13.1 Describe the classic model for electrical conduction in metals.

In the classic model for electrical conduction in metals, the outer valence electrons are assumed to move freely between atoms in the lattice which do not have valence electrons (positive-ion cores). In the absence of an electrical potential, the motion of the valence electrons is random and restricted; however, an applied electric potential allows the electrons to attain a directed drift velocity that is proportional to, but in the opposite direction of, the applied field.

13.2 Distinguish between (a) positive-ion cores and (b) valence electrons in a metallic crystal lattice such as sodium.

(a) In a metallic crystal lattice, positive-ion cores refer to atoms without their valence electrons.

(b) Valence electrons are the electrons in the outermost shell, and thus of a higher energy level, which leave the atom to form the electron gas.

13.3 Write equations for the (a) macroscopic and (b) microscopic forms of Ohm's law.

(a) The macroscopic form of Ohm's law is dependent on the conductor's geometrical shape:

$$i = \frac{V}{R}$$ where $i =$ electric current in amperes (A)

$V =$ potential difference in volts (V)
$R =$ resistance of wire in ohms (Ω)

(b) The microscopic form of Ohm's law is independent of the conductor's shape:

$$\mathbf{J} = \frac{\mathbf{E}}{\rho}$$ where $\mathbf{J} =$ current density in amperes per square meter (A/m^2)

$\mathbf{E} =$ electric field in volts per meter (V/m)
$\rho =$ electrical resistivity in ohm-meters ($\Omega \cdot$m)

13.4 How is electrical conductivity related numerically to electrical resistivity?

Electrical conductivity is inversely proportional to electrical resistivity, $\sigma = 1/\rho$.

13.5 Give two kinds of SI units for electrical conductivity.

In the SI system, electrical conductivity is expressed in (ohm-meters)$^{-1}$ = $(\Omega \cdot m)^{-1}$ or in siemens/meter = S/m.

13.6 Calculate the resistance of an iron rod 0.720 cm in diameter and 0.850 m long at 20°C. $\left[\rho_e(20°C) = 10.0 \times 10^{-6} \ \Omega \cdot cm.\right]$

The resistance is calculated with the length expressed as 85 cm,

$$R = \rho \frac{l}{A} = \left(10.0 \times 10^{-6} \ \Omega \cdot cm\right)\left[\frac{85 \ cm}{\pi (0.720 \ cm)^2 / 4}\right] = \mathbf{2.09 \times 10^{-3} \ \Omega}$$

13.7 A nichrome wire must have a resistance of 120 Ω. How long must it be (in meters) if it is 0.0015 in. in diameter? $\left[\sigma_e(nichrome) = 9.3 \times 10^5 \ (\Omega \cdot m)^{-1}.\right]$

The wire cross-sectional area in square meters is:

$$A = \pi d^2 / 4 = \pi (0.0015 \ in)^2 \left[(2.54 \ cm/in)(10^{-2} \ m/cm)\right]^2 / 4 = 1.14 \times 10^{-9} \ m^2$$

The required length of nichrome wire is thus:

$$l = R\sigma A = (120 \ \Omega)\left[9.3 \times 10^5 \ (\Omega \cdot m)\right]\left(1.14 \times 10^{-9} \ m^2\right) = \mathbf{0.127 \ m}$$

13.8 A wire 0.40 cm in diameter must carry a 25 A current.
(a) If the maximum power dissipation along the wire is 0.025 W/cm, what is the minimum allowable electrical conductivity of the wire (give answer in SI units)?
(b) What is the current density in the wire?

(a) Given $P = i^2 R$ and $R = \dfrac{l}{\sigma A}$, $P = \dfrac{i^2 l}{\sigma A}$ or $\sigma = \dfrac{i^2 l}{PA}$. The minimum conductivity is thus,

$$\sigma = \frac{i^2}{(P/l)A} = \frac{(25 \ A)^2}{(0.025 \ W/cm)(100 \ cm/m)\left[\pi(0.004 \ m)^2 / 4\right]} = \mathbf{1.99 \times 10^7 \ (\Omega \cdot m)^{-1}}$$

(b) The magnitude of the current density is the current per unit cross-sectional area,

$$J = \frac{i}{A} = \frac{25 \ A}{\pi(0.004 \ m)^2 / 4} = \mathbf{1.99 \times 10^6 \ A/m^2}$$

13.9 An iron wire is to conduct a 6.5 A current with a maximum voltage drop of 0.005 V/cm. What must be the minimum diameter of the wire in meters at (20°C)?

The diameter is related to the voltage per length and current through the resistance,

$$R = \rho\frac{l}{A} = \frac{V}{i} \quad \text{and } A = \frac{\pi}{4}d^2$$

Combining the two equations and solving for d,

$$d = \sqrt{\frac{4\rho i}{\pi(V/l)}}$$

Since the resistivity for iron is given in Table 13.3 for 0°C, we must correct for temperature,

$$\rho_T = \rho_{0°C}(1+\alpha_T T) = (9.0\times10^{-6}\ \Omega\cdot\text{cm})\left[1+\frac{0.0045}{°C}(20°C)\right] = 9.81\times10^{-6}\ \Omega\cdot\text{cm}$$

Substituting,

$$d = \sqrt{\frac{4(9.81\times10^{-6}\ \Omega\cdot\text{cm})(6.5\ \text{A})}{\pi(0.005\ \text{V/cm})}} = 0.127\ \text{cm} = \mathbf{1.27\times10^{-2}\ m}$$

13.10 Define the following quantities pertaining to the flow of electrons in a metal conductor: (a) drift velocity; (b) relaxation time; (c) electron mobility.

(a) The drift velocity, $\mathbf{v}_d$, is the average velocity achieved by an electron in the presence of a uniform electric field, $\mathbf{E}$. An average value is used since the electron's motion varies periodically, in a sawtooth manner, as the particle accelerates, collides with positive-ion cores, and reaccelerates.

(b) The relaxation time, τ, is the average time between collisions of a conduction electron with the positive-ion cores of the metal lattice.

(c) The electron mobility, μ, is the proportionality constant relating the drift velocity to the applied electric field as $\mathbf{v}_d = \mu\mathbf{E}$. The SI units for μ are $m^2/(V\cdot s)$.

13.11 What causes the electrical resistivity of a metal to increase as its temperature increases? What is a phonon?

As the temperature of a metal increases, the vibration of the positive-ion cores increases in amplitude, causing the release of phonons. Phonons are discrete units or quantum of energy that travel as waves. In the case of a heated metal conductor, these waves are thermally excited and scatter conduction electrons. Consequently, the distance between the electrons, and thus their relaxation times between collisions, decreases. As a result of the increase in collisions, the metal resistivity increases.

13.12   What structural defects contribute to the residual component of the electrical resistivity of a pure metal?

Dislocations, vacancies, grain boundaries and impurity atoms are common structural defects that contribute to the small residual component of a pure metal's electrical resistivity.

13.13   What effect do elements that form solid solutions have on the electrical resistivities of pure metals?

Elements that form solid solutions increase the electrical resistivities of pure metals by causing additional electron scattering. The extent of this increased resistance is dependent upon both the type and the weight percent of the alloying element added.

