7 Calcular las derivadas parciales de primer orden de las siguientes funciones en un punto genérico. ¿Qué condiciones debe verificar este punto?

	$\frac{\partial f}{\partial x}(x,y)$	$\frac{\partial f}{\partial y}(x,y)$	Condiciones
$(a)\ f(x,y) = \frac{x-y}{x+y}$	$\frac{2y}{(x+y)^2}$	$\frac{-2x}{(x+y)^2}$	$x+y \neq 0$
$(b)\ f(x,y) = \frac{3x^2-2y^2}{x^2+y^2}$	$\frac{10xy^2}{(x^2+y^2)^2}$	$\frac{-10x^2y}{(x^2+y^2)^2}$	$x^2+y^2 \neq 0$
$(c)\ f(x,y) = \frac{x^2y+xy^2}{xy}$	1	1	$xy \neq 0$
$(d)\ f(x,y) = \sqrt{\frac{x+y}{x-y}}$	$-\frac{y}{(x-y)^2}\sqrt{\frac{x-y}{x+y}}$	$\frac{x}{(x-y)^2}\sqrt{\frac{x-y}{x+y}}$	$x-y\ \neq 0 \quad \frac{x+y}{x-y} > 0$
$(e)\ f(x,y) = 2^{\frac{1}{x+y}}$	$-2^{\frac{1}{x+y}} Ln\ 2\ \frac{1}{(x+y)^2}$	$-2^{\frac{1}{x+y}} Ln\ 2\ \frac{1}{(x+y)^2}$	$x+y \neq 0$
$(f)\ f(x,y) = x^y + y^x$	$yx^{y-1} + y^x Ln\ y$	$x^y Ln\ x + xy^{x-1}$	$x > 0\ \ y > 0$
$(g)\ f(x,y) = arc\ tg\left(\frac{y}{x}\right) + arc\ tg\left(\frac{x}{y}\right)$	$\frac{-y/x^2}{1+(y/x)^2} + \frac{1/y}{1+(x/y)^2} = 0$	$\frac{1/x}{1+(y/x)^2} + \frac{-x/y^2}{1+(x/y)^2} = 0$	$x \neq 0\ \ y \neq 0$
$(h)\ f(x,y) = \frac{e^x}{x^2-y^2}$	$\frac{e^x(-2x+x^2-y^2)}{(x^2-y^2)^2}$	$\frac{2e^xy}{(x^2-y^2)^2}$	$x^2-y^2 \neq 0$
$(i)\ f(x,y) = \frac{x^2-y^2}{x^2+y^2}$	$\frac{4xy^2}{(x^2+y^2)^2}$	$\frac{-4x^2y}{(x^2+y^2)^2}$	$x^2+y^2 \neq 0$

	$\frac{\partial f}{\partial x}(x,y,z)$	$\frac{\partial f}{\partial y}(x,y,z)$	$\frac{\partial f}{\partial z}(x,y,z)$	Condiciones
$(j)\ f(x,y,z) = Ln\big(cos(x+y+z)\big)$	$-tg(x+y+z)$	$-tg(x+y+z)$	$-tg(x+y+z)$	$\cos(x+y+z) > 0 \rightarrow$ $\lvert x+y+z\rvert < \frac{\pi}{2} + 2k\pi \quad k \in \mathbb{Z}$
$(k)\ f(x,y,z) = cos(xz+yz)$	$-z\ sen(xz+\ yz)$	$-z\ sen\ (x\,z\ +\ y\,z)$	$-(x\ +\ y)\ sen(xz\ + y\,z)$	$\forall\ (x,y,z)\ \in \mathbb{R}^3$
$(l)\ f(x,y,z) = e^{-x} sen\ y \cos z$	$-e^{-x} sen\ y \cos z$	$e^{-x} cos\ y \cos z$	$-e^{-x} sen\ y\ sen\ z$	$\forall\ (x,y,z)\ \in \mathbb{R}^3$

8 Calcular el vector gradiente de las siguientes funciones en un punto genérico y, si es posible, en el punto que se indica:

