

## *Topic 6. Radiation Fundamentals*

**Telecommunication Systems Fundamentals**

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### **Concepts in this Chapter**

- *Antennas: definitions and classification*
- *Antenna parameters*
- *Fundamental Theorems: uniqueness and reciprocity. Images' method*
- *Friis' equation*
- *Link Budget of a Radio-Link*

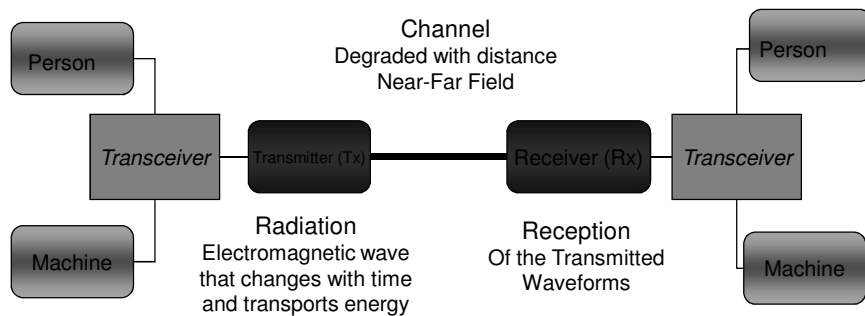
*Theory classes: 1.5 sessions (3 hours)*  
*Problems resolution: 0.5 session (1 hours)*

## Bibliography

Antenna Theory and Design. W.L. Stutzman, G.A. Thiele.  
John Wiley & Sons

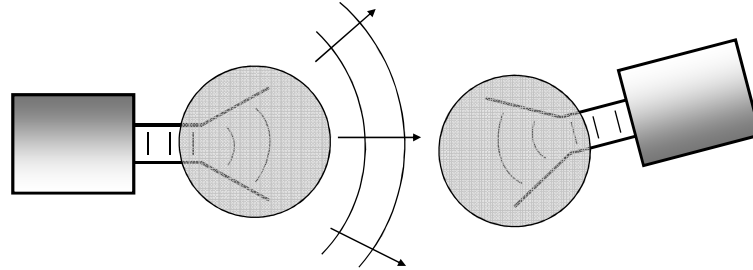
## Introduction: Radio-Telecommunication Systems

- Info transmission implies to transmit a signal (with a given energy) through a radio-channel



## Introduction: Transmitting and Receiving Antenna

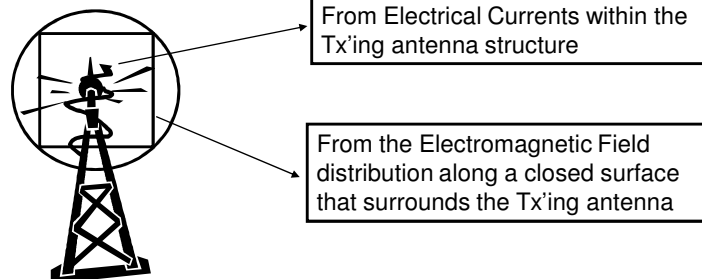
- An antenna can either transmit (radiate) energy in Transmission



- Or capture energy in Reception

## Radiation Performance of an Antenna

- Radiation is the electromagnetic energy flux outward from a source
- Basic Problem in electromagnetic theory:
  - Calculus of the electromagnetic field produced by a structure in any given space point



## Efficiency as Main Objective in Antennas

- Efficiency is the main objective when designing/ selecting an antenna
  - Maximize the electromagnetic field power in a given point given an amount of power provided to the antenna
- Which antenna parameters should we consider
  - Phase Center
  - Power Parameters
    - » Radiated power flux density
    - » Radiation intensity
    - » Directivity
    - » Power Gain
  - Gain diagram
  - Polarization
  - Bandwidth

## Power Parameters: Poynting's Theorem

- **Complex Poynting's Vector:** electromagnetic energy flux density through a given surface

$$S = \frac{1}{2} \operatorname{Re} \{ \bar{E} \times \bar{H}^* \}$$

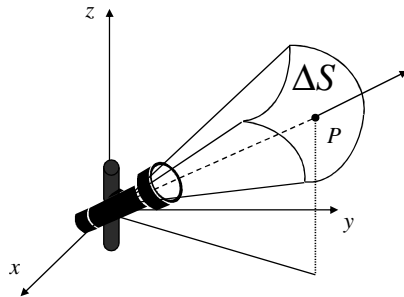
- **Average Power:** Poynting's vector flux

$$P_{media} = \iint S \cdot dS \text{ [W]}$$

## Power Parameters: Radiation Density

- Average radiated power **per surface unit** in a given direction

$$\phi(\theta, \varphi) = \frac{1}{2} \operatorname{Re}\{\bar{E} \times \bar{H}^*\} \left[ \frac{W}{m^2} \right]$$

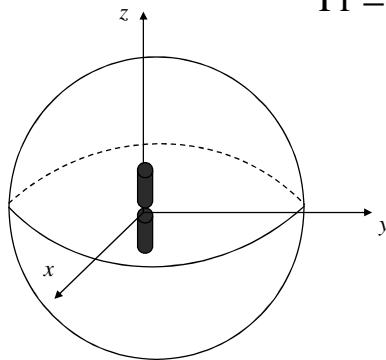


$$\phi(\theta, \varphi) = \frac{\Delta p(\theta, \varphi)}{\Delta S}$$

## Power Parameters: Radiated Power

- **Sum up** radiated flux density along a sphere surface that circumscribe the antenna