13.14   Calculate the electrical resistivity (in ohm-meters) of a silver wire 15 m long and 0.030 m in diameter at 160°C. $\left[\rho_e(\text{Fe at } 0°\text{C}) = 9.0 \times 10^{-6} \ \Omega \cdot \text{cm.}\right]$

The electrical resistivity is independent of the wire length and diameter but dependent on temperature. From Table 13.3, the resistivity for silver at 0°C is $1.47 \times 10^{-6} \ \Omega \cdot \text{cm}$. Thus, at 160°C,

$$\rho_T = \rho_{0°\text{C}}\left(1 + \alpha_T T\right) = \left(1.47 \times 10^{-6} \ \Omega \cdot \text{cm}\right)\left[1 + \frac{0.0038}{°\text{C}}(160°\text{C})\right] = \mathbf{2.36 \times 10^{-6} \ \Omega \times \text{cm}}$$

13.15   At what temperature will an iron wire have the same electrical resistivity as an aluminum one has at 35°C?

Equating the relations for resistivity for iron and aluminum and solving for the iron temperature, we find

$$\left(\rho_T\right)_{\text{Fe}} = \left(\rho_T\right)_{\text{Al}}$$

$$\left[\rho_{0°\text{C}}\left(1 + \alpha_T T\right)\right]_{\text{Fe}} = \left[\rho_{0°\text{C}}\left(1 + \alpha_T T\right)\right]_{\text{Al}}$$

$$\left(9.0 \times 10^{-6} \ \Omega \cdot \text{cm}\right)\left[1 + 0.0045\left(°\text{C}^{-1}\right)\left(T_{\text{Fe}}\right)\right] = \left(2.7 \times 10^{-6} \ \Omega \cdot \text{cm}\right)\left[1 + 0.0039\left(°\text{C}^{-1}\right)(35°\text{C})\right]$$

$$9.0 \times 10^{-6} + \left(4.050 \times 10^{-8}\right)T_{\text{Fe}} = 3.069 \times 10^{-6}$$

$$T_{\text{Fe}} = \mathbf{-146.5°C}$$

13.16   At what temperature will the electrical resistivity of an iron wire be $25.0 \times 10^{-8} \ \Omega \cdot \text{m}$?

For a resistivity of $25.0 \times 10^{-8} \ \Omega \cdot \text{m}$, an iron wire must have a temperature of:

$$\rho_T = \rho_{0^\circ C}\left(1 + \alpha_T T\right)$$

$$T = \frac{1}{\alpha_T}\left(\frac{\rho_T}{\rho_{0^\circ C}} - 1\right) = \frac{1}{0.0045\left(^\circ C^{-1}\right)}\left(\frac{25.0 \times 10^{-8}\ \Omega \cdot cm}{9.0 \times 10^{-6}\ \Omega \cdot cm}\right) = \mathbf{6.17^\circ C}$$

13.17 Why are the valence-electron energy levels broadened into bands in a solid block of a good conducting metal such as sodium?

In a solid block of a good conducting metal, the valence-electron energy levels are broadened into bands because of the Pauli exclusion principle. The valence electrons are furthest from the nucleus and, unlike the core electrons, free to interact and interpenetrate each other. While all of these valence electrons are from the same energy level, the Pauli exclusion principle prevents them from having identical energy states. Consequently, each valence electron has a slightly different energy level, and as a group, the valence electrons form a continuous energy band. For example, in a block of sodium having N atoms, there are N distinct $3s^1$ energy levels for each of the valence electrons constituting the $3s$ band.

13.18 Why don't the energy levels of the inner-core electrons of a block of sodium metal also form energy bands?

The energy levels of the inner-core electrons do not form energy bands because they are tightly bound to the nucleus and do not enter into bonding. Thus, they maintain their discrete energy levels.

13.19 Why is the $3s$ electron energy band in a block of sodium only half-filled?

The $3s$ electron energy band in a block of sodium is only half-filled because sodium only has one $3s$ electron while the Pauli exclusion principle allows for two $3s$ electrons of opposite spins.

13.20 What explanation is given for the good electrical conductivity of magnesium and aluminum even though these metals have filled outer $3s$ energy bands?

Although the outer $3s$ energy bands in magnesium and aluminum are filled, these metals have good electrical conductivity because their $3s$ bands overlap their $3p$ bands. In the case of magnesium, the empty $3p$ band combines with the $3s$ band to form a partially filled $3sp$ band. Similarly, in aluminum, the $3p$ band, which contains one electron, overlaps the full $3s$ band.

13.21 How does the energy-band model explain the poor electrical conductivity of an insulator such as pure diamond?

In an insulator, such as pure diamond, the electrons are tightly bound in covalent or ionic bonds. The energy-band model theorizes that these bound electrons fill a lower valence band which is separated by a large energy gap, E_g, from an empty outer conduction

band. A large potential is thus necessary for an electron to overcome the gap. In the case of pure diamond, approximately 6 to 7 eV are required to free an electron for conduction.

13.22 Define an intrinsic semiconductor. What are the two most important elemental semiconductors?

An intrinsic semiconductor is a pure semiconductor whose electrical conductivity is a function of the temperature and inherent conductive properties of the material, such as its energy gap and bonding structure.

13.23 What type of bonding does the diamond cubic structure have? Make a two-dimensional sketch of the bonding in the silicon lattice, and show how electron-hole pairs are produced in the presence of an applied field.

The diamond cubic structure has highly directional covalent bonds consisting of tetrahedral sp^3 hybrid bonding orbitals. In the presence of an applied field, the bonding valence electron attains a critical energy level and becomes a free conduction electron. A hole is thus produced in the lattice, as shown below in the schematic.
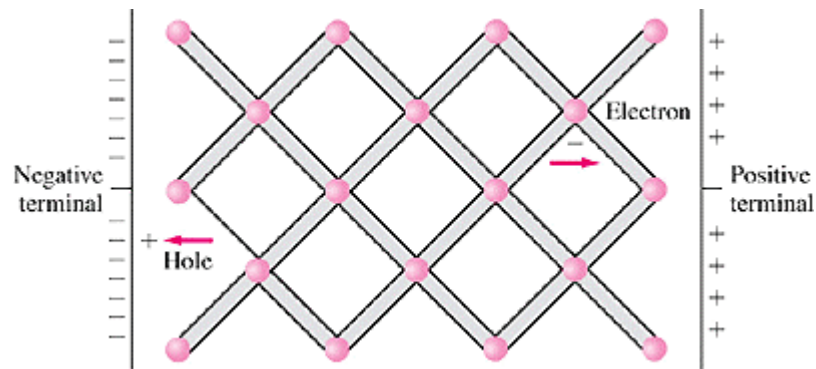
13.24 Why is a hole said to be an imaginary particle? Use a sketch to show how electron holes can move in a silicon crystal lattice.

A hole is said to be an imaginary particle because although a hole is not a real particle, it appears to behave like a positively charged carrier. As electrons move toward a positive terminal, they leave behind holes in the crystal lattice which cause the associated atoms to become positive. As depicted in the three-stage sketch on the following page, the result is an apparent motion of the holes toward the negative terminal.
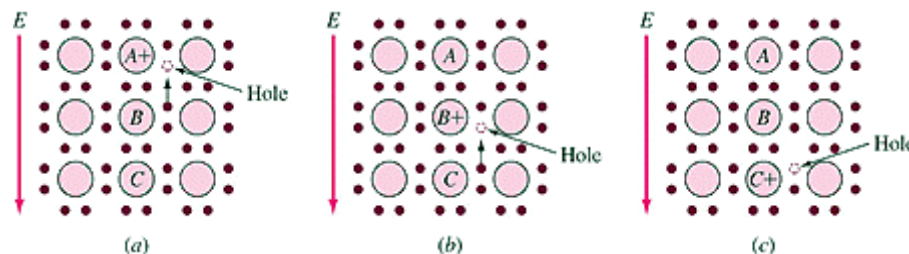
13.25 Define electron and electron hole mobility as pertains to charge movement in a silicon lattice. What do these quantities measure, and what are their SI units?