$(a)\ f(x,y) = \dfrac{x+y}{xy} \quad P(1,1)$

$$\begin{cases} D_1 f(x,y) = -\dfrac{y^2}{x^2y^2} = -\dfrac{1}{x^2} \\ D_2 f(x,y) = -\dfrac{x^2}{x^2y^2} = -\dfrac{1}{y^2} \end{cases} \rightarrow \nabla f(x,y) = \begin{pmatrix} -\dfrac{1}{x^2} \\ -\dfrac{1}{y^2} \end{pmatrix} \xrightarrow{xy\neq 0} \nabla f(1,1) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$(b)\ f(x,y) = \sqrt{x+y} \quad P(1,0)$

$$\begin{cases} D_1 f(x,y) = \dfrac{1}{2\sqrt{x+y}} \\ D_2 f(x,y) = \dfrac{1}{2\sqrt{x+y}} \end{cases} \rightarrow \nabla f(x,y) = \begin{pmatrix} \dfrac{1}{2\sqrt{x+y}} \\ \dfrac{1}{2\sqrt{x+y}} \end{pmatrix} \xrightarrow{x+y>0} \nabla f(1,0) = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$(c)\ f(x,y) = \dfrac{e^y}{x+y} \quad P(0,1)$

$$\begin{cases} D_1 f(x,y) = \dfrac{-e^y}{(x+y)^2} \\ D_2 f(x,y) = \dfrac{e^y(x+y-1)}{(x+y)^2} \end{cases} \rightarrow \nabla f(x,y) = \begin{pmatrix} \dfrac{-e^y}{(x+y)^2} \\ \dfrac{e^y(x+y-1)}{(x+y)^2} \end{pmatrix} \xrightarrow{x+y\neq 0} \nabla f(0,1) = \begin{pmatrix} -e \\ 0 \end{pmatrix}$$

$(d)\ f(x,y) = e^{-x} sen(x+y) \quad P(0,\pi)$

$$\begin{cases} D_1 f(x,y) = -e^{-x} sen(x+y) + e^{-x}\cos(x+y) \\ D_2 f(x,y) = e^{-x}\cos(x+y) \end{cases} \rightarrow$$

$$\nabla f(x,y) = \begin{pmatrix} e^{-x}\left(\cos(x+y) - sen(x+y)\right) \\ e^{-x}\cos(x+y) \end{pmatrix} \quad \forall (x,y) \in \mathbb{R}^2 \rightarrow \nabla f(0,\pi) = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$(e)\ f(x,y,z) = x^{y+z} \quad P(1,1,1)$

$$\begin{cases} D_1 f(x,y,z) = (y+z)x^{y+z-1} \\ D_2 f(x,y,z) = x^{y+z} Ln\, x \\ D_3 f(x,y,z) = x^{y+z} Ln\, x \end{cases} \rightarrow \nabla f(x,y,z) = \begin{pmatrix} (y+z)x^{y+z-1} \\ x^{y+z} Ln\, x \\ x^{y+z} Ln\, x \end{pmatrix} \xrightarrow{x>0} \nabla f(1,1,1) = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$(f)\ f(x,y,z) = 2^{\frac{z}{x+y}} \quad P(1,1,1)$

$$\begin{cases} D_1 f(x,y,z) = -2^{\frac{z}{x+y}}\ Ln2\ \dfrac{z}{(x+y)^2} \\ D_2 f(x,y,z) = -2^{\frac{z}{x+y}}\ Ln2\ \dfrac{z}{(x+y)^2} \\ D_3 f(x,y,z) = 2^{\frac{z}{x+y}}\ Ln\ 2\ \left(\dfrac{1}{x+y}\right) \end{cases} \rightarrow \nabla f(x,y,z) = \begin{pmatrix} -\dfrac{2^{\frac{z}{x+y}}\ z\ Ln2}{(x+y)^2} \\ -\dfrac{2^{\frac{z}{x+y}}\ z\ Ln\ 2}{(x+y)^2} \\ \dfrac{2^{\frac{z}{x+y}}\ Ln\ 2}{x+y} \end{pmatrix} \xrightarrow{x+y\neq 0} \nabla f(1,1,1) =$$

$$= \begin{pmatrix} \dfrac{-\sqrt{2}Ln\ 2}{4} \\ \dfrac{-\sqrt{2}Ln\ 2}{4} \\ \dfrac{\sqrt{2}Ln\ 2}{2} \end{pmatrix}$$