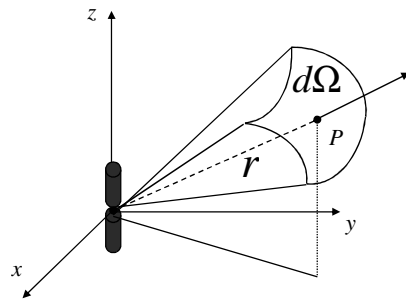
$$Pr = \iint \phi(\theta, \varphi) \cdot dS \text{ [W]}$$



## Power Parameters: Radiation Intensity

- Average radiated power **per solid angle unit** in a given direction

$$i(\theta, \varphi) = \frac{\Delta p(\theta, \varphi)}{\Delta \Omega} \left[ \frac{W}{\text{steradian}} \right]$$



$$Pr = \iint i(\theta, \varphi) \cdot d\Omega$$

## Power Parameters: Radiation Intensity

- **Independently of the distance** from the antenna

$$\Delta S = r^2 \Delta \Omega \Rightarrow \phi(\theta, \varphi) \cdot r^2 = i(\theta, \varphi)$$

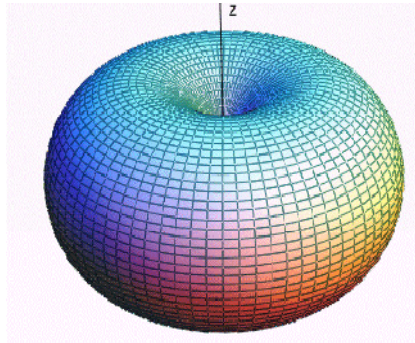
- The Power Flux Density decreases with distance inversely proportional to the area of the spherical solid angle

$$r(\theta, \varphi) = \frac{i(\theta, \varphi)}{i_{\max}}$$

- Radiation Diagram (power-wise)

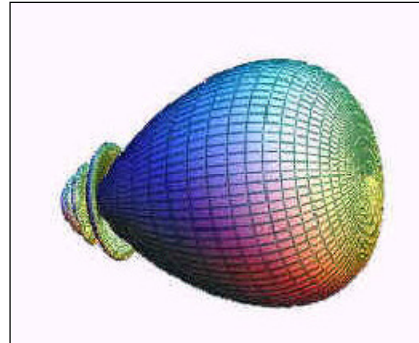
## Power Parameters: Radiation Intensity

Omnidirectional (on Azimuth)



Dipolo: typical on cellular terminals

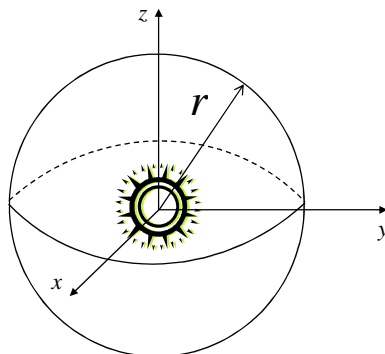
Directive



Yagi: typical for television receivers

## Power Parameters: Isotropic Antenna

- **Ideal** point source that radiates **uniformly** in all directions

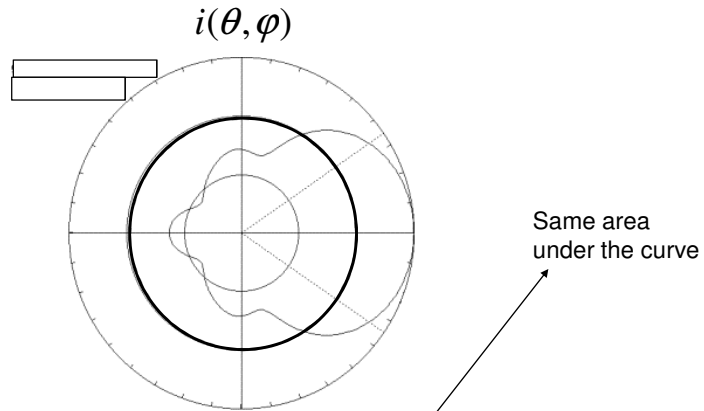


$$i_{iso} = \frac{Pr}{4\pi}$$

$$\phi_{iso} = \frac{Pr}{4\pi \cdot r^2}$$

Reference to compare  
rest of Antennas

## Power Parameters: Isotropic Antenna



Assuming the same transmitted power by both antennas...which *focalizes* better?

## Power Parameters: Directivity (function of direction)

- Ratio between the power density flux an antenna radiates and the one an isotropic (omnidirectional) antenna would do, as a function of the radiating direction

$$D(\theta, \varphi) = \frac{i(\theta, \varphi)}{i_{iso}} = 4\pi \frac{i(\theta, \varphi)}{Pr}$$

$$i(\theta, \varphi) = \frac{Pr}{4\pi} D(\theta, \varphi)$$

$$\phi(\theta, \varphi) = \frac{Pr}{4\pi \cdot r^2} D(\theta, \varphi)$$



## Power Parameters: Directivity

- Directivity is defined as the maximum value of the Directivity function

$$D = 4\pi \frac{i_{\max}}{P_r}$$

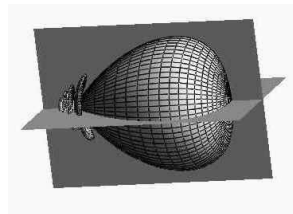
– Because  $P_r = \iint i(\theta, \varphi) \cdot d\Omega$        $r(\theta, \varphi) = \frac{i(\theta, \varphi)}{i_{\max}}$

$$D = \frac{4\pi}{\Omega_A} \quad \text{being} \quad \Omega_A = \iint r(\theta, \varphi) d\Omega$$

