged values of the pressure coefficient ($\overline{C}_{p_l}$ and $\overline{C}_{p_u}$) to estimate the mean flow velocities on the lower ($u_{e_l}$) and upper ($u_{e_u}$) surfaces.

Both the lower and upper surfaces will be further approximated as one-sided flat plates with the constant outer flow velocities just found. Assume also that the boundary layer starts with negligible thickness at the leading edge and that it is turbulent throughout.

2. (2p) Calculate the wall shear stress distribution along the chord ($\tau_{w_l}$ and $\tau_{w_u}$).

3. (2p) Integrate the wall shear stress to find the one-sided friction drag force for the lower and upper surfaces ($D_l$ and $D_u$).

4. (2p) Combine upper and lower surface results to produce the airfoil friction drag force ($D$). Calculate the friction drag coefficent ($C_D$).

5. (2p) Calculate upper and lower momentum ($\theta_{te_l}$ and $\theta_{te_u}$) thicknesses at the trailing edge. How are these thicknesses related to the friction drag coefficient?

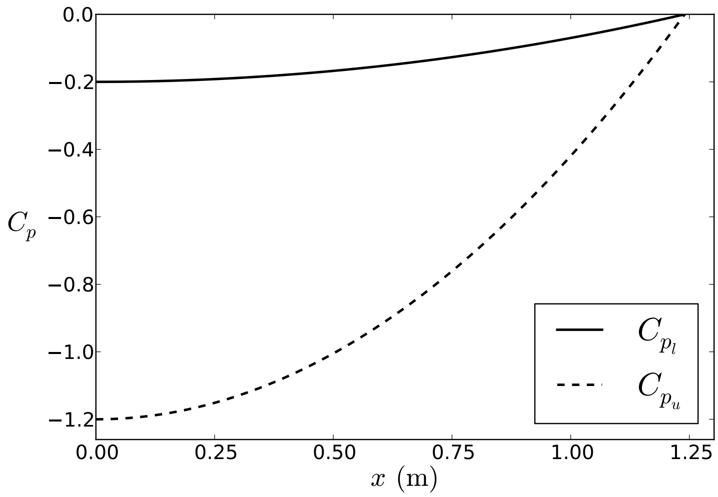


Figure 7: Assumed parabolic pressure coefficient distributions along the chord on the upper ($C_{p_u}(x)$) and lower ($C_{p_l}(x)$) surfaces for the studied airfoil. Both distributions present a suction peak right at the leading edge: $C_{p_u}(0) = -1.2$ and $C_{p_l}(0) = -0.2$.

## Solutions

1. First, we obtain a chord-averaged value for $C_p$:

$$\overline{C}_p = \frac{1}{l}\int_0^l C_p(x)\,dx = \frac{2}{3}C_{p0} \qquad \longrightarrow \qquad \overline{C}_{p_l} = -0.1\hat{3} \qquad \text{and} \qquad \overline{C}_{p_u} = -0.8$$

Using bernoulli, we obtain mean velocity values for the upper and lower surfaces:

$$C_P = \frac{p - p_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - \left(\frac{u}{U_\infty}\right)^2 \qquad \longrightarrow \qquad u = U_\infty\sqrt{(1 - C_p)}$$

$$u_{e_l} = 1.0646\,U_\infty = 85.17\,\text{m/s} \qquad \text{and} \qquad u_{e_u} = 1.3416\,U_\infty = 107.33\,\text{m/s}$$

2. Using the tabulated values for turbulent boundary layer friction coefficients we obtain:

$$\tau_w = \frac{1}{2}\rho u_e^2 C_f = \frac{1}{2}\rho u_e^2 \frac{0.0368}{Re_x^{1/6}} = 3.521\cdot\, 10^{-3}\, u_e^{11/6} x^{-1/6}$$

$$\tau_{w_l} = 12.176\, x^{-1/6} \qquad \text{and} \qquad \tau_{w_u} = 18.607\, x^{-1/6}$$

