Topic 5: Transmission Lines

Telecommunication Systems Fundamentals

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Concepts in this Chapter

- ***Mathematical Propagation Model for a guided transmission line***
  - ***Primary Parameters***
  - ***Secondary Parameters***
  - ***Bandwidth and Attenuation***
- ***Common Transmission Lines***
  - ***Copper pair***
  - ***Coaxial***
  - ***Microstrip***

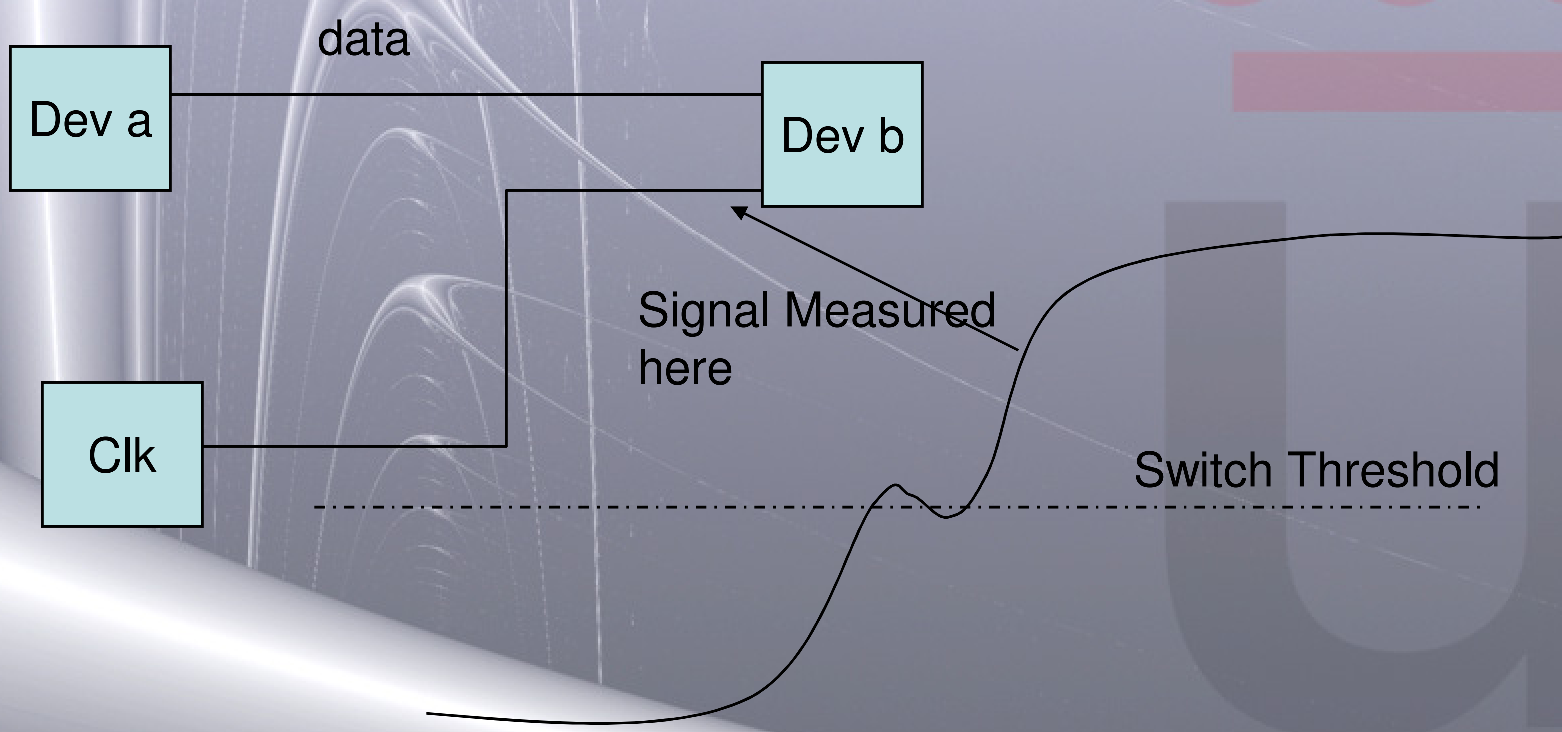
Theory classes: 1.5 sessions (3 hours)
Problems resolution: 0.5 session (1 hour)

Bibliography

- Líneas de Transmisión. Vicente E. Boria. Universidad Politécnica de Valencia.
- Sistemas de Telecomunicación. Transmisión por Línea. J. Hernando Rábanos. Servicio de Publicaciones de la ETSI Telecomunicación UPM

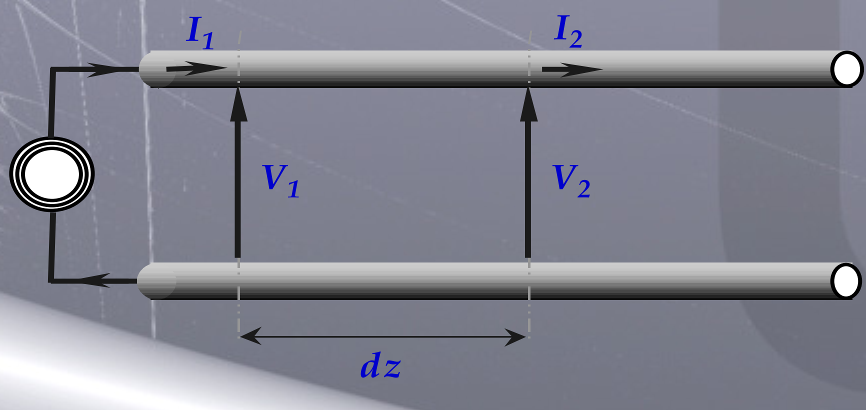
Problem Statement: a practical example

- An engineer tells you the measured clock is non-monotonic and because of this the flip flop internally may double clock the data.