## Power Parameters: Directivity

- When the Beam is narrow

$$D \approx \frac{4\pi}{\theta_1 \theta_2}$$



- Conclusions
  - Directivity provides information about how the radiated power is distributed with direction (elevation and azimuth)
  - Directivity does not provide information about the actual transmitted power

## Power Parameters: Gain Function

- **Ratio** between the power intensity radiated in a direction and the radiated intensity of an isotropic antenna, given a power available to the antenna

$$G(\theta, \varphi) = 4\pi \frac{i(\theta, \varphi)}{P_{in}}$$

being  $P_{in}$  the power available at the antenna input

## Power Parameters: Gain

- Gain is the maximum value of the Gain Function

$$G = 4\pi \frac{i_{\max}}{P_{in}}$$

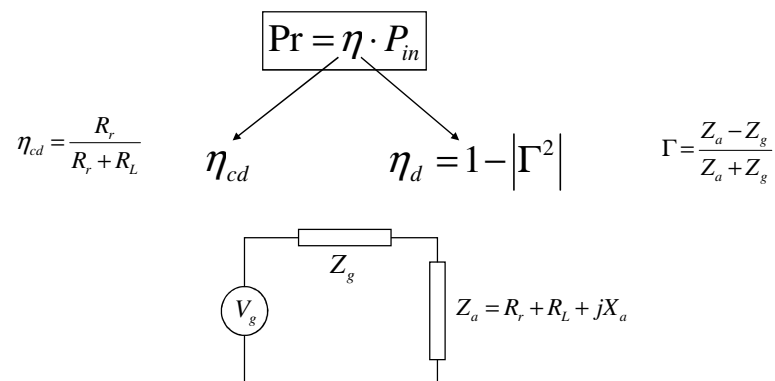
- Because it is a ratio, the units are dBs

## Power Parameters: Examples of Gain

ANTENNA TYPE	GAIN (dBi)
Isotropic	0,0
Ground Plane 1/4 wavelength	1,8
Dipole 1/2 wavelength	2,1
Monopole 5/8 wavelength	3,3
Yagui 2 elements	7,1
Yagui 3 elements	10,1
Yagui 4 elements	12,1
Yagui 5 elements	14,1

## Power Parameters: Efficiency

- $P_{in}$  and  $P_r$  are related to each other around radiating **Efficiency** of the antenna



## Power Parameters: Efficiency

- From the above definition of Efficiency, the relationship between Gain and Directivity of an antenna can be derived

$$G = \eta \cdot D$$

- Can the Gain of an antenna be increased by increasing the Directivity?

## Example

- A dipole of half wavelength without losses, with input impedance of  $73\Omega$  is connected to a transmission line with characteristic impedance of  $50\Omega$ . Assuming the radiating intensity of the antenna is

$$i(\theta) = B_0 \sin^3(\theta)$$

Compute the Gain of the Antenna

$$\begin{aligned}
 i_{\max} &= i_{\max}(\theta) = B_0 \\
 P_r &= \int_0^{2\pi} \int_0^\pi D(\theta) \sin\theta \, d\theta = 2\pi B_0 \int_0^\pi \sin^4\theta \, d\theta = B_0 \left( \frac{3\pi^2}{4} \right) \\
 D &= 4\pi \frac{i_{\max}}{P_r} = 1.697 \\
 G &= \eta \cdot D = (1 - |\Gamma|^2) D = \left( 1 - \left| \frac{73 - 50}{73 + 50} \right|^2 \right) 1.697 = 0.965 \cdot 1.697 = 1.638 = 2.14 \text{ dB}
 \end{aligned}$$

## Example Answer

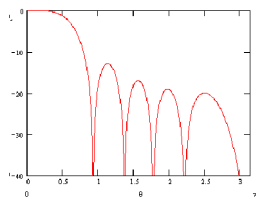
$$i_{\max} = i_{\max}(\theta) = B_0$$

$$P_r = \int_0^{2\pi} \int_0^{\pi} D(\theta) \sin \theta \, d\theta = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0 \left( \frac{3\pi^2}{4} \right)$$

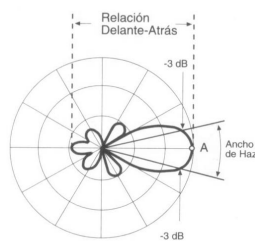
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## Radiation Diagram

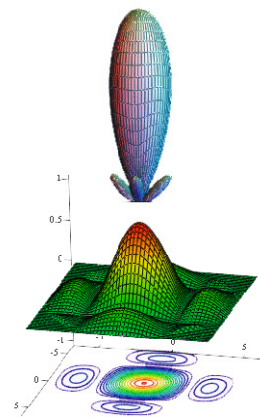


Cartesian



Polar

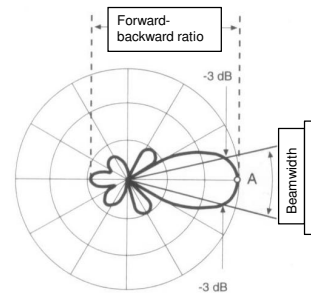
What parameter are useful?