Electron mobility, $v_n / \mathbf{E}$, and electron hole mobility, $v_p / \mathbf{E}$, respectively measure how fast the electrons and holes in semiconductors drift in the presence of an applied field. The SI units for mobility are $m^2 / (V \cdot s)$.

13.26 Explain, using an energy-band diagram, how electrons and electron holes are created in pairs in intrinsic silicon.

The creation of electrons and electron holes is depicted in the energy-band diagram below. Initially, the covalently bonded valence electrons occupy the lower energy level of the valance band. The application of thermal or electrical energy excites the valence electrons sufficiently to overcome the energy gap, E_g, and enter the empty or nearly empty conduction band. As a consequence, a hole is produced in the valence band. Thus an electron-hole pair is created.
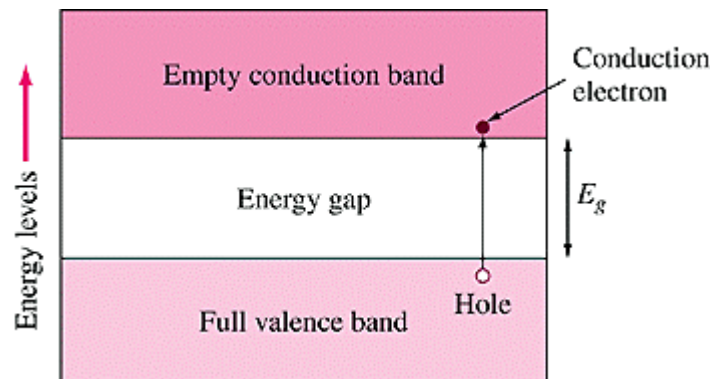
13.27 What is the ratio of the electron-to-hole mobility in silicon and germanium?

At 300 K, the ratio of the electron-to-hole mobility in silicon and germanium are:

$$\text{Si: } \frac{\mu_n}{\mu_p} = \frac{0.135}{0.048} = 2.8125 \approx \mathbf{2.81} \qquad \text{Ge: } \frac{\mu_n}{\mu_p} = \frac{0.39}{0.19} = 2.0526 \approx \mathbf{2.05}$$

13.28 Calculate the number of germanium atoms per cubic meter.

The number of germanium atoms, per cubic meter at 300 K, is:

$$\text{Density of Ge atoms} = \left\{ \frac{\text{atoms}}{m^3} \right\} = \frac{N_o \rho}{\text{atomic weight}} = \left\{ \frac{(\text{atoms/mol})(g/m^3)}{(g/\text{mol})} \right\}$$

$$= \frac{(6.02 \times 10^{23} \text{ atoms/mol})(5.32 \times 10^6 \text{ g/m}^3)}{72.60 \text{ g/mol}}$$

$$= \mathbf{4.41 \times 10^{28} \text{ atoms/m}^3}$$

13.29 Calculate the electrical resistivity of germanium at 300 K.

The electrical resistivity of geranium at 300 K can be calculated based upon the number of conduction electrons per unit volume, the electron charge, and the mobility values.

$$\rho = \frac{1}{\sigma} = \frac{1}{n_i q \left(\mu_n + \mu_p \right)}$$

$$= \frac{1}{\left(\dfrac{2.4 \times 10^{19} \text{ carriers}}{\text{m}^3} \right) \left(\dfrac{1.60 \times 10^{-19} \text{ C}}{\text{carrier}} \right) \left(\dfrac{0.39 \text{ m}^2}{\text{V} \cdot \text{s}} + \dfrac{0.19 \text{ m}^2}{\text{V} \cdot \text{s}} \right)} = \mathbf{0.45\, \Omega \cdot m}$$

13.30 Explain why the electrical conductivity of intrinsic silicon and germanium increases with increasing temperature.

The electrical conductivity of intrinsic silicon and germanium increases with increasing temperature because the additional thermal energy allows for the formation of more electron-hole pairs.

13.31 The electrical resistivity of pure germanium is 0.46 $\Omega \cdot$m at 300 K. Calculate its electrical conductivity at 425°C.

Since the resistivity, and thus the conductivity, is given for room temperature, we can use the relation $\sigma = \sigma_0 \exp \dfrac{-E_g}{2kT}$ to solve two equations simultaneously.

$$\sigma_{300 \text{ K}} = \sigma_0 \exp \left[\left(\frac{-E_g}{2k} \right) \frac{1}{T_{300 \text{ K}}} \right], \qquad \sigma_{698 \text{ K}} = \sigma_0 \exp \left[\left(\frac{-E_g}{2k} \right) \frac{1}{T_{698 \text{ K}}} \right]$$

Dividing the second equation by the first, σ_0 is eliminated. For germanium, the band energy gap is $E_g = -0.67 \text{ eV}$. Thus,

$$\frac{\sigma_{698 \text{ K}}}{\sigma_{300 \text{ K}}} = \exp \left[\frac{-0.67 \text{ eV}}{2 \left(8.62 \times 10^{-5} \text{ eV/K} \right)} \left(\frac{1}{698} - \frac{1}{300} \right) \right]$$

$$\sigma_{698 \text{ K}} = \sigma_{300 \text{ K}} \exp \left(7.3866 \right) = \left(\frac{1}{0.46 \ \Omega \cdot \text{m}} \right) \left(1614 \right) = \mathbf{3509 \ \left(\Omega \cdot m \right)^{-1}}$$

13.32 The electrical resistivity of pure silicon is 2.3×10^3 $\Omega \cdot$m at 300 K. Calculate its electrical conductivity at 325°C.

Since the resistivity is given for room temperature, we can use the relation $\sigma = \sigma_0 \exp \dfrac{-E_g}{2kT}$ to solve two equations simultaneously.

$$\sigma_{300\ K} = \sigma_0 \exp\left[\left(\frac{-E_g}{2k}\right)\frac{1}{T_{300\ K}}\right], \qquad \sigma_{598\ K} = \sigma_0 \exp\left[\left(\frac{-E_g}{2k}\right)\frac{1}{T_{598\ K}}\right]$$