Transmission Line Concept

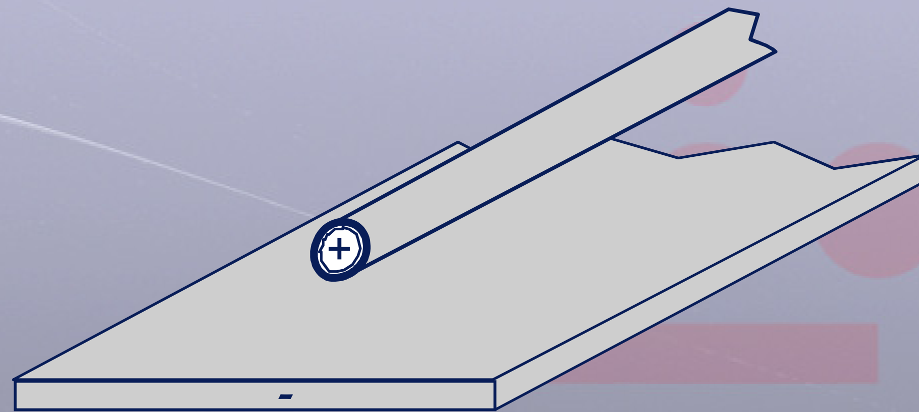
- Voltage and current on a transmission line is a function of both time and position
  - Must think in terms of position and time to understand transmission line behavior
  - This positional dependence is added when the assumption of the size of the circuit being small compared to the signaling wavelength

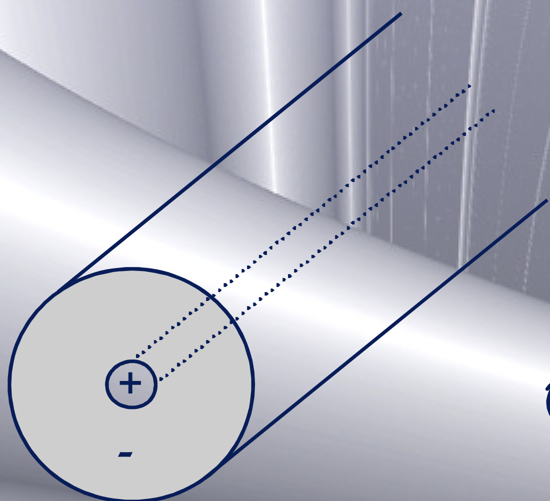


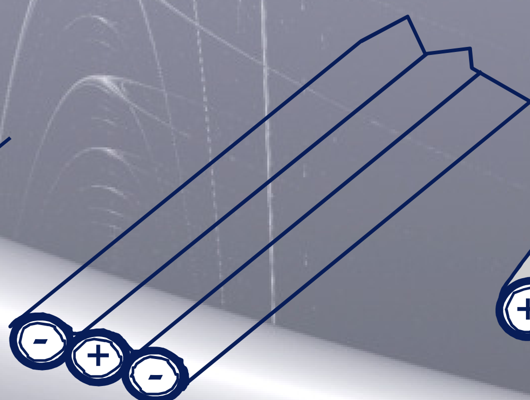
$$V = f(z,t)$$
$$I = f(z,t)$$

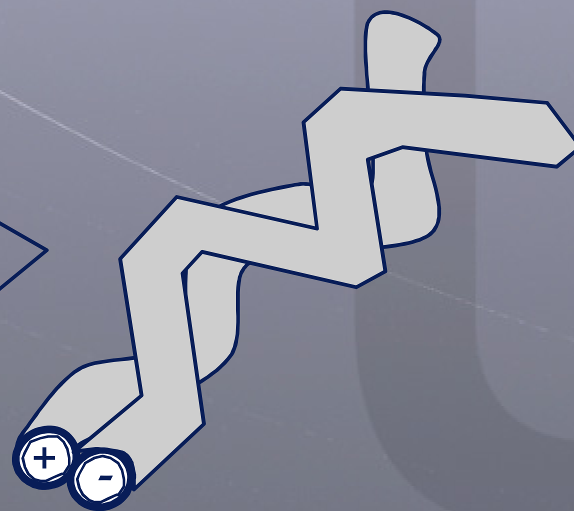
Examples of Transmission Lines

- Cables and wires
  - Coax cable
  - Wire over ground
  - Tri-lead wire
  - Twisted pair (two-wire line)
- Long distance interconnects









Transmission Line "Definition"

- General transmission line: a closed system in which power is transmitted from a source to a destination
- In this course we will study only TEM (transversal Electro-Magnetic) mode transmission lines
  - A two conductor wire system with the wires in close proximity, providing relative impedance, velocity and closed current return path to the source
  - Characteristic impedance is the ratio of the voltage and current waves at any one position on the transmission line