3. Friction drag per span unit is then computed by integrating along the chord:

$$D = \int_0^l \tau_w\,dx = \int_0^{1.24} 3.521\cdot\, 10^{-3}\, u_e^{11/6} x^{-1/6}\,dx = 5.0548\cdot\, 10^{-3}\, u_e^{11/6}$$

$$D_l = 17.5\,\text{N/m} \qquad \text{and} \qquad D_u = 26.7\,\text{N/m}$$

4. The total friction drag force is then calculated as the sum:

$$D = D_l + D_u = 44.2\,\text{N/m}$$

And the airfoil friction drag coefficient is

$$C_D = \frac{D}{\frac{1}{2}\rho l U_\infty^2} = 0.00909$$

5. From the tabulated data for a turbulent boundary layer we know that

$$\theta(x) = x \cdot \frac{0.0221}{Re_x^{1/6}} = 3.452 \cdot 10^{-3}\, u_e^{-1/6}\, x^{5/6}$$

At the trailing edge, we have

$$\theta_{te_l} = 1.969 \cdot 10^{-3}\,\text{m} \qquad \text{and} \qquad \theta_{te_u} = 1.895 \cdot 10^{-3}\,\text{m}$$

From theory, we know that for a one-sided flat plate we have

$$C_{D_{fp}} = \frac{D_{fp}}{\frac{1}{2}\rho l u_e^2} = 2\frac{\theta_{te}}{l}$$

We can relate this with the airfoil friction drag coefficient in the following way:

$$C_D = \frac{D}{\frac{1}{2}\rho l U_\infty^2} = \frac{D_l}{\frac{1}{2}\rho l U_\infty^2} + \frac{D_u}{\frac{1}{2}\rho l U_\infty^2} = C_{D_l}\left(\frac{u_{e_l}}{U_\infty}\right)^2 + C_{D_u}\left(\frac{u_{e_u}}{U_\infty}\right)^2$$

$$C_D = 2\frac{\theta_{te_l}}{l}\left(\frac{u_{e_l}}{U_\infty}\right)^2 + 2\frac{\theta_{te_u}}{l}\left(\frac{u_{e_u}}{U_\infty}\right)^2 = 3.463 \cdot 10^{-3} + 5.716 \cdot 10^{-3} = 0.00910$$

The small discrepancy is due to truncation in the tabulated values.

**Aerodynamics & Flight Mechanics** **EETAC**

**Q2**, 28/06/2013 **Final Exam**

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## Aerodynamics: Boundary layer Problem

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The velocity distribution of the flow around a symmetric airfoil at zero angle of attack as measured on a wind tunnel can be approximated by

$$u(x) = \begin{cases} u_m \left[\sin\left(\dfrac{\pi x}{2x_m}\right)\right]^{1/3.88} & x \leq x_m \\ u_m \left[\cos\left(\dfrac{\pi(x-x_m)}{2(R-x_m)}\right)\right]^{1/3.88} & x_m < x \leq L \end{cases}$$

where $L$ is the chord, $R$ is a parameter related to the recompression length, $x_m$ is the coordinate for maximum velocity and $u_m$ is the maximum velocity. The upstream velocity is $U_\infty$.

1. Nondimensionalise the velocity distro with $U_\infty$ and $L$ to express $\overline{u} = u/U_\infty$ as a function of $\overline{x} = x/L$. Use the nondimensional parameters $\overline{u}_m = u_m/U_\infty$, $\overline{x}_m = x_m/L$ and $\overline{R} = R/L$. See Fig. 8.

2. Assume a turbulent boundary layer throughout and nondimensionalise the approximate integral method so that it is expressed in terms of $\overline{\theta} = \theta/L$, $\overline{u} = u/U_\infty$ and $Re_L = U_\infty L/\nu$.

From here on use the values $\overline{x}_m = 0.25$, $\overline{u}_m = 1.2$, $\overline{R} = 1.1$ and $Re_L = 10^7$.

3. Use the integral method to find the boundary layer momentum thickness $\overline{\theta}_m$ at the coordinate for maximum velocity $\overline{x}_m$. Assume the boundary layer starts with no thickness at $\overline{x} = 0$.

4. Use the integral method to find the boundary layer momentum thickness at the trailing edge $\overline{\theta}_1 = \overline{\theta}(1)$ of the airfoil.

5. Use the Squire & Young formula to calculate the airfoil's friction drag coefficient: $C_D = 4\,\overline{\theta}_1\,\overline{u}_1^{3.2}$.

