# *Topic 2: The Communications Channel*

**Fundamental Principles of Telecommunication Systems**

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# Concepts in this Chapter

- The communications channel as a Linear Time Invariant system
  - Signal in presence of Additive White Gaussian Noise
  - Signal to Noise Ratio
- Channel degradation
  - Noise
  - Distortion
  - Interference.
- Quality metrics in communication systems
  - Noise Equivalent Temperature
  - Noise Factor
  - Noise Factor of a cascade of systems

***Theory classes: 2 sessions (4 hours)***

# Bibliography


1. Communication Systems Engineering. John. G. Proakis. Prentice Hall

2. Sistemas de Comunicación. S. Haykin. Wiley

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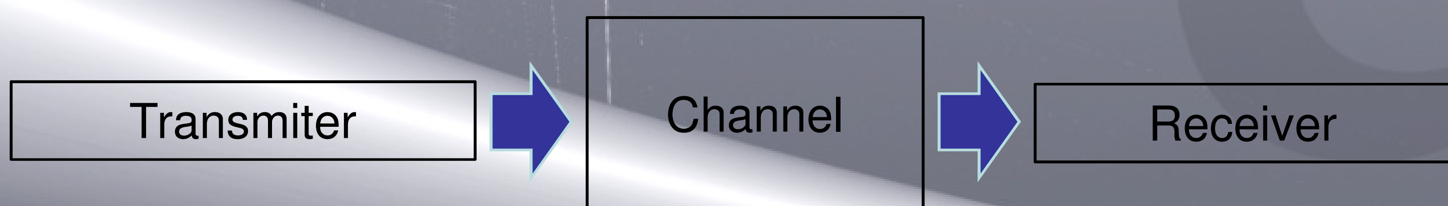
# The Communications Channel

- We will look at the communications channel as a Linear Time Invariant system
  - Stochastic Processes through a LTI Channel
  - Signal in presence of Additive White Gaussian Noise
  - Signal to Noise Ratio

# Big Picture of the Transmission Process

- Transmission of information from one point to another, regardless the nature of the info, follows the next phases:
  1. Message Generation (voice, music, video, data…)
  2. Mapping of the message to apropriate signals (electric signals, light, …)
  3. Coding of signals to adapt them to the mean of transmission
  4. Transmission itself of the coded signals
  5. Decoding of received signals
  6. Regeneration of the original message



# Rationale

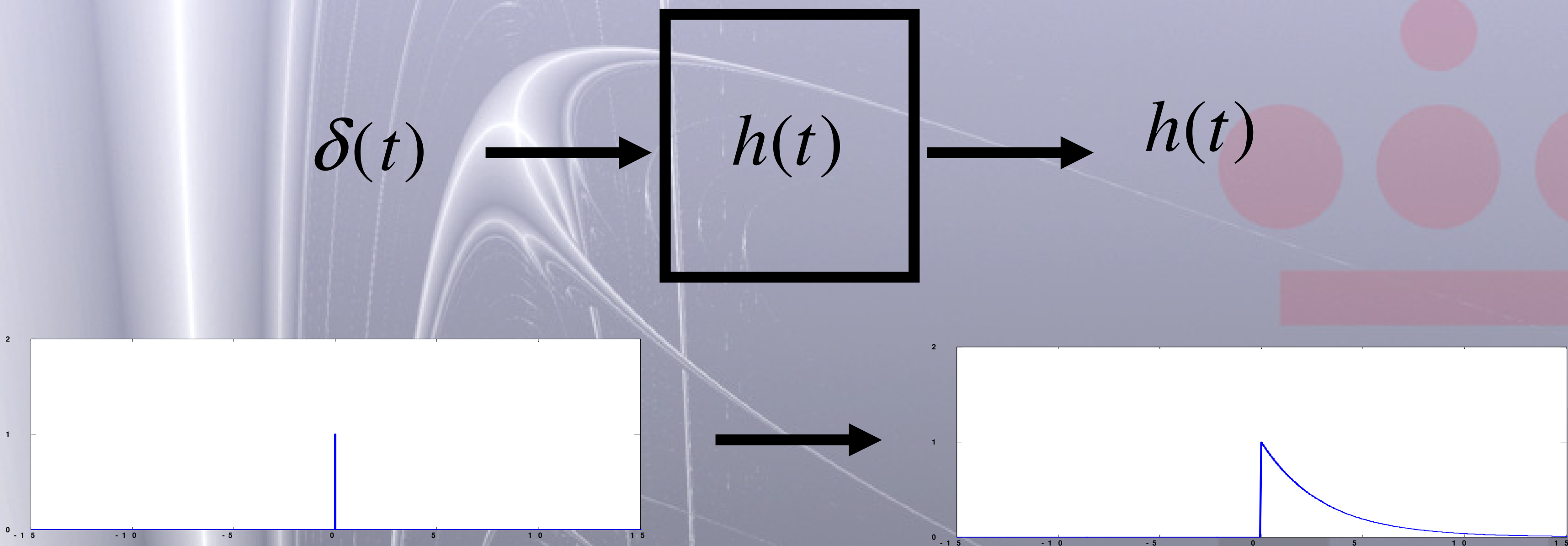
- In any transmission scenario, there are many disturbances degrading the transmitted signal
- Degradation may occur because the systems utilized for the communications (channel, filters, amplifiers, mixers, modulator, demodulators…) o because other signal co-existing with the Signal of Interest (SoI)
- Transmitted signal and rest of other degrading signals can be modeled as Stochastic Processes
- Therefore, first thing we need to know is how LTI systems affect to SP