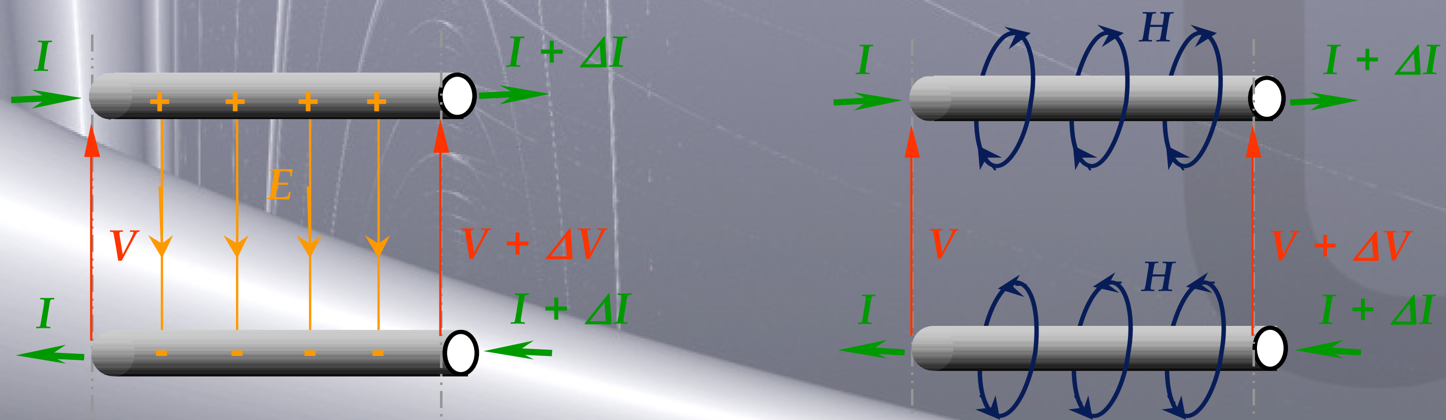
$$Z0 = \frac{V}{I}$$

  - Propagation velocity is the speed with which signals are transmitted through the transmission line in its surrounding medium

$$v = \frac{c}{\sqrt{\varepsilon_r}}$$

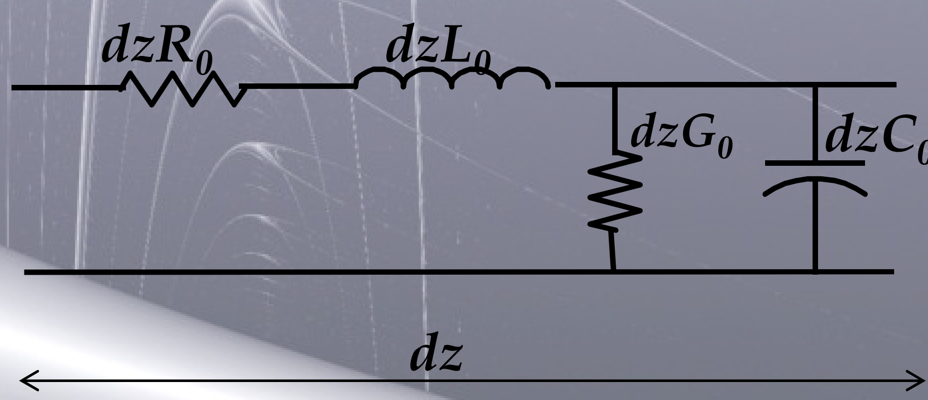
Transmission Lines as a Complex Electromagnetic Problem

- Both Electric and Magnetic fields are present in the transmission lines
  - These fields are perpendicular to each other and to the direction of wave propagation for TEM mode waves, which is the simplest mode
- Electric field is established by a potential difference between two conductors.
  - Implies equivalent circuit model must contain capacitor
- Magnetic field induced by current flowing on the line
  - Implies equivalent circuit model must contain inductor



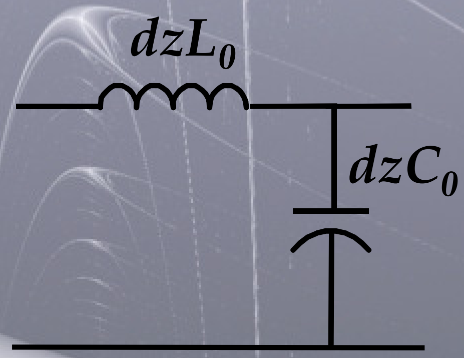
Transmission Line Equivalent Circuit

- General Characteristics (Primary Parameters)
  - Per-unit-length Capacitance (C_0) [pf/m]
  - Per-unit-length Inductance (L_0) [nf/m]
  - Per-unit-length (Series) Resistance (R_0) [Ω/m]
  - Per-unit-length (Parallel) Conductance (G_0) [1/Ωm]