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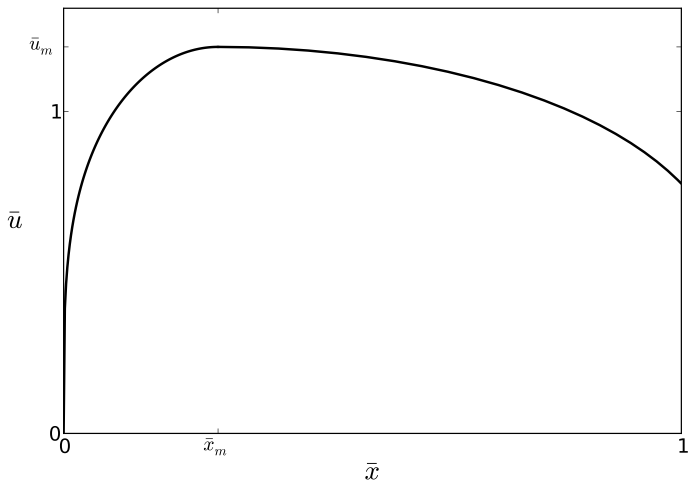


Figure 8: Nondimensional velocity distribution around the airfoil.

## Solutions

1. The velocity distribution easily nondimensionalises to

$$\overline{u}(\overline{x}) = \begin{cases} \overline{u}_m \left[\sin\left(\dfrac{\pi\overline{x}}{2\overline{x}_m}\right)\right]^{1/3.88} & \overline{x} \leq \overline{x}_m \\ \overline{u}_m \left[\cos\left(\dfrac{\pi(\overline{x}-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)\right]^{1/3.88} & \overline{x}_m < \overline{x} \leq 1 \end{cases}$$

2. To nondimensionalise the approximate integral method we procede as usual:

$$\left[\theta u^{H+2}\right]_x^{m_0+1} = \left[\theta u^{H+2}\right]_{x_0}^{m_0+1} + (m_0+1)\, b \int_{x_0}^{x} \frac{u^{(H+2)(m_0+1)}}{(u/\nu)^{m_0}}\, dx \qquad \text{with} \begin{cases} m_0 = 1/5 \\ b = 0.0086 \\ H = 1.4 \end{cases}$$

$$\overline{\theta}^{1.2}\overline{u}^{4.08} = \overline{\theta}_0^{1.2}\overline{u}_0^{4.08} + \frac{1.2\cdot 0.0086}{L^{1.2}U_\infty^{4.08}}\nu^{0.2} L U_\infty^{3.88}\int_{\overline{x}_0}^{\overline{x}} \overline{u}^{3.88}\, d\overline{x} = \overline{\theta}_0^{1.2}\overline{u}_0^{4.08} + \frac{0.01032}{Re_L^{0.2}}\int_{\overline{x}_0}^{\overline{x}} \overline{u}^{3.88}\, d\overline{x}$$

3. To find the momentum thickness at the maximum velocity point, we need only apply the integral method between the leading edge (with negligible thickness as hypothesised: $\overline{\theta}_0 = \overline{\theta}(0) = 0$) and the point we are interested in ($\overline{\theta}_m = \overline{\theta}(\overline{x}_m)$):

$$\begin{aligned} \overline{\theta}_m^{1.2}\overline{u}_m^{4.08} &= \cancel{\overline{\theta}_0^{1.2}\overline{u}_0^{4.08}} + \frac{0.01032}{Re_L^{0.2}}\int_0^{\overline{x}_m} \overline{u}^{3.88}\, d\overline{x} = \frac{0.01032}{Re_L^{0.2}}\int_0^{\overline{x}_m} \overline{u}_m^{3.88}\sin\left(\frac{\pi\overline{x}}{2\overline{x}_m}\right) d\overline{x} = \\ &= \frac{0.01032}{Re_L^{0.2}}\frac{2\overline{u}_m^{3.88}\overline{x}_m}{\pi}\left[-\cos\left(\frac{\pi\overline{x}}{2\overline{x}_m}\right)\right]_0^{\overline{x}_m} = \frac{0.00657}{Re_L^{0.2}}\overline{u}_m^{3.88}\overline{x}_m \end{aligned}$$

$$\overline{\theta}_m = \frac{0.01518}{\overline{u}_m^{1/6} Re_L^{1/6}}\overline{x}_m^{5/6} \qquad \longrightarrow \qquad \boxed{\overline{\theta}_m = 3.16\cdot 10^{-4}}$$