Dividing the second equation by the first, σ_0 is eliminated giving,

$$\frac{\sigma_{598\ K}}{\sigma_{300\ K}} = \exp\left[\frac{-1.1\ eV}{2\left(8.62\times10^{-5}\ eV/K\right)}\left(\frac{1}{598} - \frac{1}{300}\right)\right]$$
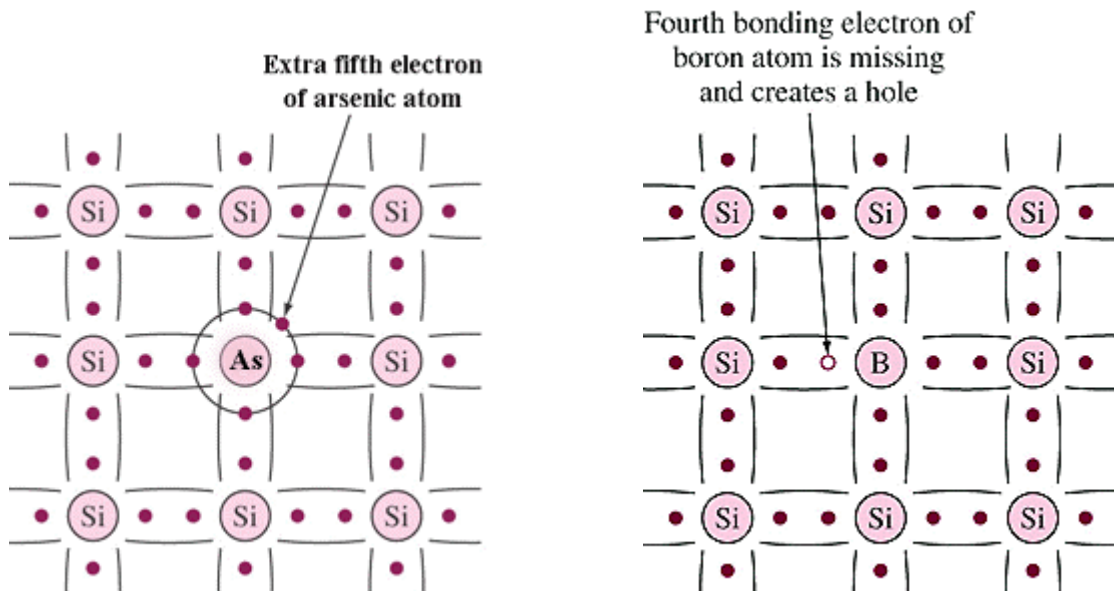
$$\sigma_{598\ K} = \sigma_{300\ K}\exp(10.5986) = \left(\frac{1}{2.3\times10^3\ \Omega\cdot m}\right)(40{,}079) = \mathbf{17.4\ (\Omega\cdot m)^{-1}}$$

13.33 Define n-type and p-type extrinsic silicon semiconductors.

An n-type (negative-type) extrinsic silicon semiconductor is a semiconducting material that was produced by doping silicon with an n-type element of Group V A, such as P, As, or Sb. Consequently, electrons are the majority charge carriers of the material. A p-type (positive-type) extrinsic silicon semiconductor is a semiconducting material that was produced by doping silicon with an p-type element of group III A, such as B, Al, or Ga. Since the dopants are acceptor atoms, holes are the majority charge carriers of the material.

13.34 Draw two-dimensional lattices of silicon of the following:
(a) n-type lattice with an arsenic impurity atom present
(b) p-type lattice with a boron impurity atom present.

Extra fifth electron of arsenic atom

Fourth bonding electron of boron atom is missing and creates a hole
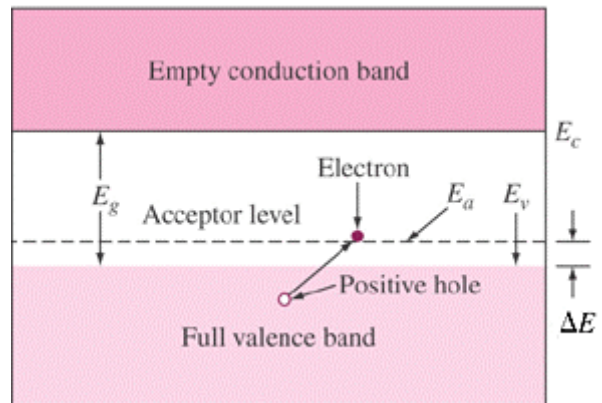
(a) N-type silicon with arsenic impurity atom (As^{5+}) has an extra electron in the Si lattice.

(b) P-type silicon with boron impurity atom (B^{3+}) contains a hole in the Si lattice.

13.35   Draw energy-band diagrams showing donor or acceptor levels for the following:
       (a) n-type silicon with phosphorus impurity atoms
       (b) p-type silicon with boron impurity atoms
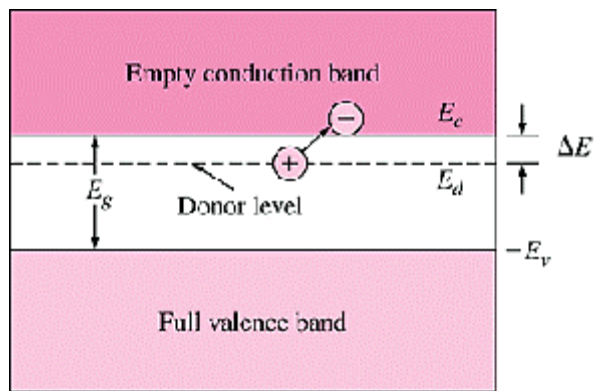
(a) Energy-band diagram for n-type extrinsic silicon with phosphorus impurity atoms. ($\Delta E = E_c - E_d$)

(b) Energy-band diagram for p-type extrinsic silicon with boron impurity atoms. ($\Delta E = E_a - E_v$)

13.36   (a) When a phosphorus atom is ionized in an n-type silicon lattice what charge does the ionized atom acquire?
       (b) When a boron atom is ionized in a p-type silicon lattice, what charge does the ionized atom acquire?

       (a) The ionized phosphorus acquires a charge of positive one (+1).
       (b) The ionized boron atom acquires a charge of negative one (−1).

13.37   What are dopants as pertains to semiconductors? Explain the process of doping by diffusion.

       As pertaining to semiconductors, dopants are impurity atoms intentionally introduced into the semiconducting material. When the dopants are added by diffusion, the doping atoms are initially deposited on a selected surface area of the semiconductor via a gaseous atmosphere. This process is conducted with the semiconductor at high temperature, typically 1100°C for silicon. Subsequently, a heat treatment in a neutral atmosphere is used to drive the layer of doping material deeper into the semiconductor and to reduce the concentration gradient of the dopant.

13.38   What are the majority and minority carriers in an n-type silicon semiconductor? In a p-type one?

       In an n-type silicon semiconductor, electrons are the majority charge carriers and holes are the minority charge carriers. Whereas inn a p-type silicon semiconductor, holes are the majority charge carriers and electrons are the minority charge carriers.

13.39 A silicon wafer is doped with 7.0×10^{21} phosphorus atoms/m³. Calculate (a) the electron and hole concentrations after doping and (b) the resultant electrical resistivity at 300 K. $\left[\text{Assume } n_i = 1.5\times10^{16} \text{ /m}^3 \text{ and } \mu_n = 0.1350 \text{ m}^2/(\text{V}\cdot\text{s}).\right]$

(a) Since the silicon is doped with phosphorus, a group V element, the semiconductor is n-type and $N_a \approx 0$. The electron and hole concentrations after doping are thus,

$$n_n = N_d = \mathbf{7.0\times10^{21}} \text{ electrons/m}^3$$

$$p_n = \frac{n_i^2}{N_d} = \frac{\left(1.5\times10^{16} \text{ carriers/m}^{-3}\right)^2}{7.0\times10^{21} \text{ holes/m}^3} = \mathbf{3.21\times10^{10}} \text{ holes/m}^3$$

(b) Assuming $n_i = 1.5\times10^{16}$ /m³ and $\mu_n = 0.1350$ m² /(V·s), the resultant resistivity at 300 K is,

$$\rho = \frac{1}{q\mu_n n_n} = \frac{1}{\left(1.60\times10^{-19} \dfrac{\text{C}}{\text{carrier}}\right)\left(0.1350 \dfrac{\text{m}^2}{\text{V}\cdot\text{s}}\right)\left(7.0\times10^{21} \dfrac{\text{electrons}}{\text{m}^3}\right)}$$

$$= \mathbf{6.61\times10^{-3}} \ \mathbf{\Omega\cdot m}$$

13.40 Phosphorus is added to make an n-type silicon semiconductor with an electrical conductivity of $250 \ (\Omega\cdot\text{m})^{-1}$. Calculate the necessary number of charge carriers required.