Joint

## Radiation Diagram

- Parameters to characterize the lobe structure
  - Beamwidth
    - Null to Null Beamwidth
    - Half Power Beamwidth (HPBW) – 3dBs
    - 10 dB Beamwidth
  - Lobes
    - Main lobe
    - Side lobes
      - First lobe
    - Backlobe
  - Forward-backward ratio

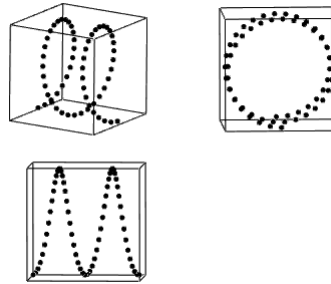


## Radiation Diagram: Classification

- Isotropic
- Omnidirectional
- Directive
- Multi-beam

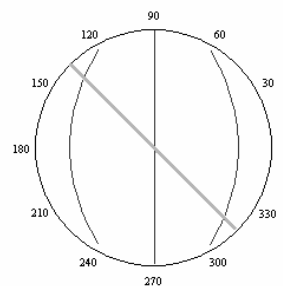
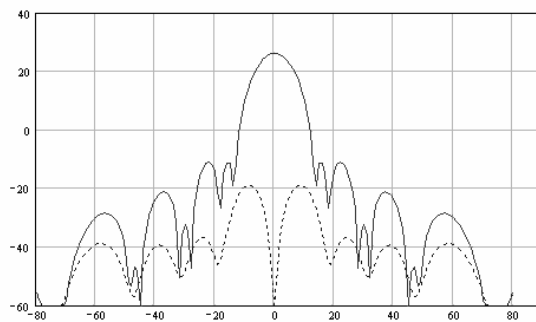
## Polarization

- Of a **Plane-wave**, it refers to the spatial orientation of the time-variation of the electric field
- Of an **antenna**, it refers to the polarization of the radiated field
  - Generally speaking polarization is defined according to the propagation direction



## Polarization

- Co-Polar and Cross-Polar components

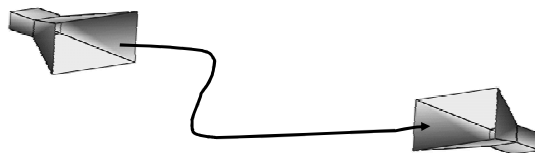


## Antenna Bandwidth

- Frequency margin where the defined parameters for the antenna remain valid (impedance, beamwidth, sidelobes ratio, etc.)
  - Narrowband Antennas (<10% central frequency)
  - Broadband Antennas (>10% central frequency)

## Antenna Radiation on Free-Space Condition

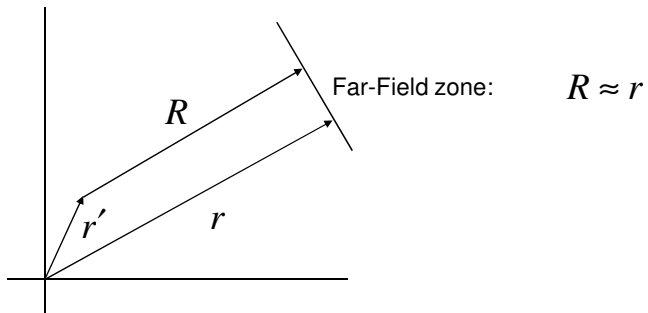
- What is Free-Space condition: no obstacles or material to influence the radiation pattern – not even the ground





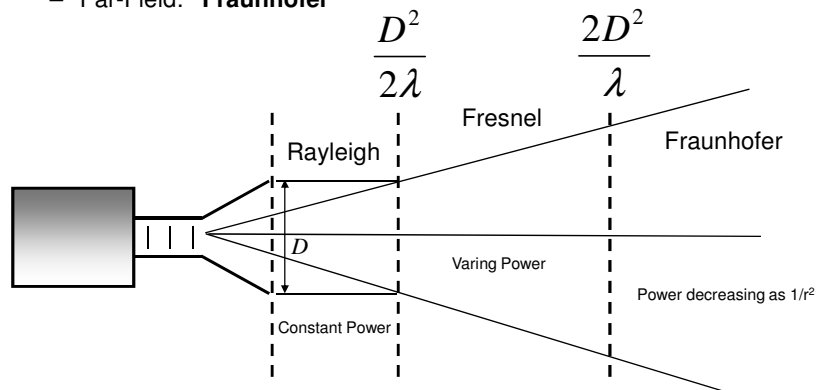
## Radiation Zones

- When the distance is much greater than the wavelength,  $R \gg \lambda$ , the observed wave behaves as a Plane-Wave.
- When can we consider we are “far enough”?



## Radiation Zones

- Simplifying but useful approach: three zones are defined:
  - Near-Field: **Rayleigh** (spheric propagation)
  - Intermediate-Field: **Fresnel** (interferences)
  - Far-Field: **Fraunhofer**



## Radiation Zones: Far-Field

- Conclusions:
  - Power decreases as **square of the distance**
  - Satisfy condition of **Plane-Wave**
    - E and H fields are perpendicular
    - Amplitude of E and H fields are related to each other through the transmission mean impedance
  - The type of the transmitting antenna affects only on the angular variation of the transmitted power flux (**Radiation Diagram**)
  - **Transmission direction of the wave propagation** coincides with **line of sight** of the transmitting antenna

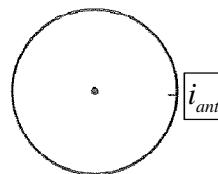
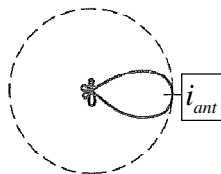
## EIRP - Equivalent Isotropic Radiated Power

- The product of the antenna Gain by the available Power

$$PIRE = G \cdot P_{in}$$

$$i_{iso} = \frac{P_{in}}{4\pi}$$

$$i_{ant} = \frac{G \cdot P_{in}}{4\pi} = \frac{PIRE}{4\pi}$$



## ***EIRP***

- Its units are dBW (or dBm)
- We will see later, this parameter (expressed on dBW) is quite useful when computing the “availability” of the radio-link
- Example: if the Gain of an antenna is 2dBi and it gets a power of 10W, How much is its EIRP?