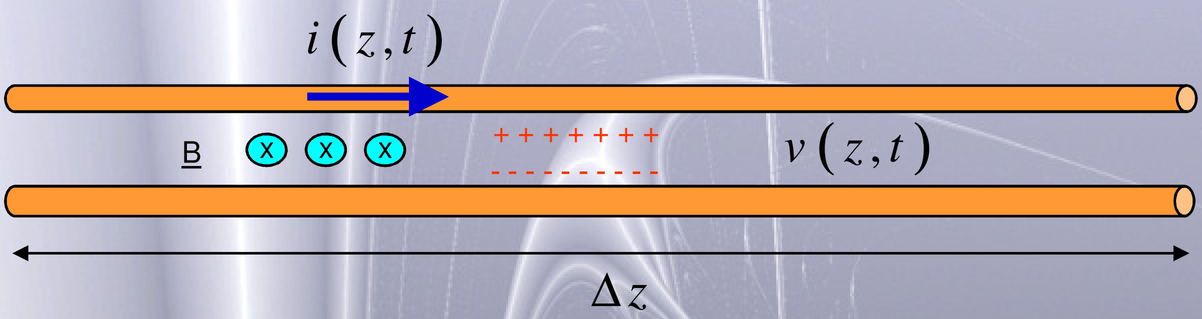


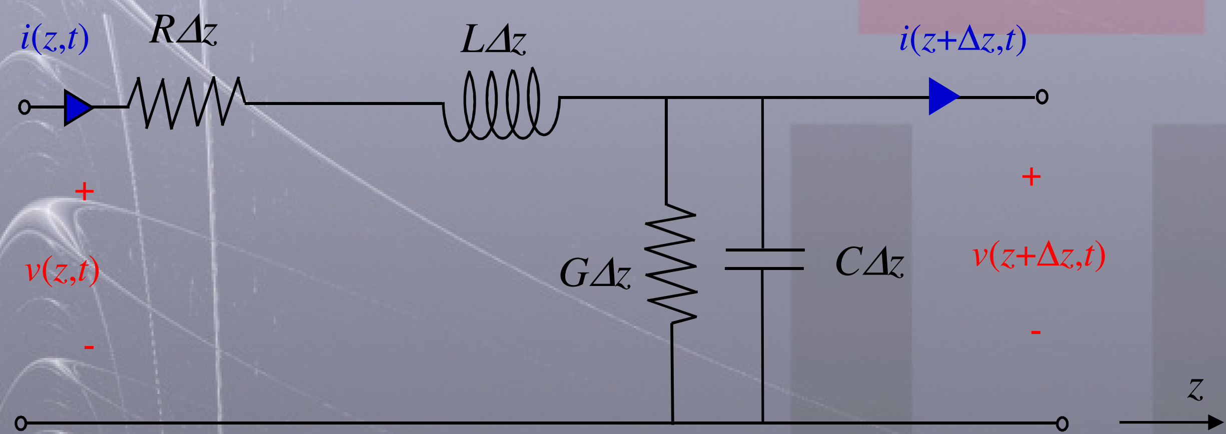
Ideal Transmission Line

- Ideal (lossless) Characteristics of Transmission Line
  - Ideal TL assumes:
    - Uniform line
    - Perfect (lossless) conductor ($R_0 \rightarrow 0$)
    - Perfect (lossless) dielectric ($G_0 \rightarrow 0$)



Transmission Line Equivalent Circuit





$$v(z,t) = v(z+\Delta z,t) + i(z,t)R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t}$$

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t}$$

Transmission Line Signal Model

Hence

$$\frac{v(z+\Delta z,t)-v(z,t)}{\Delta z}=-Ri(z,t)-L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{i(z+\Delta z,t)-i(z,t)}{\Delta z}=-Gv(z+\Delta z,t)-C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

Now let $\Delta z \rightarrow 0$

$$\frac{\partial v}{\partial z}=-Ri-L\frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z}=-Gv-C\frac{\partial v}{\partial t}$$

which are known as "Telegrapher's Equations"

Transmission Line Signal Model

- To combine these, take the derivative of the first one with respect to z:

$$
\begin{aligned}
\frac{\partial^2 v}{\partial z^2} &= -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right) \\
&= -R\frac{\partial i}{\partial z} - L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right) \\
&= -R\left[-Gv - C\frac{\partial v}{\partial t}\right] \\
&\quad - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]
\end{aligned}
$$

Switch the order of the derivatives

Transmission Line Signal Model

$$\frac{\partial^2 v}{\partial z^2} = -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

- Hence, we have

$$\frac{\partial^2 v}{\partial z^2} - \left(RG\right)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

  – The same equation also holds for i

$$\frac{d^2 V}{dz^2} - \left(RG\right)V - (RC + LG)\,j\omega V - LC(-\omega^2)V = 0$$

Transmission Line Signal Model

$$\frac{d^2V}{dz^2} = (RG)V + j\omega(RC + LG)V - (\omega^2 LC)V$$

$$RG + j\omega(RC + LG) - \omega^2 LC = (R + j\omega L)(G + j\omega C)$$

$$Z = R + j\omega L$$ = series impedance/length

$$Y = G + j\omega C$$ = parallel admittance/length

- We can re-write

$$\frac{d^2V}{dz^2} = (ZY)V$$

Transmission Line Signal Model

- Defining $\gamma^2 = ZY$
- The differential equation governing the signal is

$$\frac{d^2V}{dz^2} = (\gamma^2)V$$

- Which solution is

$$V(z) = Ae^{-\gamma z} + Be^{+\gamma z}$$

- Where

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\sqrt{z} = \sqrt{|z|}\, e^{j\theta/2}$$

$$-\pi < \theta < \pi$$

$$\gamma = \alpha + j\beta$$

$$\alpha \geq 0,\ \beta \geq 0$$

$\alpha =$ attenuation contant

$\beta =$ phase constant

Transmission Line Signal Model

- Forward travelling wave (a wave traveling in the positive z direction):

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$v^+(z,t) = \text{Re}\left\{\left(V_0^+ e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\}$$

$$= \text{Re}\left\{\left(\left|V_0^+\right| e^{j\phi} e^{-\alpha z} e^{-j\beta z}\right) e^{j\omega t}\right\}$$

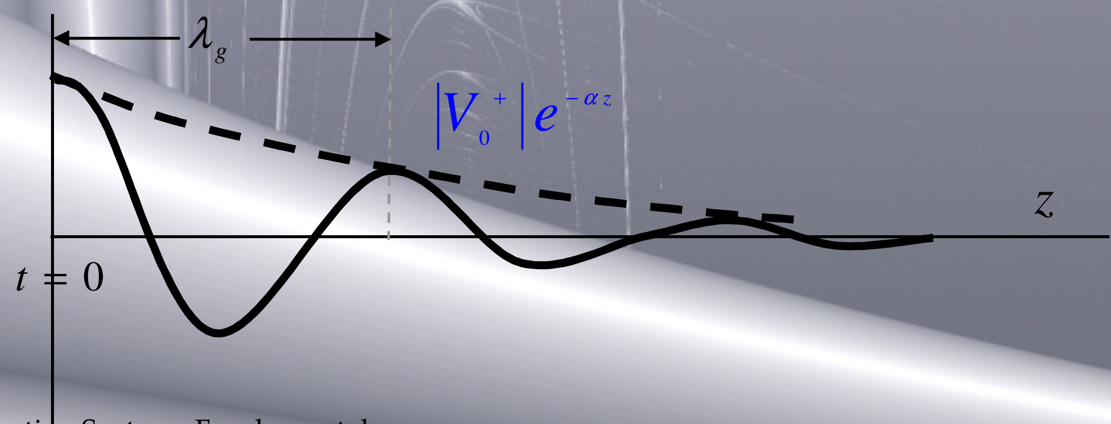
$$= \left|V_0^+\right| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