4. The integral method can be applied again downstream from the maximum velocity point to calculate the momentum thickness at the trailing edge:

$$\begin{aligned}
\overline{\theta}_1^{1.2}\overline{u}_1^{4.08} &= \overline{\theta}_m^{1.2}\overline{u}_m^{4.08} + \frac{0.01032}{Re_L^{0.2}}\int_{\overline{x}_m}^{1}\overline{u}^{3.88}\,d\overline{x} = \overline{\theta}_m^{1.2}\overline{u}_m^{4.08} + \frac{0.01032}{Re_L^{0.2}}\int_{\overline{x}_m}^{1}\overline{u}_m^{3.88}\cos\left(\frac{\pi(\overline{x}-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)d\overline{x} = \\
&= \frac{0.00657}{Re_L^{0.2}}\overline{u}_m^{3.88}\overline{x}_m + \frac{0.01032}{Re_L^{0.2}}\frac{2\overline{u}_m^{3.88}(\overline{R}-\overline{x}_m)}{\pi}\left[\sin\left(\frac{\pi(\overline{x}-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)\right]_{\overline{x}_m}^{1} = \\
&= \frac{0.00657}{Re_L^{0.2}}\overline{u}_m^{3.88}\left[\overline{x}_m + (\overline{R}-\overline{x}_m)\sin\left(\frac{\pi(1-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)\right]
\end{aligned}$$

$$\overline{\theta}_1 = \frac{0.01518}{\overline{u}_m^{1/6}Re_L^{1/6}}\frac{\left[\overline{x}_m + (\overline{R}-\overline{x}_m)\sin\left(\frac{\pi(1-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)\right]^{5/6}}{\left[\cos\left(\frac{\pi(1-\overline{x}_m)}{2(\overline{R}-\overline{x}_m)}\right)\right]^{3.4/3.88}} \qquad \longrightarrow \qquad \boxed{\overline{\theta}_1 = 4.74\cdot 10^{-3}}$$

5. Finally, the Squire & Young formula (for a symmetric airfoil at $\alpha = 0$) allows calculation of the friction drag coefficient for this airfoil:

$$C_D = 4\,\overline{\theta}_1\,\overline{u}_1^{3.2} \qquad \longrightarrow \qquad \boxed{C_D = 8.40\cdot 10^{-3}}$$

$\infty$. For the sake of completeness, the momentum thickness distribution can be easily computed from previous paragraphs and is shown in Fig. 9. The momentum thickness grows slowly until the coordinate for maximum velocity due to the flow being accelerated. From that point on, the recompression starts and the boundary layer grows much faster, especially towards the last quarter, where the recompression becomes stronger.

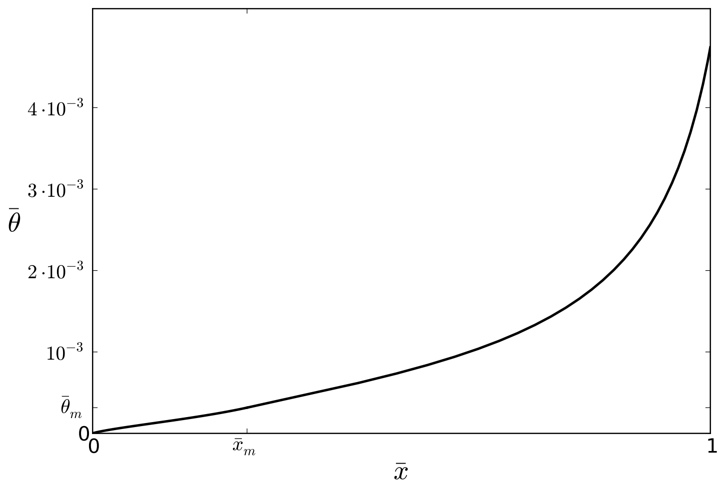


Figure 9: Nondimensional momentum thickness distribution along the airfoil chord.