1) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ be an orthonormal reference frame on a Euclidean plane A.

a) Let $\vec{f}:\vec{A}\to\vec{A}$ be a linear map such that

$\vec{f}(\vec{e}_1)=-2\vec{e}_1+\vec{e}_2$, $\vec{f}(\vec{e}_2)=\vec{e}_1-2\vec{e}_2$. Is $\vec{f}$ a symmetric tensor? Explain your answer..

b) If f is an affine transformation on A with $\vec{f}$ as its associated linear map and such that f carries the point $(P)_R=(1,1)$ to $(P')_R=(4,2)$, is f isometry? If so, identify which kind of isometry is and find its defining geometric objects.

c)

d) If g is given in the frame R by $\begin{cases} x'=-\dfrac{5}{13}x-\dfrac{12}{13}y+3 \\ y'=-\dfrac{12}{13}x+\dfrac{5}{13}y-2 \end{cases}$, is g isometry? If so, classify it and find its characteristic elements.

2) Let A be a Euclidean plane and consider the reference frame $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ such that $m(\cdot,B)=\begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}$.Is the affine map with equations in R $\begin{cases} x'=x-\dfrac{6}{5}y-6 \\ y'=-y+2 \end{cases}$ an isometry? If so, classify it and find its characteristic elements.

3) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ be a reference frame on a Euclidean plane A such that $\begin{cases} \vec{e}_2.\vec{e}_2=2 \\ (\vec{e}_1+\vec{e}_2).\vec{e}_2=1 \\ (2\vec{e}_1+\vec{e}_2)(\vec{e}_1+\vec{e}_2)=1 \end{cases}$. Is the affine map with equations in R $\begin{cases} x_1'=-x_1-x_2+4 \\ x_2'=2x_1+x_2+1 \end{cases}$ an isometry? If so, identify which isometry is and find its defining geometric objects.

4) Let A be a Euclidean plane and consider the reference frame $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ such that $m(\cdot,B)=\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$.Is the affine map with equations in R $\begin{cases} x'=-x+2y+5 \\ y'=-x+y-1 \end{cases}$ an isometry? If so, classify it and find its defining geometric elements.

5) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ be an orthonormal reference frame on a Euclidean plane A If g is given in the frame R by $\begin{cases} x'=\dfrac{-3}{5}x-\dfrac{4}{5}y+3 \\ y'=-\dfrac{4}{5}x+\dfrac{3}{5}y-1 \end{cases}$, is g isometry? If so, identify which isometry is and find its defining geometric objects.

6) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ be a reference frame on a Euclidean plane A such that $\begin{cases}(\vec{e}_1-\vec{e}_2)(\vec{e}_1+\vec{e}_2)=-1\\ (\vec{e}_1+\vec{e}_2).2\vec{e}_1=2\\ \vec{e}_2.\vec{e}_2=3\end{cases}$.

Is the affine transformation f given in R by $\begin{cases}x_1'=\dfrac{17}{13}x_1-\dfrac{12}{13}x_2+2\\ x_2'=\dfrac{10}{13}x_1-\dfrac{17}{13}x_2+5\end{cases}$ an isometry? If so, classify it and find its characteristic elements.

7) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ be an orthonormal reference frame on a Euclidean plane A and let g be an affine transformation given (in the frame R) by $\begin{cases}x'=\dfrac{-4}{5}x-\dfrac{3}{5}y+3\\ y'=\dfrac{3}{5}x-\dfrac{4}{5}y-1\end{cases}$

a) Is g isometry? In the affirmative case, determine its type and its defining elements.

b) Sketch the image under g of the circle $x^2+y^2=9$ Justify your answer.

c) Find the equations of the glide(reflection) symmetry, f, with axis the line that goes through $P=(3,1)$ and $Q=(1,3)$ and satisfying $f(P)=Q$

8) Consider an orthonormal reference frame on a Euclidean plane A and the line $r:\ 2x-y=5$

a) If f is the (reflection) symmetry with respect to the line r, find the image $P'=f(P)$ under of $P=(2,1)$.

b) Let g be the (affine) rotation of centre $P=(2,1)$ and angle $\dfrac{3\pi}{2}$. Find the equations of the composition $h=g\circ f$ Is h an isometry?

9) Consider an orthonormal reference frame on a Euclidean plane A. Find the equations of the glide (reflection) symmetry with axis the line that goes through $P=(3,1)$ and $Q=(1,3)$ that carries P to Q

10) Let A be a Euclidean plane and consider the reference frame $R=\{O;B=\{\vec{e}_1,\vec{e}_2\}\}$ such that $m(\cdot,B)=\begin{pmatrix}3&1\\1&2\end{pmatrix}$ and consider the line $r:\ x-y=2$. Find the equations of the glide (reflection) symmetry with axis r and translation vector $\vec{u}=(-2,-2)$.

11) Let $R=\{O;B=\{\vec{e}_1,\vec{e}_2,\vec{e}_3\}\}$ be an orthonormal reference frame for a Euclidean space A. If f is the affine map given in R by $\begin{cases}x'=\dfrac{2}{3}x+\dfrac{1}{3}y+\dfrac{2}{3}z+1\\ y'=-\dfrac{2}{3}x+\dfrac{2}{3}y+\dfrac{1}{3}z+1\\ z'=\dfrac{1}{3}x+\dfrac{2}{3}y-\dfrac{2}{3}z+1\end{cases}$, is f isometry? If so, identify which isometry is and find its defining geometric objects.