The number of electron charge carriers required is found based upon the resistivity. Assuming an electron mobility of $\mu_n = 0.1350$ m² /(V·s),

$$\rho = \frac{1}{\sigma} = \frac{1}{q\mu_n n_n}$$

$$n_n = \frac{\sigma}{q\mu_n} = \frac{\left[250 \ (\Omega\cdot\text{m})^{-1}\right]}{(1.60\times10^{-19} \text{ C})\left[0.1350 \text{ m}^2/(\text{V}\cdot\text{s})\right]} = \mathbf{1.16\times10^{22}} \text{ electrons/m}^3$$

13.41 A semiconductor is made by adding boron to silicon to give an electrical resistivity of $1.90 \ \Omega\cdot\text{m}$. Calculate the concentration of carriers per cubic meter in the material. $\left[\text{Assume } \mu_p = 0.048 \text{ m}^2/(\text{V}\cdot\text{s}).\right]$

For this p-type semiconductor, the concentration of holes per cubic meter is:

$$p_p = \frac{1}{\rho q\mu_p} = \frac{1}{(1.90 \ \Omega\cdot\text{m})(1.60\times10^{-19} \text{ C})\left[0.048 \text{ m}^2/(\text{V}\cdot\text{s})\right]} = \mathbf{6.85\times10^{19}} \text{ holes/m}^3$$

13.42   A silicon wafer is doped with 2.50×10^{16} boron atoms/cm^3 plus 1.60×10^{16} phosphorus atoms/ cm^3 at 27°C. Calculate (a) the electron and hole concentrations (carriers per cubic centimeter), (b) the electron and hole mobilities (use Fig. 13.26), and (c) the electrical resistivity of the material.

(a) *Majority-carrier concentration:* The net concentration of holes, the majority carriers, is the difference between acceptor and donor ion concentrations.

$$P_p \; ; \;\; N_a - N_d = 2.5 \times 10^{16} \text{ B atoms/cm}^3 - 1.6 \times 10^{16} \text{ P atoms/cm}^3 = \mathbf{9.0 \times 10^{15} \text{ holes/cm}^3}$$

Minority-carrier concentration: The concentration of the electrons is thus,

$$n_p = \frac{n_i^2}{P_p} = \frac{\left(1.5 \times 10^{10} \text{ cm}^{-3}\right)^2}{9.0 \times 10^{15} \text{ carriers/cm}^3} = \mathbf{2.50 \times 10^4 \text{ electrons/cm}^3}$$

(b) *Mobility of electrons and holes:* Obtain the electron and hole mobilities from Fig. 13.26 for a total ionized impurity concentration of $C_T = 4.1 \times 10^{16}$ ions/cm^3.

$$\mu_n \approx \mathbf{850 \text{ cm}^2/(V \cdot s)} \quad \text{and} \quad \mu_p \approx \mathbf{250 \text{ cm}^2/(V \cdot s)}$$

(c) *Electrical resistivity:* Since the doped semiconductor is p-type, we have

$$\rho = \frac{1}{q\mu_p P_p} = \frac{1}{\left(1.60 \times 10^{-19} \text{ C/hole}\right)\left[250 \text{ cm}^2/(V \cdot s)\right]\left(9.0 \times 10^{15} \text{ holes/cm}^3\right)}$$

$$= \mathbf{2.78 \; \Omega \cdot cm}$$

13.43   A silicon wafer is doped with 2.50×10^{15} phosphorus atoms/cm^3, 3.00×10^{17} boron atoms/ cm^3, and 3.00×10^{17} arsenic atoms/ cm^3. Calculate (a) the electron and hole concentrations (carriers per cubic centimeter), (b) the electron and hole mobilities (use Fig. 13.26), and (c) the electrical resistivity of the material.

(a) *Majority-carrier concentration:* The net concentration of electrons, the majority carriers, is the difference between donor and acceptor ion concentrations.

$$n_n \; ; \;\; N_d - N_a = 2.5 \times 10^{15} \text{ P atoms/cm}^3 + 3.0 \times 10^{17} \text{ As atoms/cm}^3 - 3.0 \times 10^{17} \text{ B atoms/cm}^3$$

$$= \mathbf{2.5 \times 10^{15} \text{ electrons/cm}^3}$$

Minority-carrier concentration: The concentration of the holes is thus,

$$p_n = \frac{n_i^2}{n_n} = \frac{\left(1.5 \times 10^{10} \text{ cm}^{-3}\right)^2}{2.5 \times 10^{15} \text{ carriers/cm}^3} = \mathbf{9.0 \times 10^4 \text{ holes/cm}^3}$$

(b) *Mobility of electrons and holes:* Obtain the electron and hole mobilities from Fig. 13.26 for a total ionized impurity concentration of $C_T = 6.025 \times 10^{17}$ ions/cm^3.

$$\mu_n \approx 400 \text{ cm}^2/(V \cdot s) \quad \text{and} \quad \mu_p \approx 125 \text{ cm}^2/(V \cdot s)$$

(c) *Electrical resistivity:* Since the doped semiconductor is n-type, we have

$$\rho = \frac{1}{q\mu_n n_n} = \frac{1}{(1.60 \times 10^{-19} \text{ C/electron})[400 \text{ cm}^2/(V \cdot s)](2.5 \times 10^{15} \text{ electrons/cm}^3)}$$

$$= 6.25 \, \Omega \cdot \text{cm}$$

13.44 An arsenic doped silicon wafer has an electrical resistivity of 7.5×10^{-4} $\Omega \cdot$cm at 27°C. Assume intrinsic carrier mobilities and complete ionization.
(a) What is the majority-carrier concentration (carriers per cubic centimeter)?
(b) What is the ratio of arsenic to silicon atoms in this material?

(a) Assuming the mobility of the carriers is equal to that of electrons in silicon at 300 K, the electron concentration for this n-type semiconductor is:

$$n_n = \frac{1}{\rho q \mu_n}$$

$$= \frac{1}{(7.5 \times 10^{-4} \, \Omega \cdot \text{cm})(1.60 \times 10^{-19} \text{ C/electron})[0.1350 \text{ m}^2/(V \cdot s)](10^4 \text{ cm}^2/\text{m}^2)}$$

$$= 6.17 \times 10^{18} \text{ electrons/cm}^3$$

(b) To determine the ratio of arsenic to silicon atoms, the concentration of Si atoms must first be calculated.

$$\text{Si atoms/cm}^3 = \frac{\rho N_0}{\text{atomic weight}} = \frac{(2.33 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{28.09 \text{ g/mol}}$$

$$= 4.99 \times 10^{22} \text{ atoms/cm}^3$$

Thus,

$$\frac{\text{As atoms}}{\text{Si atoms}} = \frac{6.17 \times 10^{18} \text{ atoms/cm}^3}{4.99 \times 10^{22} \text{ atoms/cm}^3} = 1.24 \times 10^{-4}$$

13.45 A boron-doped silicon wafer has an electrical resistivity of 5.0×10^{-4} $\Omega \cdot$cm at 27°C. Assume intrinsic carrier mobilities and complete ionization.
(a) What is the majority-carrier concentration (carriers per cubic centimeter)?
(b) What is the ratio of boron to silicon atoms in this material?