$(g)\ f(x,y,z) = \sqrt{xyz} \quad P(1,1,1)$

$$\begin{cases} D_1 f(x,y,z) = \dfrac{yz}{2\sqrt{xyz}} \\ D_2 f(x,y,z) = \dfrac{xz}{2\sqrt{xyz}} \\ D_3 f(x,y,z) = \dfrac{xy}{2\sqrt{xyz}} \end{cases} \rightarrow \nabla f(x,y,z) = \begin{pmatrix} \dfrac{yz}{2\sqrt{xyz}} \\ \dfrac{xz}{2\sqrt{xyz}} \\ \dfrac{xy}{2\sqrt{xyz}} \end{pmatrix} \xrightarrow{xyz>0} \nabla f(1,1,1) = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$(h)\ f(x,y,z,t) = (xt)^{yz} \quad P(1,1,1,1)$

$$\begin{cases} D_1 f(x,y,z,t) = yz(xt)^{yz-1}t \\ D_2 f(x,y,z,t) = (xt)^{yz}Ln(xt)z \\ D_3 f(x,y,z,t) = (xt)^{yz}Ln(xt)y \\ D_4 f(x,y,z,t) = yz(xt)^{yz-1}x \end{cases} \rightarrow \nabla f(x,y,z,t) = \begin{pmatrix} yz(xt)^{yz-1}t \\ (xt)^{yz}Ln(xt)z \\ (xt)^{yz}Ln(xt)y \\ yz(xt)^{yz-1}x \end{pmatrix} \xrightarrow{xt>0} \nabla f(1,1,1,1) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

9. Calcular el vector gradiente y la matriz hessiana de las siguientes funciones en un punto genérico y, si es posible, en el punto P que se indica:

$$(a)\ f(x,y)=\frac{x^3y-xy^3}{x^2+y^2}\quad P(1,1)$$

Derivadas de primer orden:

$$\begin{cases}\dfrac{\partial f}{\partial x}(x,y)=\dfrac{(3x^2y-y^3)(x^2+y^2)-2x(x^3y-xy^3)}{(x^2+y^2)^2}=\dfrac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2}\\[2ex] \dfrac{\partial f}{\partial y}(x,y)=\dfrac{(x^3-3xy^2)(x^2+y^2)-2y(x^3y-xy^3)}{(x^2+y^2)^2}=\dfrac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}\end{cases}\xrightarrow{x^2+y^2\neq 0}$$

$$\nabla f(x,y)=\begin{pmatrix}\dfrac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2}\\ \dfrac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}\end{pmatrix}\rightarrow \nabla f(1,1)=\begin{pmatrix}1\\-1\end{pmatrix}$$

Derivadas de segundo orden:

$$\boxed{\frac{\partial f}{\partial x}(x,y)=\frac{x^4y+4x^2y^3-y^5}{(x^2+y^2)^2}}\rightarrow$$

$$\frac{\partial^2 f}{\partial x^2}(x,y)=\frac{(4x^3y+8xy^3)(x^2+y^2)^2-2(x^2+y^2)2x(x^4y+4x^2y^3-y^5)}{(x^2+y^2)^4}=\frac{-4x^3y^3+12xy^5}{(x^2+y^2)^3}$$

$$\frac{\partial^2 f}{\partial y\partial x}(x,y)=\frac{(x^4+12x^2y^2-5y^4)(x^2+y^2)^2-2(x^2+y^2)2y(x^4y+4x^2y^3-y^5)}{(x^2+y^2)^4}$$
$$=\frac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3}$$

$$\boxed{\frac{\partial f}{\partial y}(x,y)=\frac{x^5-4x^3y^2-xy^4}{(x^2+y^2)^2}}\rightarrow$$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y)=\frac{(5x^4-12x^2y^2-y^4)(x^2+y^2)^2-2(x^2+y^2)2x(x^5-4x^3y^2-xy^4)}{(x^2+y^2)^4}=\frac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3}$$

$$\frac{\partial^2 f}{\partial y^2}(x,y)=\frac{(-8x^3y-4xy^3)(x^2+y^2)^2-2(x^2+y^2)2y(x^5-4x^3y^2-xy^4)}{(x^2+y^2)^4}=\frac{-12x^5y+4x^3y^3}{(x^2+y^2)^3}$$

$$Hf(x,y)=\begin{pmatrix}\dfrac{-4x^3y^3+12xy^5}{(x^2+y^2)^3} & \dfrac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3}\\ \dfrac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3} & \dfrac{-12x^5y+4x^3y^3}{(x^2+y^2)^3}\end{pmatrix}\xrightarrow{x^2+y^2\neq 0} Hf(1,1)=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$(b)\ f(x,y) = e^{xy} \quad P(1,0)$