## Review of the concept of Impulse Response of a LTI

- Remember that a LTI system is fully characterized by its impulse response, h(t).
- The impulse response is the output of the system when we apply an impulse (delta) to the input
- For any other signal, the output of a LTI system can be calculated as the sum of each instantaneous value of the input – the convolution of the input and the impulse response

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

# Review of the Impulse Response of a LTI




***The impulse response of a channel allows us to characterize a communications channel***

# Review of the Frequency Response of a LTI

- The Fourier Transform of the impulse responde describes the Frequency Response of an LTI

$$H(f) = F\{h(t)\}$$
$$= \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt$$

  - Its complex value indicates the attenuation/amplification (module) and phase/delay (phase) for each frequency component entering the LTI

# Autocorrelation and ESD in a LTI System

- The autocorrelation $R_{hh}(\tau)$ of the impulse response $h(t)$ of a LTI system is defined as:

$$R_{hh}(\tau) = h(\tau) * h(-\tau)$$
$$= \int_{-\infty}^{\infty} h(t)h(t-\tau)dt$$

- And therefore its Energy Spectral Density can be defined as:

$$G_{hh}(f) = F\{R_{hh}(\tau)\} = F\{h(\tau) * h(-\tau)\}$$
$$= F\{h(\tau)\}F\{h(-\tau)\} = \left|H(f)\right|^2$$

***The Energy Spectral Density corresponds to the module of the frequency response squared.***

# Autocorrelation of the Output of a LTI System

- The autocorrelation of the output $y(t)$ of a LTI system with impulse response $h(t)$ when the input is $x(t)$ is computed as:

$$
\begin{aligned}
R_{yy}(\tau) &= y(\tau) * y(-\tau) = \\
&= x(\tau) * h(\tau) * x(-\tau) * h(-\tau) = x(\tau) * x(-\tau) * h(\tau) * h(-\tau) \\
&= R_{xx}(\tau) * R_{hh}(\tau)
\end{aligned}
$$

- For power defined signals the following relationship is still of application:

$$R_{yy}(\tau) = R_{xx}(\tau) * R_{hh}(\tau)$$

# Autocorrelation of the Output of a LTI System

- Although of less practical interest, cross-correlation between input and output of a LTI system is:

$$R_{xy}(\tau) = x(\tau) * y(-\tau)$$
$$= x(\tau) * x(-\tau) * h(-\tau)$$
$$= R_{xx}(\tau) * h(-\tau)$$