$$G = 2 \text{ dB}$$

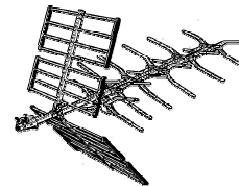
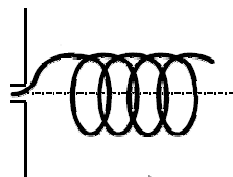
$$P_{in} = 10 \cdot \log(10) = 10 \text{ dB}$$

$$PIRE = 12 \text{ dB}$$

## **Lineal Antennas**

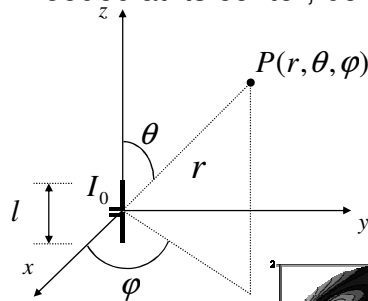
- Antennas built with thin electrically conductive wires (very small diameter compared to  $\lambda$ ).
- They are used extensively in the MF, HF, VHF and UHF bands, and mobile communications.
- Among others:

- Dipole
- Monopole
- Yagi antenna
- Loops
- Helixes



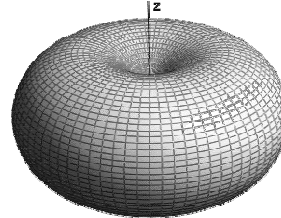
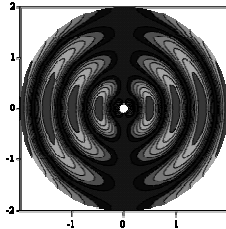
## Lineal Antennas: Infinitesimal Dipole

- Formed by two short conductive wires simetrically feeded at its center, being  $l \ll \lambda$ .



$$E_{\theta} = jZ_0 \frac{l \cdot I_0}{2\pi \cdot \lambda \cdot r} \text{sen}(\theta) \cdot e^{-j\frac{2\pi}{\lambda}r}$$

With Linear Polarization  
And radiation diagram



## Lineal Antennas: Infinitesimal Dipole

- Computing the magnetic field from the electrical, calculating the power flux and integrating for all  $\theta$  we get

$$P_r = Z_0 \left( \frac{2\pi}{3} \right) \left( \frac{l \cdot I_0}{\lambda} \right)^2 = I_0^2 R_r$$

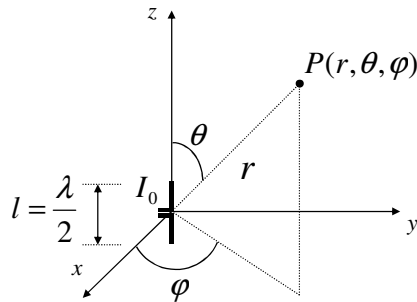
$$R_r = 80\pi^2 \left( \frac{l}{\lambda} \right)^2$$

– and consequently  $D = \frac{3}{2}$

- What is the Gain of this antenna?

## Lineal Antennas: Half-Wavelength Dipole

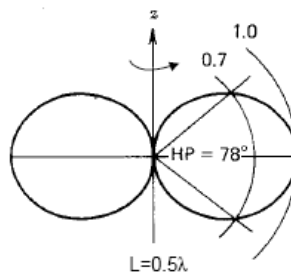
- Very common antenna, with a “convenient” radiation impedance of  $73\Omega$



$$E_{\theta} = jZ_0 \frac{I_0 e^{-j\frac{2\pi}{\lambda}r}}{2\pi \cdot r} \left[ \frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right]$$

## Lineal Antennas: Half-Wavelength Dipole

- Radiation parameters



Directividad:  $D_0 = 1,64 = 2,15 \text{ dBi}$

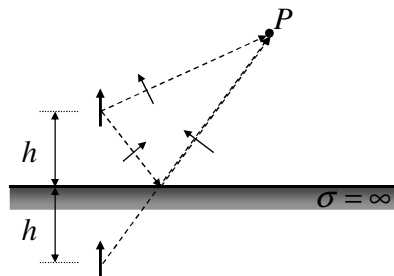
$R_{\text{radiación}}: R_{\text{rad}} = 73 \Omega$

## Radiated Field over a Perfect Conductor

- Up to this point we have assumed the antennas are in the free-space environment. However they usually are close to the ground
- When the distance to ground is comparable to the wavelength, and the beamwidth is large, antenna radiation is heavily affected by the presence of the ground
- For these scenarios, we will assume the ground is a perfect conductor, infinite and plane

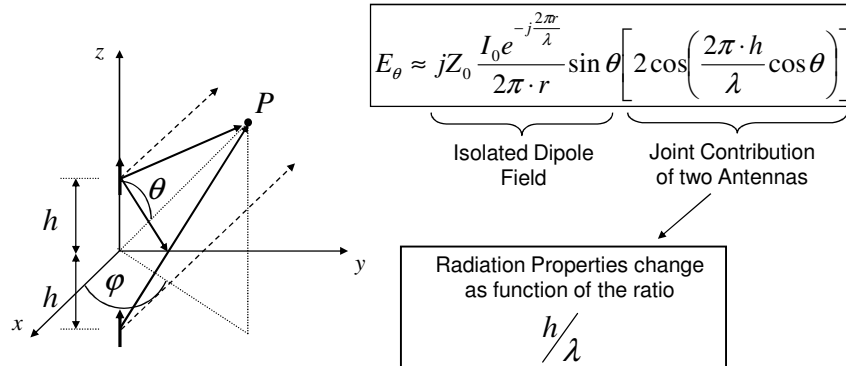
## Image Theory

- Intuitively, the field is reflected on the ground
  - Perfect conductor: the transmitted wave is reflected
  - The field  $P$  is the result of the sum of the direct and reflected waves
  - Field  $P$  is the result of the primary and image waves in the equivalent free-space scenario
    - Which is valid only for the upper half semi-space