The wave "repeats" when

$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$



Transmission Line Secondary Parameters

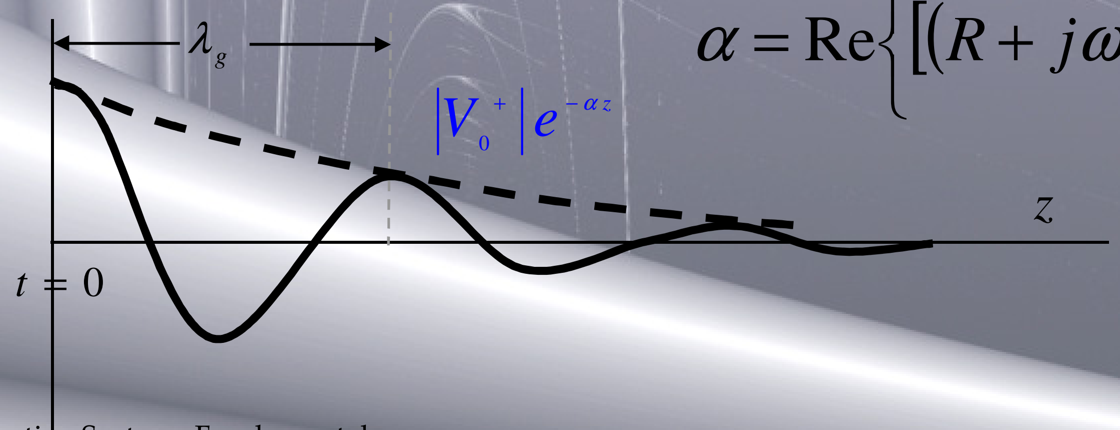
- Attenuation

$$V^+(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

$$|V^+(z)| = |V^+{}_0 e^{-\gamma z}| = |V^+{}_0||e^{-az}||e^{-j\beta z}| = |V^+{}_0||e^{-az}|$$

$$|e^{-az}|$$ = attenuation

$$\alpha = \mathrm{Re}\left\{\left[(R + j\omega L)(G + j\omega C)\right]^{\frac{1}{2}}\right\}$$

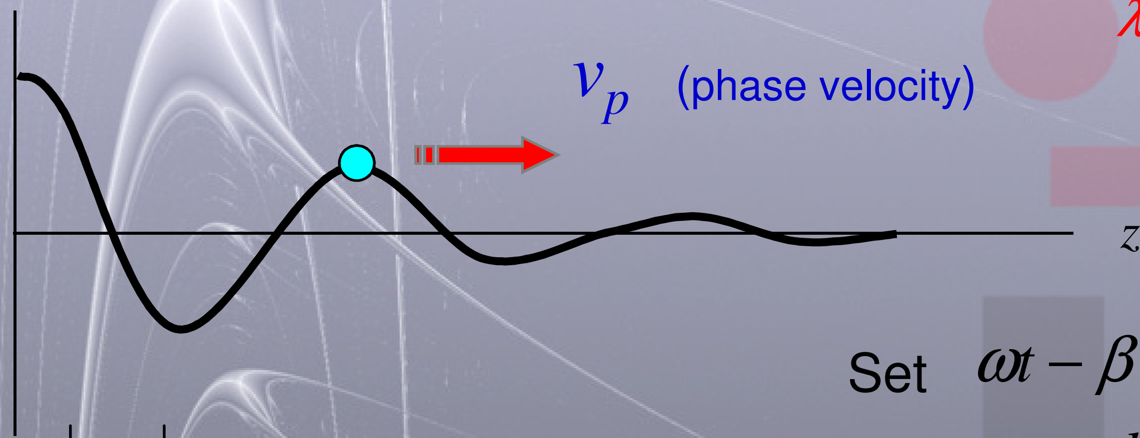


Transmission Line Secondary Parameters

- Phase Velocity

guided wavelength $\equiv \lambda_g$

$$\lambda_g = \frac{2\pi}{\beta}\,[\mathrm{m}]$$



$$v^+(z,t) = \left|V_0^+\right| e^{-\alpha z}\cos(\omega t - \beta z + \phi)$$

Set $\omega t - \beta z = \text{constant}$

$$\omega - \beta\frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$

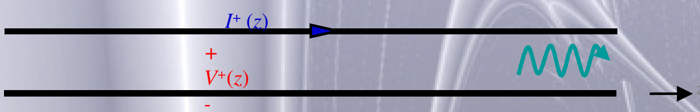
Hence $v_p = \dfrac{\omega}{\beta}$

In expanded form:

$$v_p = \frac{\omega}{\mathrm{Im}\left\{\left[(R + j\omega L)(G + j\omega C)\right]^{1/2}\right\}}$$

Transmission Line Secondary Parameters

- Characteristic Impedance Z_0



A wave is traveling in the positive *z* direction.

$$Z_0 \equiv \frac{V^+(z)}{I^+(z)}$$

$$V^+(z) = V_0^+ e^{-\gamma z}$$
$$I^+(z) = I_0^+ e^{-\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+}$$

(Z_0 is a number, not a function of position)

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{Z}{\gamma} = \left(\frac{Z}{Y}\right)^{1/2}$$

$$Z = R + j\omega L$$
$$Y = G + j\omega C$$

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\frac{dV}{dz} = -RI - j\omega LI$$
$$= -ZI$$

$$-\gamma V_0^+ e^{-\gamma z} = -ZI_0^+ e^{-\gamma z}$$

$$Z_0 = \left(\frac{R + j\omega L}{G + j\omega C}\right)^{1/2}$$

Power-wise

- Power is proportional to the voltage squared

$$P(d) \propto e^{-2\alpha d}$$

- So, attenuation dependence with distance is

$$At = K\ e^{2\alpha d}$$

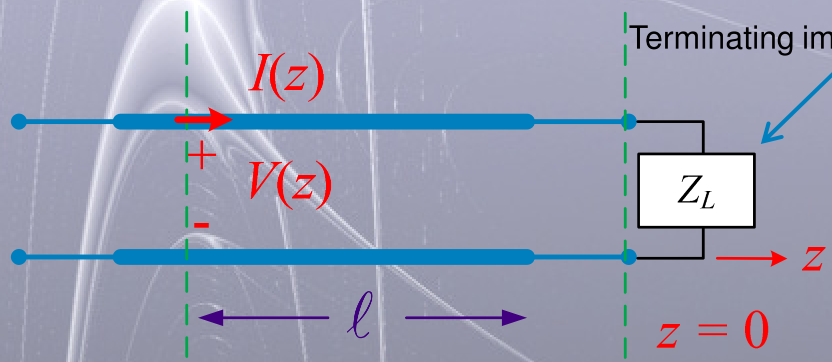
  – When expressing it on dBs

$$At(dB) = K_1 + K_2 x$$

(constants vary with frequency)