Derivadas de primer orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = y\,e^{xy} \\ \dfrac{\partial f}{\partial y}(x,y) = x\,e^{xy} \end{cases} \forall (x,y) \in \mathbb{R}^2 \to \nabla f(x,y) = \begin{pmatrix} y\,e^{xy} \\ x\,e^{xy} \end{pmatrix} \to \nabla f(1,0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = y\,e^{xy} \to \begin{cases} \dfrac{\partial^2 f}{\partial x^2}(x,y) = y^2 e^{xy} \\ \dfrac{\partial^2 f}{\partial y \partial x}(x,y) = e^{xy}(xy+1) \end{cases} \\ \dfrac{\partial f}{\partial y}(x,y) = x\,e^{xy} \to \begin{cases} \dfrac{\partial^2 f}{\partial x \partial y}(x,y) = e^{xy}(xy+1) \\ \dfrac{\partial^2 f}{\partial y^2}(x,y) = x^2 e^{xy} \end{cases} \end{cases} \quad \forall (x,y) \in \mathbb{R}^2 \to$$

$$Hf(x,y) = \begin{pmatrix} y^2 e^{xy} & e^{xy}(xy+1) \\ e^{xy}(xy+1) & x^2 e^{xy} \end{pmatrix} \to Hf(1,0) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$(c)\ f(x,y) = xLn(2y) \quad P = \left(0, \frac{e}{2}\right)$

Derivadas de primer orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = Ln(2y) \\ \dfrac{\partial f}{\partial y}(x,y) = x\dfrac{2}{2y} \end{cases} con\ y > 0 \to \nabla f(x,y) = \begin{pmatrix} Ln(2y) \\ \dfrac{x}{y} \end{pmatrix} \to \nabla f\left(0, \frac{e}{2}\right) = \begin{pmatrix} Ln\ e \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y) = Ln(2y) \to \begin{cases} \dfrac{\partial^2 f}{\partial x^2}(x,y) = 0 \\ \dfrac{\partial^2 f}{\partial y \partial x}(x,y) = \dfrac{2}{2y} = \dfrac{1}{y} \end{cases} \\ \dfrac{\partial f}{\partial y}(x,y) = \dfrac{x}{y} \to \begin{cases} \dfrac{\partial^2 f}{\partial x \partial y}(x,y) = \dfrac{1}{y} \\ \dfrac{\partial^2 f}{\partial y^2}(x,y) = \dfrac{-x}{y^2} \end{cases} \end{cases} \xrightarrow{y>0} Hf(x,y) = \begin{pmatrix} 0 & \dfrac{1}{y} \\ \dfrac{1}{y} & \dfrac{-x}{y^2} \end{pmatrix} \to Hf\left(0, \frac{e}{2}\right) = \begin{pmatrix} 0 & \dfrac{2}{e} \\ \dfrac{2}{e} & 0 \end{pmatrix}$$

$(d)\ f(x,y)=cos(xy)+sen(x^2y)\quad P\left(1,\frac{\pi}{2}\right)$

Derivadas de primer orden:

$$\begin{cases}\frac{\partial f}{\partial x}(x,y)=-sen(xy)y+cos(x^2y)2xy\\ \frac{\partial f}{\partial y}(x,y)=-sen(xy)x+cos(x^2y)x^{\,2}\end{cases}\xrightarrow{\forall(x,y)\in\mathbb{R}^2}\nabla f(x,y)=\begin{pmatrix}-sen(xy)y+cos(x^2y)2xy\\ -sen(xy)x+cos(x^2y)x^{\,2}\end{pmatrix}\to$$

$$\nabla f\left(1,\frac{\pi}{2}\right)=\begin{pmatrix}-\underbrace{sen\left(\frac{\pi}{2}\right)}_{1}\frac{\pi}{2}+\underbrace{cos\left(\frac{\pi}{2}\right)}_{0}2\frac{\pi}{2}\\ -\underbrace{sen\left(\frac{\pi}{2}\right)}_{1}+\underbrace{cos\left(\frac{\pi}{2}\right)}_{0}\end{pmatrix}=\begin{pmatrix}-\frac{\pi}{2}\\ -1\end{pmatrix}$$