(a) Assuming the mobility of the hole charge carriers is equal to 0.048 , the hole concentration for this p-type semiconductor is:

$$p_p = \frac{1}{\rho q \mu_p}$$

$$= \frac{1}{(5.0 \times 10^{-4}\ \Omega \cdot cm)(1.60 \times 10^{-19}\ C/electron)\left[0.048\ m^2/(V \cdot s)\right](10^4\ cm^2/m^2)}$$

$$= 2.60 \times 10^{19}\ \textbf{holes/cm}^3$$

(b) To determine the ratio of boron to silicon atoms, the concentration of Si atoms must first be calculated.

$$\text{Si atoms/cm}^3 = \frac{\rho N_0}{\text{atomic weight}} = \frac{(2.33\ g/cm^3)(6.02 \times 10^{23}\ atoms/mol)}{28.09\ g/mol}$$

$$= 4.99 \times 10^{22}\ \text{atoms/cm}^3$$

Thus,

$$\frac{\text{As atoms}}{\text{Si atoms}} = \frac{2.60 \times 10^{19}\ \text{atoms/cm}^3}{4.99 \times 10^{22}\ \text{atoms/cm}^3} = 5.21 \times 10^{-4}$$

13.46 Describe the origin of the three stages that appear in the plot of $\ln \sigma$ versus $1/T$ for an extrinsic silicon semiconductor (going from low to high temperatures). Why does the conductivity decrease just before the rapid increase due to intrinsic conductivity?

In the first stage of the plot, the intrinsic range, high temperatures impart sufficient energy for the electrons to hurdle the semiconductor gap, E_g . Consequently, intrinsic conduction dominates and the natural log of the conductivity varies inversely with $1/T$ as $-E_g/2k$. The second stage is respectively called the exhaustion range and the saturation range for n-type and p-type semiconductors. Within this moderate temperature range, n-type donor atoms or p-type acceptor atoms become completely ionized. As a result, the electrical conduction decreases as the intrinsic range is approached. In the third stage, the extrinsic rage, low temperatures are sufficient to excite a donor electron into the conduction band of an n-type extrinsic semiconductor $(E_c - E_d)$ or a valence electron into the acceptor level of a p-type extrinsic semiconductor $(E_a - E_v)$. The slope of the curve in this region, is $-(E_c - E_d)/k$ and $-(E_a - E_v)/k$ for n-type and p-type extrinsic semiconductors, respectively.

13.47 Define the term *microprocessor*.

A microprocessor is a "computer on a chip" and is commonly referred to as the Central Processing Unit (CPU) of a personal computer. This chip is made of silicon and incorporates millions of transistors which constitute a computing engine.

13.48 Describe the movement of majority carriers in a pn junction diode at equilibrium. What is the depletion region of a pn junction?

In a pn junction diode at equilibrium, there is no movement of majority carriers due to the potential difference established within the depletion region. This depletion region, formed at the junction of the n-type and p-type semiconductors by the diffusion and recombination of majority carriers, consists of large heavy negatively charged ions on the p-type border and positively charged ions on the n-type border. The opposing charges create an electrical potential and repel the motion of majority carriers.

13.49 Describe the movement of the majority and minority carriers in a pn junction diode under reverse bias.

Under reverse bias, majority carriers move away from the pn junction, and thus increase the depletion width, while minority carriers flow toward the junction and create a very small leakage current on the order of microamperes.

13.50 Describe the movement of the majority carriers in a pn junction diode under forward bias.

Under forward bias, the majority carriers in a pn junction diode flow toward the junction. As a result, the energy barrier of the depletion region is reduced, allowing some electrons and holes to cross the junction and recombine. Consequently, a significant current flow is established.

13.51 Describe how a pn junction diode can function as a current rectifier.

A pn junction diode can function as a current rectifier when an AC signal is applied such that the p region has a positive voltage applied and the n region is subjected to a negative voltage. The resulting positive half-wave rectification produced can be smoothed by other electronic devices and circuits such that the final output is a steady DC signal.

13.52 What is zener diode? How does this device function? Describe a mechanism to explain its operation.

A zener diode, or breakdown diode, is a rectifier diode subjected to reverse-bias voltage which is used for voltage-limiting and voltage-stabilizing applications. As the applied voltage is increased, the small leakage current remains fairly constant until the "breakdown" or zener voltage is reached and surpassed. The current then increases dramatically. Theoretically, this avalanche effect is initiated by electrons escaping the covalent lattice bonds by the attractive force of the applied electric field, and then amplified through the collisions, and thus energy exchange, of the free electrons with bonded lattice electrons.

13.53 What are the three basic elements of a bipolar junction transistor?

The three basic elements of a bipolar junction transistor are the emitter, the base, and the collector.

13.54   Describe the flow of electrons and holes when an npn bipolar junction transistor functions as a current amplifier.

When an npn bipolar junction transistor functions as a current amplifier, the large majority of electrons, 95 to 99 percent, flow from the emitter through the base and into the collector. The remaining electrons recombine with the holes that flow from the collector to the base. Very few holes flow into the emitter.

13.55   What fabrication techniques are used to encourage electrons from the emitter of an npn bipolar transistor to go right through to the collector?

To encourage electrons to flow from the emitter of an npn bipolar transistor directly through to the collector, the emitter is heavily doped with electrons and the base is made very thin with a light doping of holes.

13.56   Why is a bipolar junction transistor called bipolar?

A bipolar junction transistor is called bipolar because both majority charge carriers, the electrons and the holes, are involved in its operation.

13.57   Describe the structure of a planar npn bipolar transistor.

A microelectronic planar bipolar transistor is fabricated by repeatedly doping the surface of a single crystal of silicon. First p-type impurities are introduced throughout the silicon substrate. Next, large islands of n-type silicon are formed. Within these islands, smaller p- and n-type islands are created to serve as the emitter, base and collector.

13.58   Describe how the planar bipolar transistor can function as a current amplifier.

Similar to macroscopic bipolar transistors, the microscopic version has a forward-biased emitter-base junction and a reverse-biased base-collector junction. Thus, the bulk of electrons flow into the collector while 1 to 5 percent recombine with holes from the base terminal. Thus the microelectronic device is capable of amplifying current through the flow of emitter charge carriers.

13.59   Describe the structure of an n-type metal oxide semiconductor field-effect transistor (NMOS).

In an n-type NMOS or MOSFET, two islands of n-type silicon, formed within a p-type silicon substrate, serve as contacts respectively called the source and the drain. In the p-type region between these islands, a layer of silicon dioxide is created on the surface that acts as an electrical insulator. A polysilicon or metal layer is then deposited on this glass layer to form the third contact, referred to as the gate. A schematic depicting the structure of the NMOS device is shown on the following page.
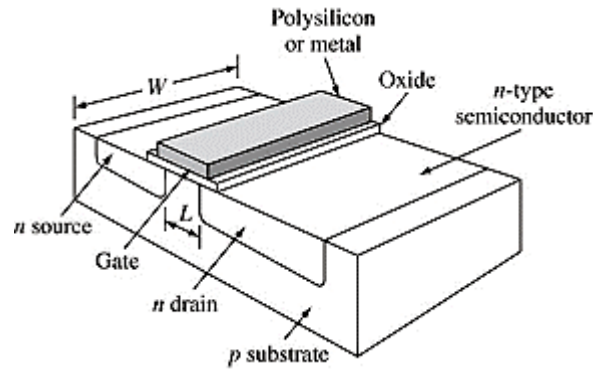
Schematic of NMOS structure

13.60 How do NMOSs function as current amplifiers?

When a positive voltage is applied to the gate of an NMOS device, its electric field attracts electrons from the n^+ source and drain regions into the thin region of silicon beneath the insulating silicon dioxide layer. As a result, this thin channel region becomes n-type silicon in which electrons are the majority carriers. If a positive voltage difference exists between the drain and source, the channel serves as a conducting path for electron flow between the drain and source.
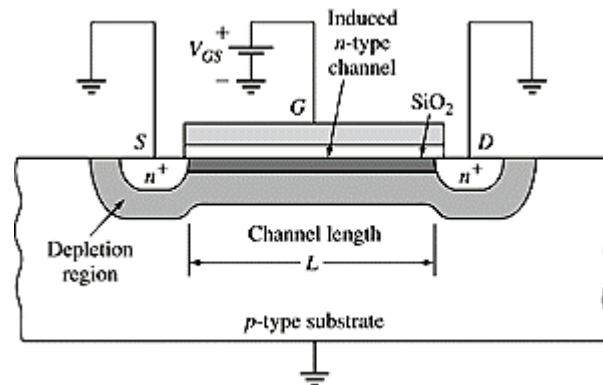

13.61 Describe the photolithographic steps necessary to produce a pattern of an insulating layer of silicon dioxide on a silicon surface.