Transmission Line Model

- When forward and backward traveling waves



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V_0^+ e^{-\gamma z}\left(1 + \frac{V_0^-}{V_0^+} e^{2\gamma z}\right) = V_0^+ e^{-\gamma z}\left(1 + \Gamma_L e^{2\gamma z}\right)$$

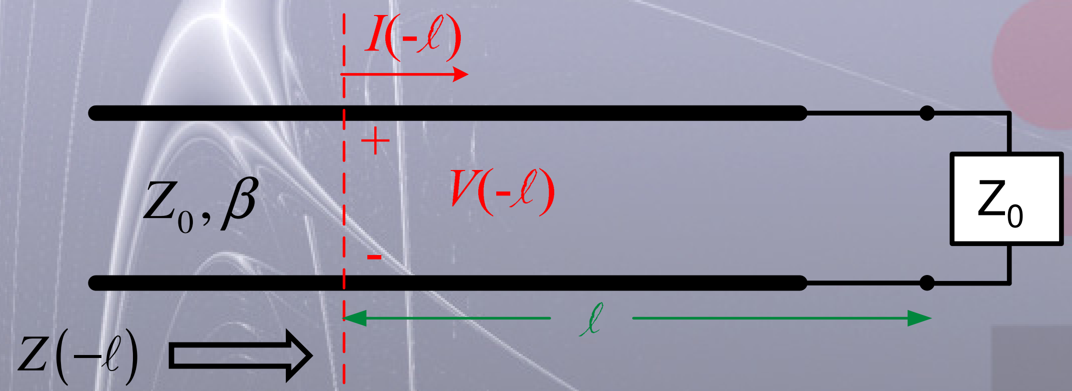
$$I(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right]$$

$\Gamma_L \equiv$ Load reflection coefficient

$$\Rightarrow \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Impedance Matching in Transmission Lines

- Case 1: Matched load: $(Z_L = Z_0)$



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$ ← No reflection from the load

$$\Rightarrow V(-\ell) = V_0^+ e^{+j\beta\ell}$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell}$$

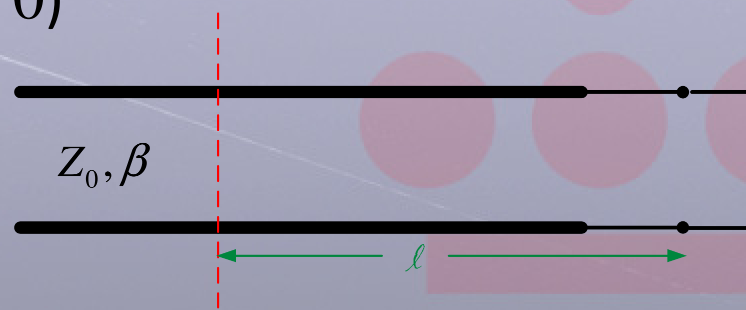
$$\Rightarrow Z(-\ell) = Z_0$$ For any ℓ

Impedance Matching in Transmission Lines

- Case 2: Short circuit load: ($Z_L = 0$)

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$Z(-\ell) = jZ_0 \tan(\beta\ell)$$

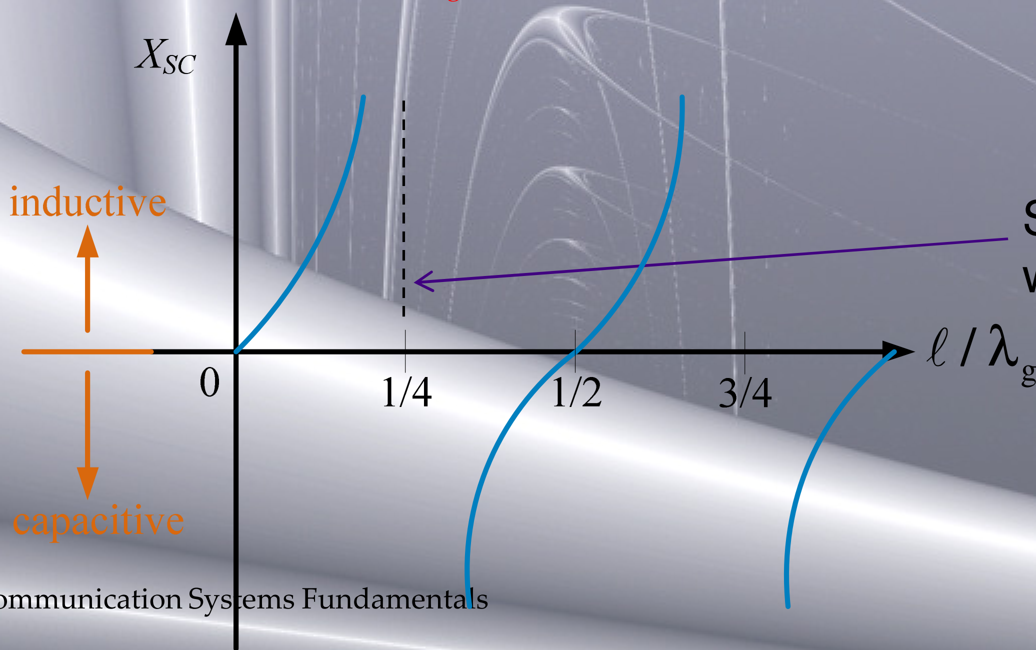


Note: $\beta\ell = 2\pi \frac{\ell}{\lambda_g}$

Always imaginary!