Derivadas de segundo orden:

$$\frac{\partial f}{\partial x}(x,y)=-sen(xy)y+cos(x^2y)2xy\to$$

$$\begin{cases}\frac{\partial^2 f}{\partial x^2}(x,y)=-cos(xy)\,y^2-sen(x^2y)2xy\ 2xy+cos(x^2y)2y\\ \\ \frac{\partial^2 f}{\partial y\partial x}(x,y)=-cos(xy)\,yx-sen(xy)-sen(x^2y)x^2 2xy+cos(x^2y)2x\end{cases}$$

$$\frac{\partial f}{\partial y}(x,y)=-sen(xy)x+cos(x^2y)x^{\,2}\to$$

$$\begin{cases}\frac{\partial^2 f}{\partial x\partial y}(x,y)=-cos(xy)\,yx-sen(xy)-sen(x^2y)2xyx^2+2xcos(x^2y)\\ \frac{\partial^2 f}{\partial y^2}(x,y)=-xcos(xy)\,x-sen(x^2y)\,x^2\,x^2\end{cases}$$

$$\xrightarrow{\forall(x,y)\in\mathbb{R}^2}\quad Hf(x,y)=$$

$$=\begin{pmatrix}-cos(xy)\,y^2-sen(x^2y)4x^2y^2+cos(x^2y)2y & -cos(xy)\,yx-sen(xy)-sen(x^2y)2x^3y+2xcos(x^2y)\\ -cos(xy)\,yx-sen(xy)-sen(x^2y)2x^3y+2xcos(x^2y) & -xcos(xy)\,x-sen(x^2y)\,x^2\,x^2\end{pmatrix}$$

$$Hf\left(1,\frac{\pi}{2}\right)=\begin{pmatrix}-\pi^2 & -1-\pi\\ -1-\pi & -1\end{pmatrix}$$

$(e)\ f(x,y,z)=\dfrac{x+z}{y-z}\quad P(0,1,-1)$

Derivadas de primer orden:

$$\begin{cases}\dfrac{\partial f}{\partial x}(x,y,z)=\dfrac{1}{y-z}\\ \dfrac{\partial f}{\partial y}(x,y,z)=\dfrac{-(x+z)}{(y-z)^2}\\ \dfrac{\partial f}{\partial z}(x,y,z)=\dfrac{(y-z)-(-1)(x+z)}{(y-z)^2}=\dfrac{y+x}{(y-z)^2}\end{cases}\xrightarrow{y-z\neq 0}\quad \nabla f(x,y,z)=\begin{pmatrix}\dfrac{1}{y-z}\\ \dfrac{-(x+z)}{(y-z)^2}\\ \dfrac{y+x}{(y-z)^2}\end{pmatrix}\to\nabla f(0,1,-1)=\begin{pmatrix}1/2\\1/4\\1/4\end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases}\dfrac{\partial f}{\partial x}(x,y,z)=\dfrac{1}{y-z}\to\begin{cases}\dfrac{\partial^2 f}{\partial x^2}(x,y,z)=0\\ \dfrac{\partial^2 f}{\partial y\partial x}(x,y,z)=\dfrac{-1}{(y-z)^2}\\ \dfrac{\partial^2 f}{\partial z\partial x}(x,y,z)=\dfrac{1}{(y-z)^2}\end{cases}\\ \dfrac{\partial f}{\partial y}(x,y,z)=\dfrac{-(x+z)}{(y-z)^2}\to\begin{cases}\dfrac{\partial^2 f}{\partial x\partial y}(x,y,z)=\dfrac{-1}{(y-z)^2}\\ \dfrac{\partial^2 f}{\partial y^2}(x,y,z)=\dfrac{2(y-z)(x+z)}{(y-z)^4}=\dfrac{2(x+z)}{(y-z)^3}\\ \dfrac{\partial^2 f}{\partial z\partial y}(x,y,z)=-\dfrac{(y-z)^2+2(y-z)(x+z)}{(y-z)^4}=-\dfrac{(y-z)+2(x+z)}{(y-z)^3}\end{cases}\\ \dfrac{\partial f}{\partial z}(x,y,z)=\dfrac{y+x}{(y-z)^2}\to\begin{cases}\dfrac{\partial^2 f}{\partial x\partial z}(x,y,z)=\dfrac{1}{(y-z)^2}\\ \dfrac{\partial^2 f}{\partial y\partial z}(x,y,z)=\dfrac{(y-z)^2-2(y-z)(y+x)}{(y-z)^4}=\dfrac{(y-z)-2(y+x)}{(y-z)^3}\\ \dfrac{\partial^2 f}{\partial z^2}(x,y,z)=\dfrac{2(y-z)(y+x)}{(y-z)^4}=\dfrac{2(y+x)}{(y-z)^3}\end{cases}\end{cases}$$