The photolithographic steps necessary to produce a pattern of an insulating layer of silicon dioxide on a silicon surface are:

1. Oxidize the surface of a single crystal silicon wafer to form a layer of SiO_2.
2. Coat the silicon dioxide with a layer of a light-sensitive material called a photoresist which becomes insoluble to organic solvents after exposure to radiation.
3. Expose selected areas of the photoresist to ultraviolet radiation through a photomask that defines the desired pattern or circuit.

4. Apply a developer solution (an organic solvent) to remove the regions of the photoresist that were not exposed to the ultraviolet radiation.
5. Immerse the wafer in a solution of hydrofluoric acid to selectively attack the portions of the silicon dioxide layer not protected by the photoresist pattern.
6. Remove the remaining photoresist through a chemical treatment.
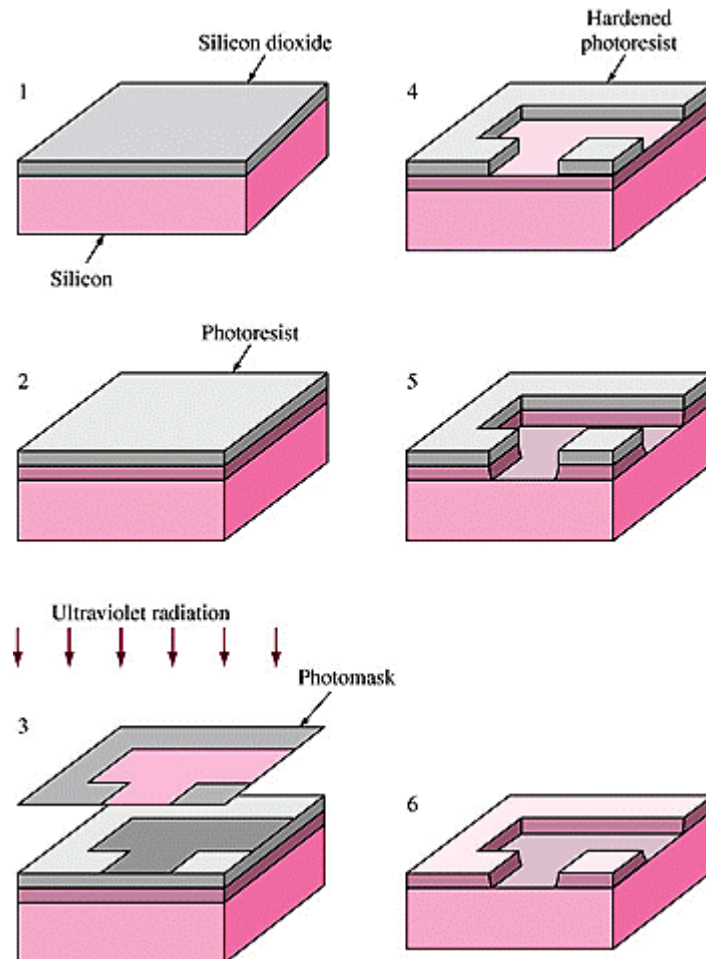
13.62 Describe the diffusion process for the introduction of dopants into the surface of a silicon wafer.

Dopants can be selectively introduced into specific regions of a silicon substrate through a high temperature diffusion process in which silicon dioxide patterns serve as masks. The glass masks prevent the dopant atoms from penetrating the silicon beneath them during a two-step diffusion process: first a rack of wafers are placed in a furnace at 1000 to 1100°C and exposed to an atmosphere containing the dopant atoms; then the rack is placed in a second furnace, set at a higher temperature to drive the dopant atoms to a specific depth below the wafer surface.

13.63   Describe the ion implantation process for introducing dopants into the surface of a silicon wafer.

In the ion implantation process, specific areas of silicon wafers are selectively doped at room temperature using photoresist or silicon dioxide masks and high speed dopant ions. The dopant atoms are first ionized and then accelerated to high energies through a high potential field of 50 to 100 kV. Upon striking the silicon wafer, the ions are embedded to depths corresponding to the mass and energy of the particles. Ion implantation allows for highly controlled doping levels and for the creation of NMOS and PMOS transistors on the same wafer.

13.64   Describe the general process for fabricating NMOS integrated circuits on a silicon wafer.

The general process for fabricating NMOS integrated circuits on a silicon wafer are as follows:

1.   A chemical vapor deposition (CVD) process deposits a thin layer of silicon nitride (Si_3N_4) on the entire wafer surface. A photolithographic step defines the transistor areas and the remaining silicon nitride is removed by chemical etching. The exposed regions are doped with boron ions and then oxidized to suppress conduction between the transistor sites.
2.   The Si_3N_4 is removed by an etchant that does not attack SiO_2. A clean thermal oxide is grown to about 0.1 μm thickness in the transistor areas. A CVD process then deposits polycrystalline silicon (poly) over the entire wafer. A second photolithographic process then defines the desired patterns for the gate electrodes. Subsequently, the undesired poly is removed by etching and an n-type dopant is introduced, through thermal diffusion or ion implantation, into the regions that will serve as the transistor source and drain.
3.   Another CVD process deposits an insulating layer over the entire wafer. This third masking step defines the areas for transistor contacts. The bare silicon or poly in these areas is exposed through chemical or plasma etching.
4.   Aluminum is deposited over the entire wafer by evaporation from a hot crucible in a vacuum evaporator. A fourth masking step patterns the Al for the desired circuit connections.
5.   A protective passivating layer is deposited over the entire surface and a final masking step is performed to remove this insulating layer from the pads where contacts will be made. Circuits are tested by using needlelike probes on the contact pads. Defective units are marked and the wafer is sawed into individual chips. Good chips are packaged and tested a second time.

13.65   Why is silicon nitride (Si_3N_4) used in producing NMOS integrated circuits on a silicon wafer?

Silicon nitride is used in producing NMOS integrated circuits on a silicon wafer because it is soluble to etchants which do not attack silicon dioxide; thus, the Si_3N_4 can be

removed from selected areas for additional processing while areas with a SiO$_2$ layer remain protected.

13.66 What are complementary metal oxide semiconductor (CMOS) devices? What are the advantages of CMOS devices over the NMOS or PMOS devices?

Complementary metal oxide semiconductor (CMOS) devices are circuits containing both NMOS and PMOS devices. Their advantages over the NMOS or PMOS devices are: the MOS devices can be arranged to achieve lower power consumption; and a higher density of transistors can fit on a single wafer.