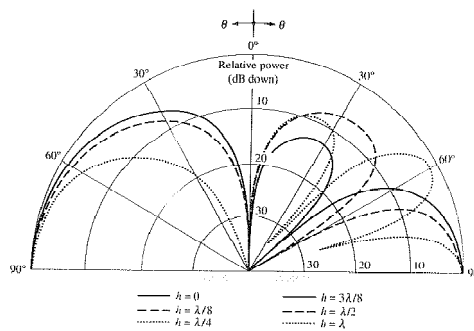
## Image Theory

- Example: field produced by a Infinitesimal dipole



## Image Theory

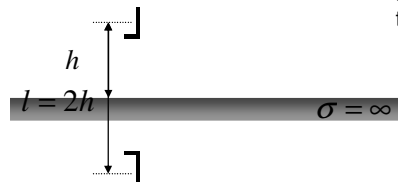
- Example (cont.)
  - If  $\lambda \gg h$  then, directivity increases by 3dB! -> decrease the size
  - Otherwise....



## Monopole

- Vertical Dipole divided to its half, that is fed between wire end and a conductive plane

- Applying Image theory, it can be proven that a monopole above a conductive plane exhibit the same behavior than a dipole with a length twice the height of the monopole



Consider only  $z > 0$  and  $\lambda \gg h$

$$Pr_{mono} = \frac{1}{2} Pr_{dipolo}$$

$$R_{r,mono} = \frac{1}{2} R_{r,dipolo}$$

$$i_{mono} = i_{dipolo}$$

$$D_{mono} = 2D_{dipolo}$$

## Monopole

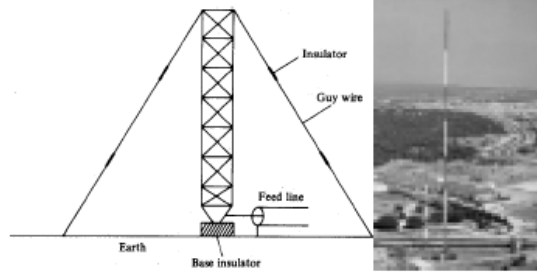
- Example:  $\lambda/4$  monopole
  - As shown before, the monopole exhibit the same performances than a  $\lambda/2$  dipole and therefore its directivity is

$$D = 5.15 \text{ dBi}$$

- At low frequencies, this antenna has quite large physical dimensions
  - Example: the standard AM transmitter for frequency carrier around 1MHz, corresponds to a wavelength of 300 m, and therefore this  $\lambda/4$  monopole has a height of 75 m.

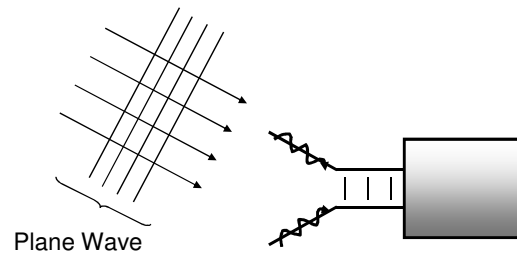


## Monopole



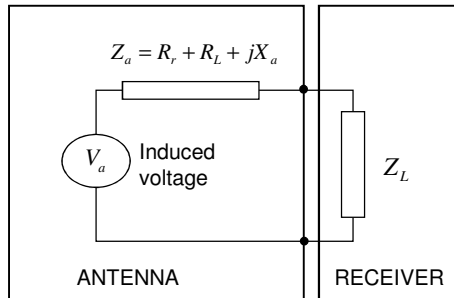
## Reception of an Electromagnetic Field

- If an electromagnetic wave, with a plane-wave like propagation, runs over a conductor (antenna) it generates a current distribution over it



## Equivalent Circuit Model for a Receiving Antenna

- An antenna at reception is designed to optimize the power handed out at its terminal



- Available power at antenna

$$P_a = \frac{|V_a|^2}{8R_a}$$

- Power handed out

$$P_L = \frac{1}{2} |I_L|^2 R_L = P_a (1 - |\Gamma|^2)$$

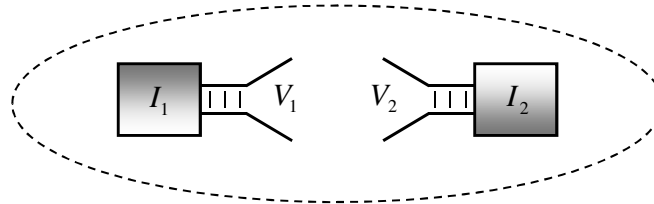
$$\Gamma = \frac{Z_a - Z_L}{Z_a + Z_L}$$

## Reciprocity Theorem

- It allows to relate the properties of an antenna when receiving and transmitting
- “The relationship between an oscillating current and the resulting electric field is unchanged if one interchanges the points where the current is placed and where the field is measured”
  - For the specific case of an electrical network, it is sometimes phrased as the statement that voltages and currents at different points in the network can be interchanged”

