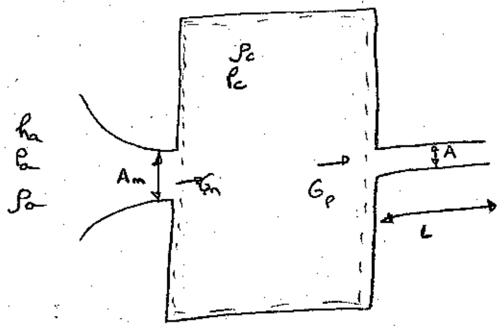
Problema 1 :



1.) $\frac{d}{dt}\int \rho \cdot dV + \int \rho \cdot \vec{v}\cdot\vec{n}\, d\sigma = 0$

$$\frac{d}{dt}\left(V \cdot \rho_c\right) = G_m - G_p$$

$$\frac{d}{dt}\left(\rho\left(e + \cancel{\frac{v^2}{2}}^{\,0}\right)V\right) = G_m\, h_{o,m} - G_p\, h_{o,p}$$ (1)

$$\Downarrow$$

$$\frac{d}{dt}\left(\frac{V}{\gamma-1}\cdot p_c\right) = G_m \cdot h_a - G_p \cdot h_{o,c}$$

Cálculo gasto de salida.

$\frac{p_c}{p_o} \to \infty \quad \to$ Tobera Bloqueada $\left.\begin{matrix} M(L) = 1 \\ f\, L/D = 1 \end{matrix}\right\} \Rightarrow M(0) = 0'6$ (1)

$$G_p = \rho(0)\cdot u(0)\cdot A = \rho(L)\cdot u(L)\cdot A$$

$$G_p = \rho_c \cdot a_c \cdot A \cdot \frac{\rho(0)}{\rho_c}\cdot\frac{u(0)}{a_c} = \rho_c\cdot a_c \cdot A \cdot \underbrace{M(0)\cdot\left(1 + \frac{\gamma-1}{2}M^2(0)\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}_{G = 0'487}$$

Cálculo gasto de entrada.

$\frac{p_a}{p_c} > \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \to G_m = G_m^* = \rho_a \cdot a_a \cdot A_m \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ Bloqueada (1)

$\frac{p_a}{p_c} < \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \to G_m = \rho_a \cdot a_a \cdot A_m \left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{p_a}{p_c}\right)^{-\frac{1+\gamma}{2\gamma}}\left[\left(\frac{p_a}{p_c}\right)^{\frac{\gamma-1}{\gamma}}-1\right]^{1/2}$ No bloqueada

Resolvemos para encontrar $p_c, \rho_c, G_m, G_p \to$

$$V\cdot\frac{d\rho_c}{dt} = \rho_a a_a A_m\left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{p_a}{p_c}\right)^{-\frac{1+\gamma}{2\gamma}}\left[\left(\frac{p_a}{p_c}\right)^{\frac{\gamma-1}{\gamma}}-1\right]^{1/2} - \rho_c a_c A\, G$$

$$\frac{V}{\gamma-1}\cdot\frac{dp_c}{dt} = h_a\, \rho_a a_a A_m\left(\frac{2}{\gamma-1}\right)^{1/2}\left(\frac{p_a}{p_c}\right)^{-\frac{1+\gamma}{2\gamma}}\left[\left(\frac{p_a}{p_c}\right)^{\frac{\gamma-1}{\gamma}}-1\right]^{1/2} - h_c\, \rho_c a_c A\, G$$

$t = 0 \quad \rho_c = \rho_c(t=0)$
$\qquad\;\; p_c = p_c(t=0)$

2.) $t \to \infty$ Estacionario $\quad G_m = G_p \quad h_a = h_{c,0} \Rightarrow \boxed{T_a = T_c}$ (1)

$$\rho_a \cdot a_a \cdot A_m\left(\frac{2}{\gamma-1}\right)^{1/2}\cdot\left(\frac{p_a}{p_c}\right)^{-\frac{1+\gamma}{2\gamma}}\left[\left(\frac{p_a}{p_c}\right)^{\frac{\gamma-1}{\gamma}}-1\right]^{1/2} = \rho_c \cdot a_c \cdot A \cdot G$$ (1)