# Power/Energy Spectral Density and LTI System


$$y(t) = x(t) * h(t)$$

**Note we are assuing that Fourier Transform of both x(t) and h(t) can be computed**

$$Y(f) = X(f)H(f)$$

# Energy Spectral Density and LTI Systems

- The output signal auto-correlation is:

$$R_{yy}(\tau) = R_{xx}(\tau) * R_{hh}(\tau)$$

- Therefore, computing its Fourier Transform, its Energy Spectral Density is:

$$\begin{aligned} G_{yy}(f) &= \left|Y(f)\right|^2 = \left|X(f)\right|^2 \left|H(f)\right|^2 \\ &= G_{xx}(f)\left|H(f)\right|^2 \\ &= G_{xx}(f) G_{hh}(f) \end{aligned}$$

# Energy Spectral Density and LTI Systems

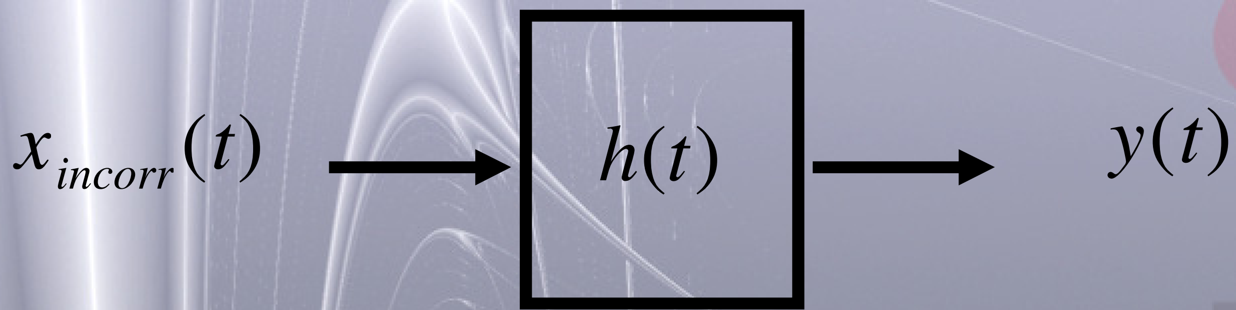
- Similarly to Energy defined signals, for power defined signals the autocorrelation is:

$$R_{yy}(\tau) = R_{xx}(\tau) * R_{hh}(\tau)$$

- And therefore the Power Spectral Density becomes:

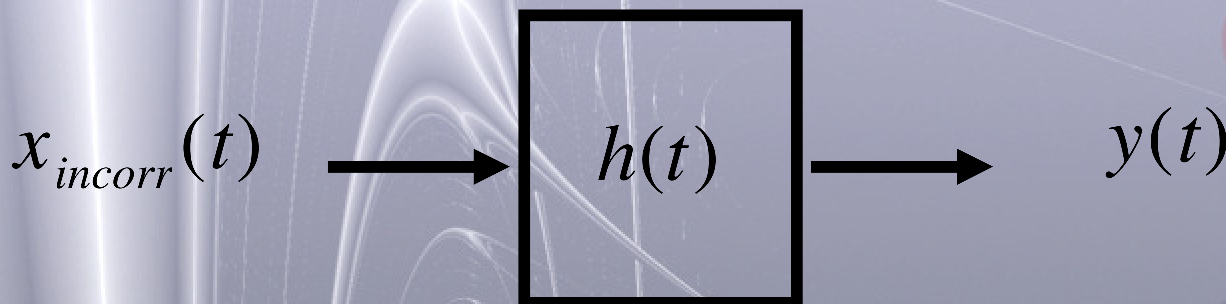
$$S_{yy}(f) = S_{xx}(f) G_{hh}(f)$$
$$= S_{xx}(f) \left| H(f) \right|^2$$

# Case of Study: White Noise




$$R_{xx}(\tau) = \frac{N_0}{2}\delta(\tau) \longrightarrow R_{yy}(\tau) = \frac{N_0}{2}R_{hh}(\tau)$$

# Case of Study: White Noise




$$S_{xx}(f) = \frac{N_0}{2} \longrightarrow S_{yy}(f) = \frac{N_0}{2} G_{hh}(f) = \frac{N_0}{2} \left| H(f) \right|^2$$