$$Z(-\ell) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\beta\ell)$$



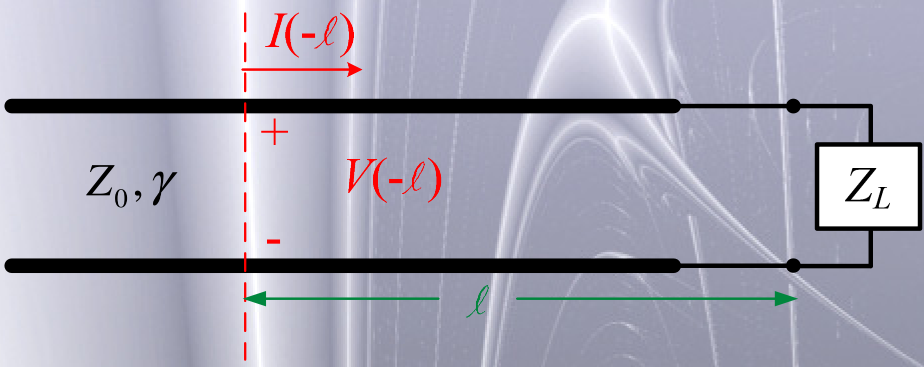
S.C. can become an O.C. with a $\lambda_g/4$ trans. line

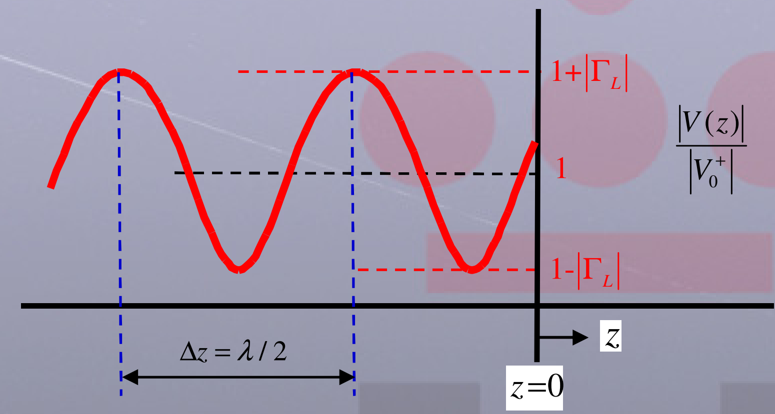
Impedance Matching in Transmission Lines

- Case 3: Open Circuit ($Z_L = \infty$)
  - Homework

Impedance Matching in Transmission Lines

- Voltage Standing Wave Ratio





$$V_{max} = |V_0^+|(1+|\Gamma_L|)$$

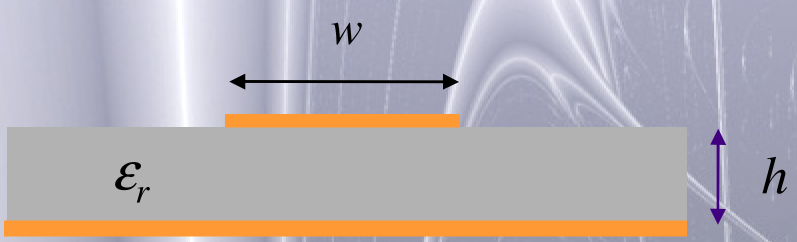
$$V_{min} = |V_0^+|(1-|\Gamma_L|)$$

$$\text{Voltage Standing Wave Ratio (VSWR)} = \frac{V_{max}}{V_{min}}$$

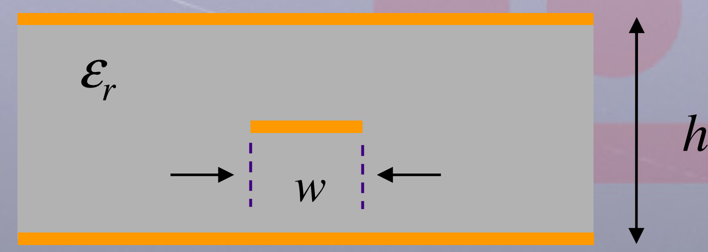
$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Transmission Lines

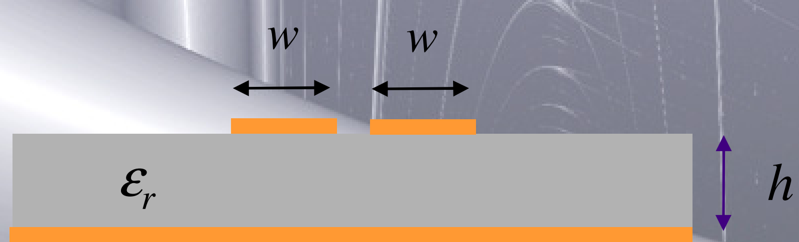
- Transmission lines commonly met on printed-circuit boards



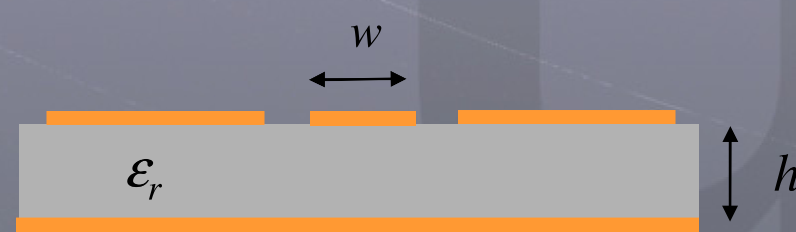
Microstrip



Stripline



Coplanar strips



Coplanar waveguide (CPW)

Transmission Lines

ε is electric permittivity

ε_0= 8.85 X 10^{-12} *F/m* (free space)