## Reciprocity Theorem

- Suppose an anechoic chamber (no echo) in which are placed two antennas, both can transmit and receive, and operating at the same frequency



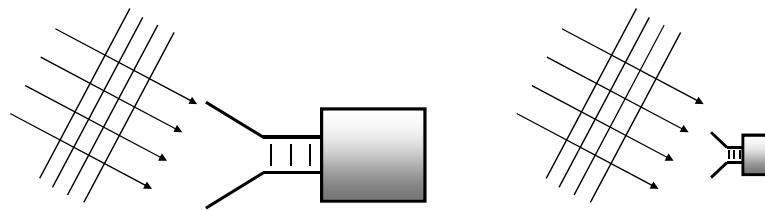
- The roles of sending and receiving can be exchanged. Thus, the radiation patterns of transmitting and receiving are the same

$$\text{If } I_1 \rightarrow V_{2,ca} \Rightarrow V_{1,ca} = V_{2,ca} \text{ when } I_2 \rightarrow V_{1,ca}$$

$$I_1 = I_2$$

## Effective Aperture

- Effective Aperture of an antenna characterizes the electromagnetic energy that it is able to capture
- Intuitively a large antenna captures more power, as it has more area



## Effective Aperture

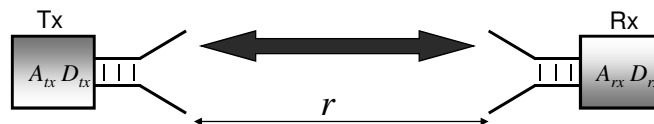
- Equivalent aperture is defined as

$$A_{ef} = \frac{\text{Potencia entregada}}{\text{Densidad de potencia incidente}} = \frac{P_L}{\phi_i} = \frac{|I_L|^2 R_L / 2}{\phi_i}$$

- The value does not have to match the dimensions (physical) of the antenna.
- When the antenna is flat, the physical relationship between the opening ( $A_f$ ) and the effective aperture ( $A_{eff}$ ) is known as aperture efficiency, verifying that:

$$A_{eff} = \varepsilon_{ap} A_f \quad \text{con } 0 \leq \varepsilon_{ap} \leq 1$$

## Directivity vs Maximum Effective Aperture



$$\phi_{tx} = \frac{P_{tx}}{4\pi \cdot r^2} D_{tx}$$

$$P_{rx} = \phi_{tx} A_{rx} = \frac{P_{tx} D_{tx} A_{rx}}{4\pi \cdot r^2} \longrightarrow D_{tx} A_{rx} = \frac{P_{rx}}{P_{tx}} (4\pi r^2)$$

$$D_{rx} A_{tx} = \frac{P_{tx}}{P_{rx}} (4\pi r^2)$$

## Directivity vs Maximum Effective Aperture

$$\frac{D_{tx}}{A_{tx}} = \frac{D_{rx}}{A_{rx}} \Rightarrow \frac{D_{tx}}{A_{tx,m}} = \frac{D_{rx}}{A_{rx,m}}$$

- The above solution is valid for any antenna. For an infinitesimal dipole it can be proof that

$$A_{ef} = \frac{3\lambda^2}{8\pi} = \frac{\lambda^2}{4\pi} D$$

## Directivity vs Maximum Effective Aperture

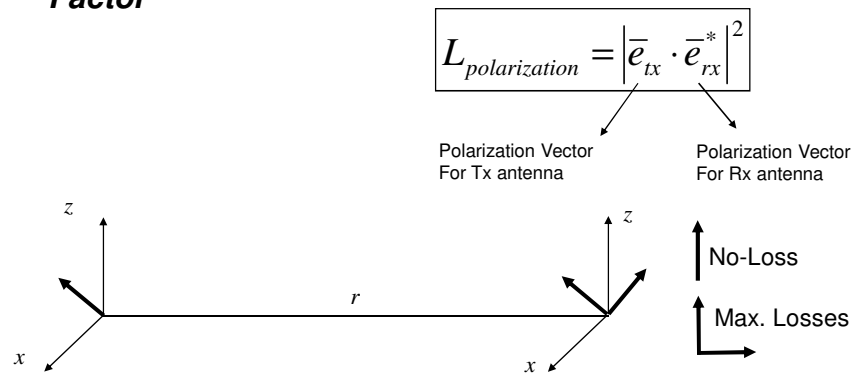
- In case of losses associated to the antenna, the maximum effective aperture is

$$A_{ef} = \eta_{cd} (1 - |\Gamma|^2) \frac{\lambda^2}{4\pi} D = \frac{\lambda^2}{4\pi} G$$

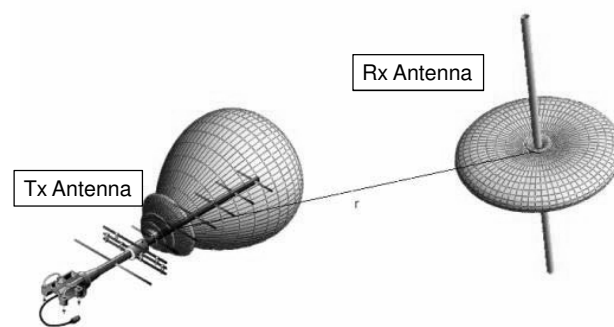
$$G = \frac{4\pi}{\lambda^2} A_{ef}$$

## Polarization Mismatch

- The difference of polarization between transmitting and receiving antennas, it is known as **Polarization Loss Factor**