# Noise

- In the telecommunications field, we define noise to any random fluctuation of the received signal that deteriorates it
- Different types of noise:
  - Shot-noise and flicker-noise
  - Thermal noise
  - Atmospheric noise
  - Human made noise: electric motors, engines, power network, etc.
- Noise is characterized as a Stochastic Process:
  - Power Spectral Density
  - Probability Density Function of the amplitude

# White Thermal Noise

- Thermal noise is best modeled by a Gaussian amplitude distribution (pdf) and uncorrelated (autocorrelation equal to a delta), and therefore a flat PSD: White

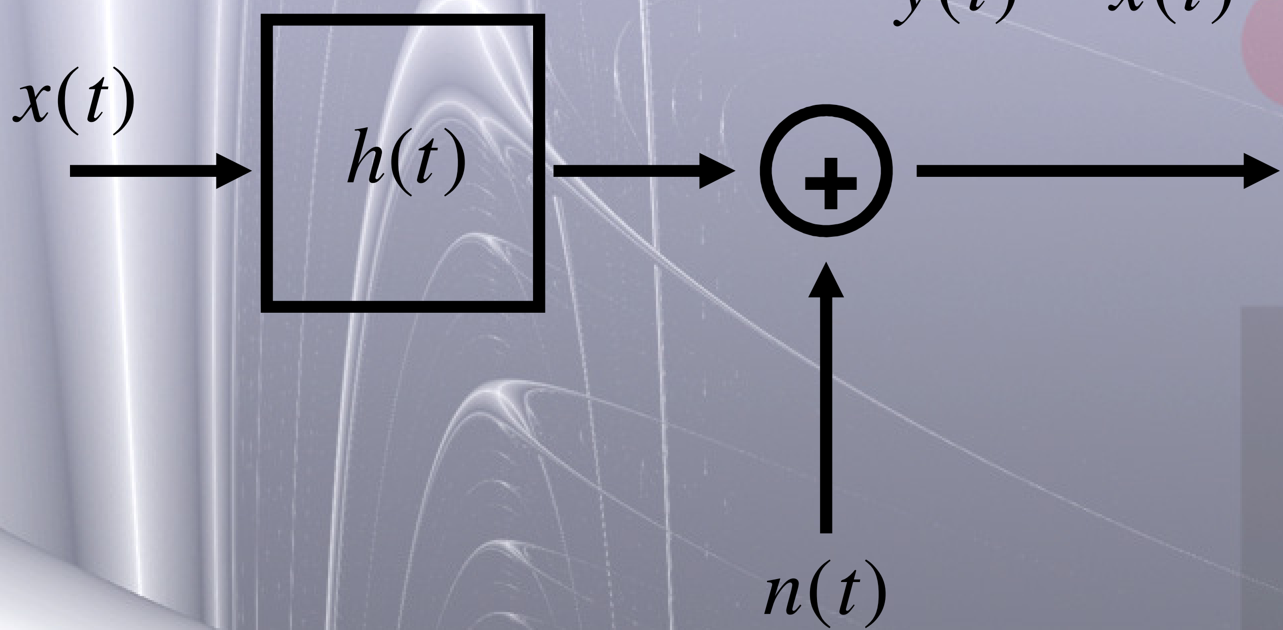
$$S_{noise}(f) = \frac{N_0}{2}$$

- In a first approach, we accept infinite bandwidth for the model

- Noise PSD changes when filtering it, then we call it "colored"