13.67 Calculate the intrinsic electrical conductivity of GaAs at 125°C. $\left[E_g = 1.47 \text{ eV}; \right.$

$\mu_n = 0.720 \text{ m}^2 / (\text{V} \cdot \text{s}); \mu_p = 0.020 \text{ m}^2 / (\text{V} \cdot \text{s}); n_i = 1.4 \times 10^{12} \text{ m}^{-3}.\left.\right]$

First determine σ_0, the intrinsic electrical conductivity, for GaAs at 27°C (300 K):

$$\sigma = \sigma_0 = n_i q(\mu_n + \mu_p)$$
$$= (1.4 \times 10^{12} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})\left[0.720 \text{ m}^2 / (\text{V} \cdot \text{s}) + 0.020 \text{ m}^2 / (\text{V} \cdot \text{s})\right]$$
$$= 1.66 \times 10^{-7} \ (\Omega \cdot \text{m})^{-1}$$

Next, calculate σ at 125°C (398 K) for the energy gap of 1.47 eV:

$$\sigma = \sigma_0 e^{-E_g / 2kT}$$
$$\frac{\sigma_{398 \text{ K}}}{\sigma_{300 \text{ K}}} = \frac{\exp\left\{-1.47 \text{ eV} / \left[(2)(8.62 \times 10^{-5} \text{ eV/K})(398 \text{ K})\right]\right\}}{\exp\left\{-1.47 \text{ eV} / \left[(2)(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})\right]\right\}}$$
$$\sigma_{398 \text{ K}} = \sigma_{300 \text{ K}} \left(e^{6.998}\right) = \left[1.66 \times 10^{-7} (\Omega \cdot \text{m})^{-1}\right](1094.9)$$
$$= \mathbf{1.82 \times 10^{-4} \ (\Omega \cdot m)^{-1}}$$

13.68 Calculate the intrinsic electrical conductivity of InSb at 60 and at 70°C. $\left[E_g = 0.17 \text{ eV}; \right.$

$\mu_n = 8.00 \text{ m}^2 / (\text{V} \cdot \text{s}); \mu_p = 0.045 \text{ m}^2 / (\text{V} \cdot \text{s}); n_i = 1.35 \times 10^{22} \text{ m}^{-3}.\left.\right]$

First determine σ_0, the intrinsic electrical conductivity, for InSb at 27°C (300 K):

$$\sigma = \sigma_0 = n_i q(\mu_n + \mu_p)$$
$$= (1.35 \times 10^{22} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})\left[8.0 \text{ m}^2 / (\text{V} \cdot \text{s}) + 0.045 \text{ m}^2 / (\text{V} \cdot \text{s})\right]$$
$$= 1.74 \times 10^4 \ (\Omega \cdot \text{m})^{-1}$$

Next, calculate σ at 60°C (333 K) and 70°C (343 K) for the energy gap of 0.17 eV:

$$\frac{\sigma_{333\,K}}{\sigma_{300\,K}} = \exp\left[\frac{-0.17\text{ eV}}{2\left(8.62\times10^{-5}\text{ eV/K}\right)}\left(\frac{1}{333}-\frac{1}{300}\right)\right]$$

$$\sigma_{333\,K} = \sigma_{300\,K}\exp(0.3257) = \left[1.74\times10^4\ (\Omega\cdot m)^{-1}\right](1.385) = \mathbf{2.41\times10^4}\ \mathbf{(\Omega\cdot m)^{-1}}$$

$$\frac{\sigma_{343\,K}}{\sigma_{300\,K}} = \exp\left[\frac{-0.17\text{ eV}}{2\left(8.62\times10^{-5}\text{ eV/K}\right)}\left(\frac{1}{343}-\frac{1}{300}\right)\right]$$

$$\sigma_{343\,K} = \sigma_{300\,K}\exp(0.41206) = \left[1.74\times10^4\ (\Omega\cdot m)^{-1}\right](1.51) = \mathbf{2.63\times10^4}\ \mathbf{(\Omega\cdot m)^{-1}}$$

13.69 Calculate the intrinsic electrical conductivity of (a) GaAs and (b) InSb at 75°C.

(a) From Prob. 13.67, $\sigma_0 = 1.66\times10^{-7}\ (\Omega\cdot m)^{-1}$ and $E_g = 1.47$ eV for GaAs at 27°C (300 K). Thus, for 75°C (348 K), the intrinsic electrical conductivity is:

$$\frac{\sigma_{348\,K}}{\sigma_{300\,K}} = \exp\left[\frac{-1.47\text{ eV}}{2\left(8.62\times10^{-5}\text{ eV/K}\right)}\left(\frac{1}{348}-\frac{1}{300}\right)\right]$$

$$\sigma_{348\,K} = \sigma_{300\,K}\exp(3.9203) = \left[1.66\times10^{-7}\ (\Omega\cdot m)^{-1}\right](50.42) = \mathbf{8.37\times10^{-6}}\ \mathbf{(\Omega\cdot m)^{-1}}$$

(b) From Prob. 13.68, $\sigma_0 = 1.74\times10^4\ (\Omega\cdot m)^{-1}$ and $E_g = 0.17$ eV for InSb at 27°C (300 K). Thus, for 75°C (348 K), the intrinsic electrical conductivity is:

$$\frac{\sigma_{348\,K}}{\sigma_{300\,K}} = \exp\left[\frac{-0.17\text{ eV}}{2\left(8.62\times10^{-5}\text{ eV/K}\right)}\left(\frac{1}{348}-\frac{1}{300}\right)\right]$$

$$\sigma_{348\,K} = \sigma_{300\,K}\exp(0.45337) = \left[1.74\times10^4\ (\Omega\cdot m)^{-1}\right](1.57) = \mathbf{2.73\times10^4}\ \mathbf{(\Omega\cdot m)^{-1}}$$

13.70 What fraction of the current is carried by (a) electrons and (b) holes in (i) InSb, (ii) InB, and (iii) InP at 27°C?

(a) The fraction of current carried by electrons in InSb and InP are:

i. InSb: $\dfrac{\mu_n}{\mu_n+\mu_p} = \dfrac{8.000}{8.000+0.045} = \mathbf{0.994}$

ii. InP: $\dfrac{\mu_n}{\mu_n+\mu_p} = \dfrac{0.460}{0.460+0.010} = \mathbf{0.979}$

(b) The fraction of current carried by holes in InSb and InP are:

   i. InSb: $\dfrac{\mu_p}{\mu_n + \mu_p} = \dfrac{0.045}{8.000 + 0.045} = \mathbf{0.006}$

   ii. InP: $\dfrac{\mu_p}{\mu_n + \mu_p} = \dfrac{0.010}{0.460 + 0.010} = \mathbf{0.021}$

13.71 What fraction of the current is carried by (a) electrons and (b) holes in (i) GaSb and (ii) GaP at 27°C?

(a) The fraction of current carried by electrons in GaSb and GaP are:

   i. GaSb: $\dfrac{\mu_n}{\mu_n + \mu_p} = \dfrac{0.500}{0.500 + 0.100} = \mathbf{0.833}$

   ii. GaP: $\dfrac{\mu_n}{\mu_n + \mu_p} = \dfrac{0.030}{0.030 + 0.015} = \mathbf{0.667}$

(b) The fraction of current carried by holes in GaSb and GaP are:

   iii. GaSb: $\dfrac{\mu_p}{\mu_n + \mu_p} = \dfrac{0.100}{0.500 + 0.100} = \mathbf{0.167}$

   iv. GaP: $\dfrac{\mu_p}{\mu_n + \mu_p} = \dfrac{0.015}{0.030 + 0.015} = \mathbf{0.333}$