ε_r is relative dielectric constant

Tipical ε_r : 3.29 paper; 1 air;

3 to 4 polietilene and 4 to 6 PVC

$$\mu = \mu_r \mu_0 ; \qquad \varepsilon = \varepsilon_r \varepsilon_0$$

μ is magnetic permeability

μ_0= 4p X 10^{-7} *H/m* (free space)

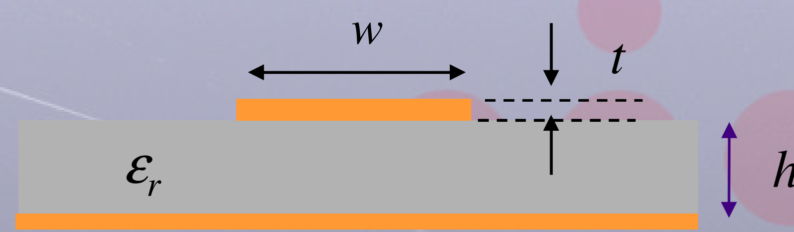
μ_r is relative permeability

For most of dielectric materials, μ_r is close to one

ρ is resistivity:

Cooper: 17.4; Bronze 19; Iron 120 (x 10^{-8} Ω/m)

Microstrip Transmission Line



$$(w/h \geq 1)$$

$$Z_0(f) = Z_0(0)\left(\frac{\varepsilon_r^{eff}(f)-1}{\varepsilon_r^{eff}(0)-1}\right)\sqrt{\frac{\varepsilon_r^{eff}(0)}{\varepsilon_r^{eff}(f)}}$$

$$Z_0(0) = \frac{120\pi}{\sqrt{\varepsilon_r^{eff}(0)}\left[(w'/h)+1.393+0.667\ln\left((w'/h)+1.444\right)\right]}$$

$$w' = w + \frac{t}{\pi}\left(1+\ln\left(\frac{2h}{t}\right)\right)$$

$$\varepsilon_r^{eff}(f) = \left(\sqrt{\varepsilon_r^{eff}(0)} + \frac{\sqrt{\varepsilon_r}-\sqrt{\varepsilon_r^{eff}(0)}}{1+4F^{-1.5}}\right)^2$$

$$\varepsilon_r^{eff}(0) = \frac{\varepsilon_r+1}{2} + \left(\frac{\varepsilon_r-1}{2}\right)\left(\frac{1}{\sqrt{1+12(h/w)}}\right) - \left(\frac{\varepsilon_r-1}{4.6}\right)\left(\frac{t/h}{\sqrt{w/h}}\right)$$

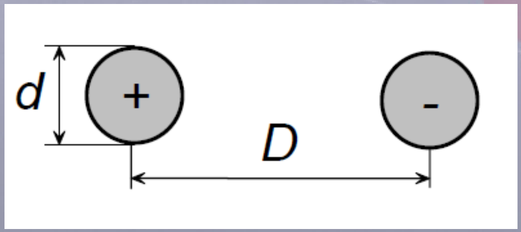
Copper Pair / Twisted Pair

- Primary parameters of a Copper Pair

$$L = 2.3\frac{\mu}{\pi}\log\left(2\frac{D}{d}\right)$$

$$R = 2\frac{\rho}{\frac{\pi}{4}d^2}$$

$$C = 0{,}43\frac{\pi\varepsilon}{\log\left(\frac{2D}{d}\right)}$$



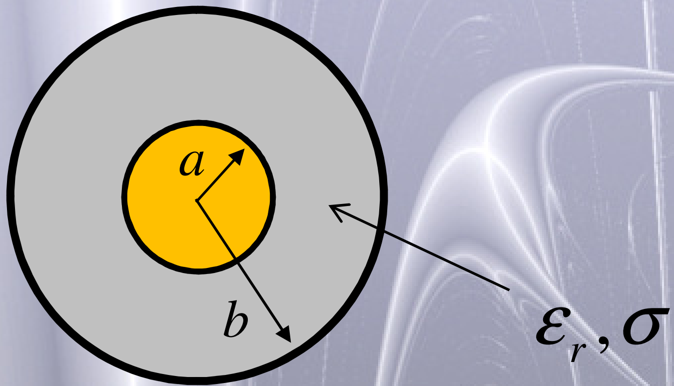
- D: Distance between cables
- d: diameter of each cable

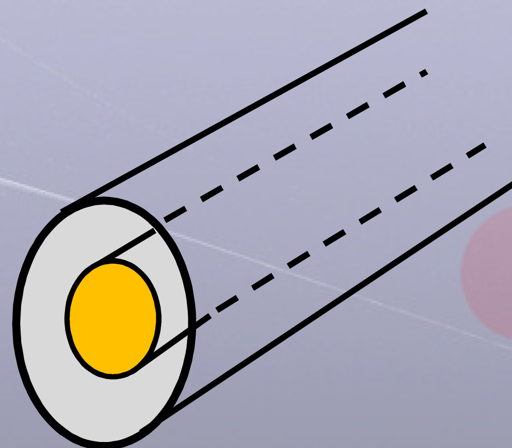
$$Z_0 = 120\cosh^{-1}\left(\frac{D}{d}\right)$$

$$\approx 276\log\left(\frac{2D}{d}\right)$$

G: conductance depends on the dialectric, usually negligible but depending on the frequency

Coaxial Cable





$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad \text{[F/m]}$$

$$R = \rho\left(\frac{1}{2\pi a} + \frac{1}{2\pi b}\right)$$

$$L = \frac{\mu_0\mu_r}{2\pi}\ln\left(\frac{b}{a}\right) \quad \text{[H/m]}$$

$$G = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)} \quad \text{[S/m]}$$

Summary of Concepts in this Chapter

- What are the primary parameters of the transmission lines
- Calculation of attenuation, and propagation velocity for transmission lines
- Calculate above parameters for common transmisión lines