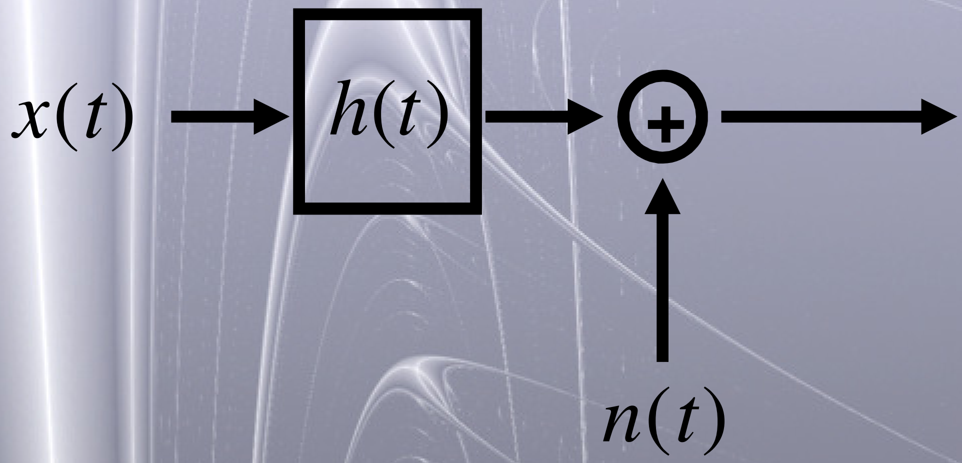
# Model of LTI Channel plus Noise

$$y(t) = x(t) * h(t) + n(t)$$




(Assuming power defined signals)

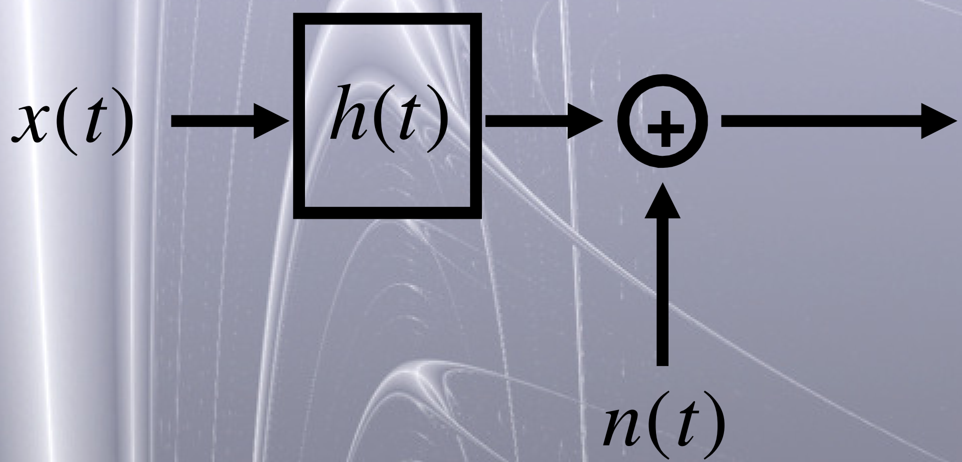
# Model of LTI Channel plus Noise: autocorrelation




$$y(t) = x(t) * h(t) + n(t)$$

$$R_{yy}(\tau) = R_{xx}(\tau) * R_{hh}(\tau) + R_{nn}(\tau)$$

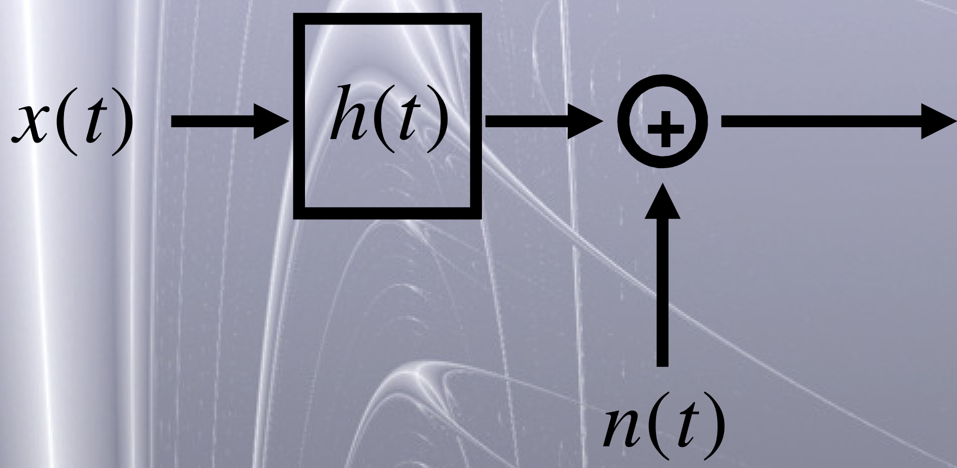
# Model of LTI Channel plus Noise: Average Power