$$Hf(x,y,z)=\begin{pmatrix}0&\dfrac{-1}{(y-z)^2}&\dfrac{1}{(y-z)^2}\\ \dfrac{-1}{(y-z)^2}&\dfrac{2(x+z)}{(y-z)^3}&\dfrac{-2x-y-z}{(y-z)^3}\\ \dfrac{1}{(y-z)^2}&\dfrac{-2x-y-z}{(y-z)^3}&\dfrac{2(y+x)}{(y-z)^3}\end{pmatrix}\xrightarrow{y-z\neq 0}Hf(0,1,-1)=\begin{pmatrix}0&\dfrac{-1}{4}&\dfrac{1}{4}\\ \dfrac{-1}{4}&\dfrac{-1}{4}&0\\ \dfrac{1}{4}&0&\dfrac{1}{4}\end{pmatrix}$$

Problemas resueltos derivadas de orden superior. Matriz hessiana. Matemáticas I.

$$(f)\ f(x,y,z) = Ln\left(\frac{x+y}{z}\right) \qquad P(1,1,2)$$

Derivadas de primer orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y,z) = \dfrac{z}{x+y}\,\dfrac{1}{z} = \dfrac{1}{x+y} \\ \dfrac{\partial f}{\partial y}(x,y,z) = \dfrac{z}{x+y}\,\dfrac{1}{z} = \dfrac{1}{x+y} \\ \dfrac{\partial f}{\partial z}(x,y,z) = \dfrac{z}{x+y}\,\dfrac{-(x+y)}{z^2} = \dfrac{-1}{z} \end{cases} \xrightarrow{z\neq 0,\ \frac{x+y}{z}>0} \nabla f(x,y,z) = \begin{pmatrix} \dfrac{1}{x+y} \\ \dfrac{1}{x+y} \\ \dfrac{-1}{z} \end{pmatrix} \to \nabla f(1,1,2) = \begin{pmatrix} 1/2 \\ 1/2 \\ -1/2 \end{pmatrix}$$

Derivadas de segundo orden:

$$\begin{cases} \dfrac{\partial f}{\partial x}(x,y,z) = \dfrac{1}{x+y} \to \begin{cases} \dfrac{\partial^2 f}{\partial x^2}(x,y,z) = \dfrac{-1}{(x+y)^2} \\ \dfrac{\partial^2 f}{\partial y \partial x}(x,y,z) = \dfrac{-1}{(x+y)^2} \\ \dfrac{\partial^2 f}{\partial z \partial x}(x,y,z) = 0 \end{cases} \\ \dfrac{\partial f}{\partial y}(x,y,z) = \dfrac{1}{x+y} \to \begin{cases} \dfrac{\partial^2 f}{\partial x \partial y}(x,y,z) = \dfrac{-1}{(x+y)^2} \\ \dfrac{\partial^2 f}{\partial y^2}(x,y,z) = \dfrac{-1}{(x+y)^2} \\ \dfrac{\partial^2 f}{\partial z \partial y}(x,y,z) = 0 \end{cases} \\ \dfrac{\partial f}{\partial z}(x,y,z) = \dfrac{-1}{z} \to \begin{cases} \dfrac{\partial^2 f}{\partial x \partial z}(x,y,z) = 0 \\ \dfrac{\partial^2 f}{\partial y \partial z}(x,y,z) = 0 \\ \dfrac{\partial^2 f}{\partial z^2}(x,y,z) = \dfrac{1}{z^2} \end{cases} \end{cases} \xrightarrow{z\neq 0,\ \frac{x+y}{z}>0}$$

$$Hf(x,y,z) = \begin{pmatrix} \dfrac{-1}{(x+y)^2} & \dfrac{-1}{(x+y)^2} & 0 \\ \dfrac{-1}{(x+y)^2} & \dfrac{-1}{(x+y)^2} & 0 \\ 0 & 0 & \dfrac{1}{z^2} \end{pmatrix} \xrightarrow[z\neq 0]{\frac{x+y}{z}>0} Hf(1,1,2) = \begin{pmatrix} \dfrac{-1}{4} & \dfrac{-1}{4} & 0 \\ \dfrac{-1}{4} & \dfrac{-1}{4} & 0 \\ 0 & 0 & \dfrac{1}{4} \end{pmatrix}$$