## Gains in the Radio Link: Tx and Rx Gains



$$\phi_{tx} = \frac{P_{tx} G_{tx}}{4\pi r^2} = \frac{PIRE}{4\pi r^2}$$

$$P_{rx} = \phi_{tx} A_{rx} = \phi_{tx} \frac{\lambda^2}{4\pi} G_{rx} = \frac{P_{tx} G_{tx} G_{rx} \lambda^2}{(4\pi r)^2}$$

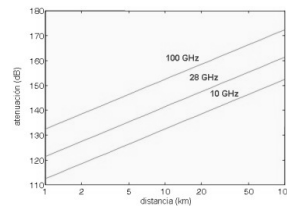
## Friis Transmission Equation

- For Isotropic antennas and free-space propagation
  - The basic losses are

$$\left. \begin{aligned} P_{Rx} &= \phi_{iso} \cdot S_{eq_{iso}} = \phi_{iso} \cdot \frac{\lambda^2}{4\pi} \\ \phi_{iso} &= \frac{P_{Tx}}{4\pi d^2} \end{aligned} \right\} \Rightarrow l_{bf} = \frac{P_{Tx}}{P_{Rx}} = \left( \frac{4\pi d}{\lambda} \right)^2$$

$$L_{bf} (dB) = 32,45 + 20 \log f (MHz) + 20 \log d (km)$$

$$L_{bf} (dB) = 92,45 + 20 \log f (GHz) + 20 \log d (km)$$



## Friis Transmission Equation

- For any pair of antennas and free-space propagation
  - The basic losses are

$$\left. \begin{aligned} l_f &= \frac{P_{et}}{P_{dr}} \quad \left\{ \begin{array}{l} P_{et} : \text{power handed in to Tx antenna} \\ P_{dr} : \text{power handed in to Rx antenna} \end{array} \right. \\ P_{dr} &= \phi \cdot S_{eq} = \phi \cdot \frac{\lambda^2}{4\pi} \cdot g_{Rx} \\ \phi &= \frac{P_{et}}{4\pi d^2} \cdot g_{Tx} \end{aligned} \right\} \Rightarrow l_f = \frac{P_{Tx}}{P_{Rx}} = \left( \frac{4\pi d}{\lambda} \right)^2 \cdot \frac{1}{g_{Tx} \cdot g_{Rx}} = \frac{l_{bf}}{g_{Tx} \cdot g_{Rx}}$$

$$P_{Rx} = \frac{P_{Tx} g_{Tx} \cdot g_{Rx}}{\left( \frac{4\pi d}{\lambda} \right)^2}$$

$$L_{lf} (dB) = L_{bf} (dB) - G_{Tx} (dB) - G_{Rx} (dB)$$

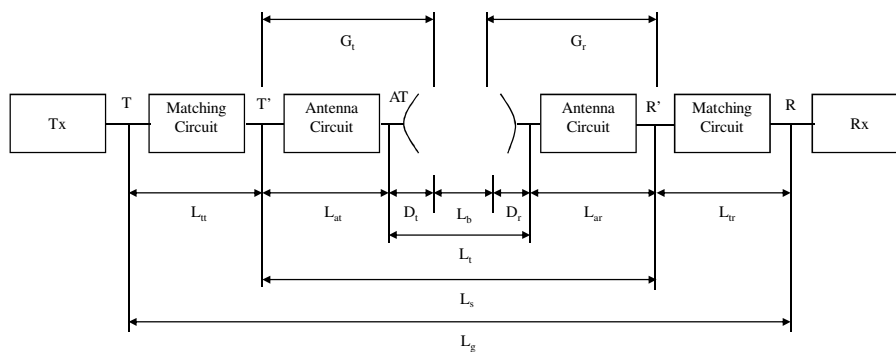
## Friis Transmission Equation

- Sumarizing, for any antenna

$$l_t = \frac{p_{Tx}}{p_{Rx}} = a_e \cdot \left( \frac{4\pi d^2}{\lambda} \right) \cdot \frac{1}{g_{Tx} \cdot g_{Rx}} \quad \boxed{L_t = L_{bf} + A_e - G_t - G_r = L_b - G_t - G_r}$$

	Usual Notation	Definition	
		Antennas	Mean
Free-Space Basic Loss	$L_{bf}$	Isotropic	Free-Space
Basic Loss	$L_b$	Isotropic	Any
Free-Space Transmission Loss	$L_{ff}$	Any	Free-Space
Transmission Loss	$L_t$	Any	Any

## Friis Transmission Equation





## Link Budget

- Link Budget = expression for available power at the receiver as a function of
  - Transmitted Power
  - Rx and Tx Antenna Gains
  - All the losses in the link

$$P_{Rx} = P_{Tx} - L_{bf} + G_{Tx} + G_{Rx} - A_e$$

## Link Budget

- Other factors affecting the link
  - Normalized Noise Power
    - SNR
    - Power-limited Systems
      - Minimum Received Power (Sensibility) + Fading Margin
      - The maximum distance between Tx and Rx is calculated by the Link Budget

$$p_n = k \cdot T_0 \cdot b \cdot f_{sis} \quad P_n (dBm) = F_{sis} (dB) + 10 \log b (Hz) - 174$$

- Interference
  - C/I; SINR
  - Interference-limited Systems
- Performances: BER, PER,  $P_{out} \dots$

$$p_n = k \cdot T_{eq} \cdot b$$

## **Important Concepts in this Topic**

- Poynting Vector
- Radiated Power Flux Density
- Antenna Directivity
- Antenna Gain
- Antenna Efficiency
- Antenna Effective Aperture
- Polarization
- Reciprocity Theorem
- Most common simple antennas
- Friis Equation and Link Budget
- Free-Space Basic Propagation Loss