$$y(t) = x(t) * h(t) + n(t)$$

$$R_{yy}(0) = \{R_{xx} * R_{hh}\}(0) + R_{nn}(0)$$

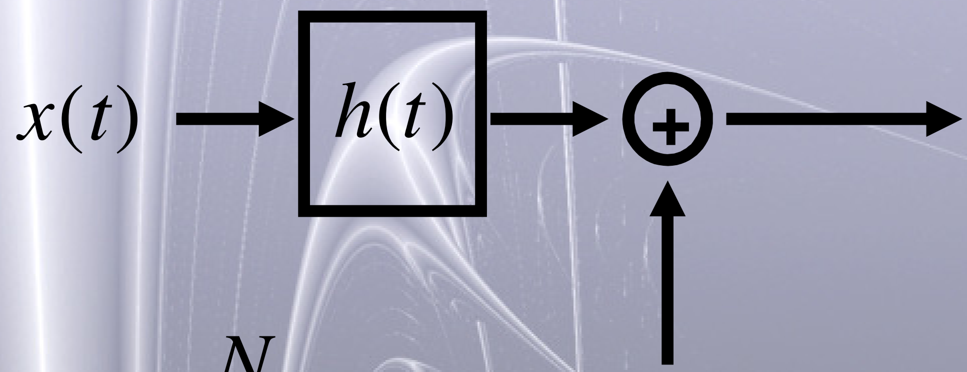
# Model of LTI Channel plus Noise: Spectrum




$$y(t) = x(t) * h(t) + n(t)$$

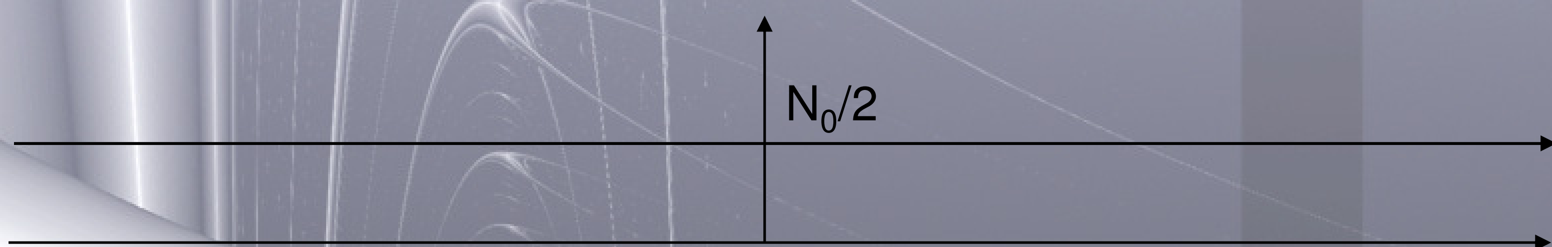
$$S_{yy}(f) = S_{xx}(f)\left|H(f)\right|^2 + S_{nn}(f)$$