$\Rightarrow$ Despejamos $p_c \Rightarrow$ Calculamos ρ_c

$$\frac{p_c}{\rho_c} = R_g \cdot T_c = R_g \cdot T_a = \frac{p_a}{\rho_a}$$

Problema 2:

1.) $\rho \cdot \frac{U^2}{L} \gg \mu \frac{U}{R_0^2}$ $\qquad$ $\rho \cdot \frac{U}{t_c} \ll \rho \cdot \frac{U^2}{L}$

$$\boxed{Re \sim \frac{\rho U R_0}{\mu} \cdot \frac{R_0}{L} \gg 1}$$

$$\boxed{S_t \sim \frac{\rho \cdot U \cdot t}{\rho L} \sim \frac{\cancel{\rho} \cdot V_0 / \cancel{\rho} A_0}{L} \sim \frac{V_0}{L \cdot A_0} \gg 1}$$

1

2.) $\cancel{p_a} + \cancel{\rho} \cdot g \cdot H = \cancel{p_a} - \cancel{\rho} \cdot g \cdot h + \frac{1}{2} \cancel{\rho} \cdot U^2$ $\qquad h = \sqrt{L^2 - R_0^2}$

$$\frac{U^2}{2} = g \cdot (H + h)$$

1

$$U = \left[2g \cdot \left(H + \left(L^2 - R_0^2\right)^{1/2}\right)\right]^{1/2}$$

$$\boxed{Q = U \cdot A_0 = A_0 \cdot \left[2g \cdot \left(H + \left(L^2 - R_0^2\right)^{1/2}\right)\right]^{1/2}}$$

3) $\frac{d}{dt} \int \rho \cdot dV = -\rho Q$

$$\frac{d}{dt}\left(\cancel{\rho} \cdot \pi R^2 H\right) = -A_0 \cancel{\rho} \left[2g \cdot \left(H + \left(L^2 - R_0^2\right)^{1/2}\right)\right]^{1/2}$$

1,5

$$\int dH \cdot \left(H + (L^2 - R_0^2)^{1/2}\right)^{-1/2} = \int -\frac{A_0}{\pi R^2} \sqrt{2g}\, dt$$

$$2 \cdot \sqrt{H + (L^2 - R_0^2)^{1/2}} \Bigg|_{H_0}^{H} = -\frac{A_0}{\pi R^2} \sqrt{2g}\, t$$

$$\boxed{2 \cdot \left(\sqrt{H + (\,)^{1/2}} - \sqrt{H_0 + (\,)^{1/2}}\right) = -\frac{A_0}{\pi R^2} \sqrt{2g}\, t}$$

4) $\cancel{p_a} + \rho g H = \cancel{p_a} - \rho g h + \frac{1}{2} U^2 \rho - \rho \frac{\Omega^2 \cdot R_0^2}{2}$

$$\boxed{U^2 = 2g\left(H + (L^2 - R_0^2)^{1/2}\right) + \Omega^2 \cdot R_0^2}$$

$$Q = A_0 \cdot \left[2g\left(H + (L^2 - R_0^2)^{1/2}\right) + \Omega^2 R_0^2\right]^{1/2}$$

1,5

$$\frac{d}{dt}\left(\pi R^2 H\right) = -A_0 \cdot \left[2g\left(H + (\,)^{1/2}\right) + \Omega^2 R_0^2\right]^{1/2}$$

$$\boxed{2\left[\sqrt{H + (\,)^{1/2} + \frac{\Omega^2 R_0^2}{2g}} - \sqrt{H_0 + (\,)^{1/2} + \frac{\Omega^2 R_0^2}{2g}}\right] = -\frac{A_0 \sqrt{2g}}{\pi R^2}\, t}$$

$$(\,)^{1/2} = \left(L^2 - R_0^2\right)^{1/2}$$