# Model of LTI Channel plus Additive White Gaussian Noise (AWGN)



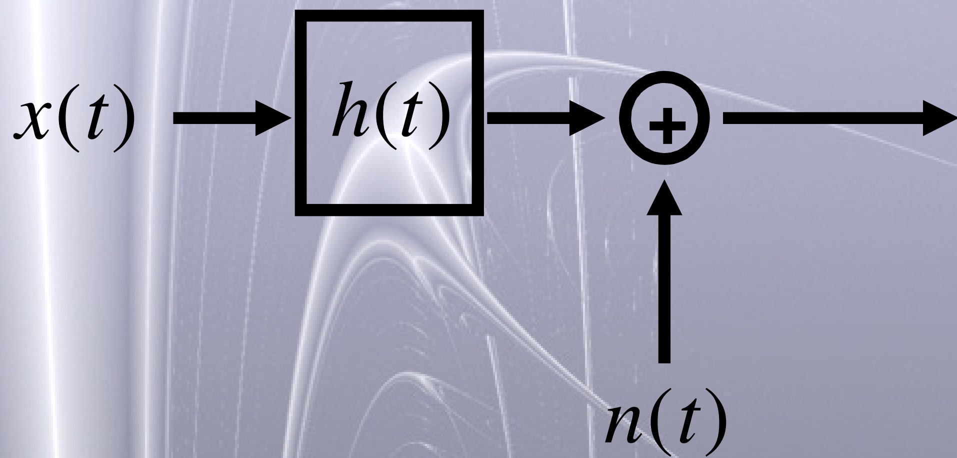

$$y(t) = x(t) * h(t) + n(t)$$

$$R_{nn}(\tau) = \frac{N_0}{2}\delta(\tau) \qquad n(t) \qquad S_{nn}(f) = \frac{N_0}{2}$$




**White Noise in first approach: infinite power → no realistic, but useful model when noise bandwidth is much larger than signal bandwidth. Which usually happens in telecommunications**

# Model of LTI Channel plus AWGN




$$y(t) = x(t) * h(t) + n(t)$$

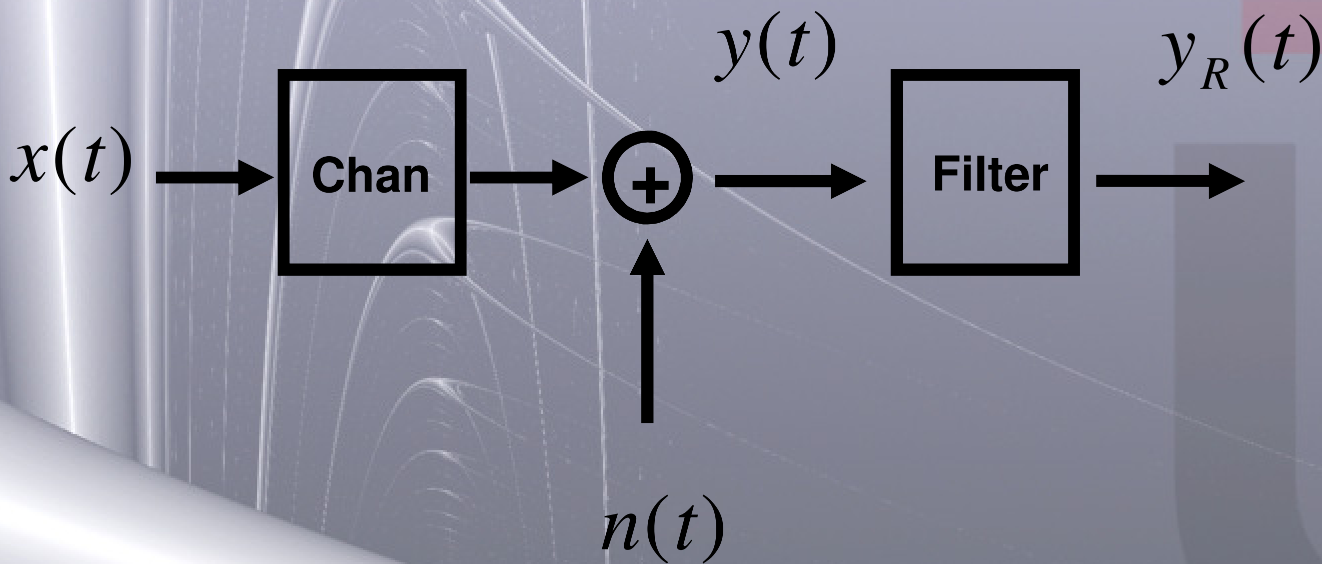
$$R_{yy}(\tau) = R_{xx}(\tau) * R_{hh}(\tau) + \frac{N_0}{2}\delta(\tau)$$

$$S_{yy}(f) = S_{xx}(f)\left|H(f)\right|^2 + \frac{N_0}{2}$$

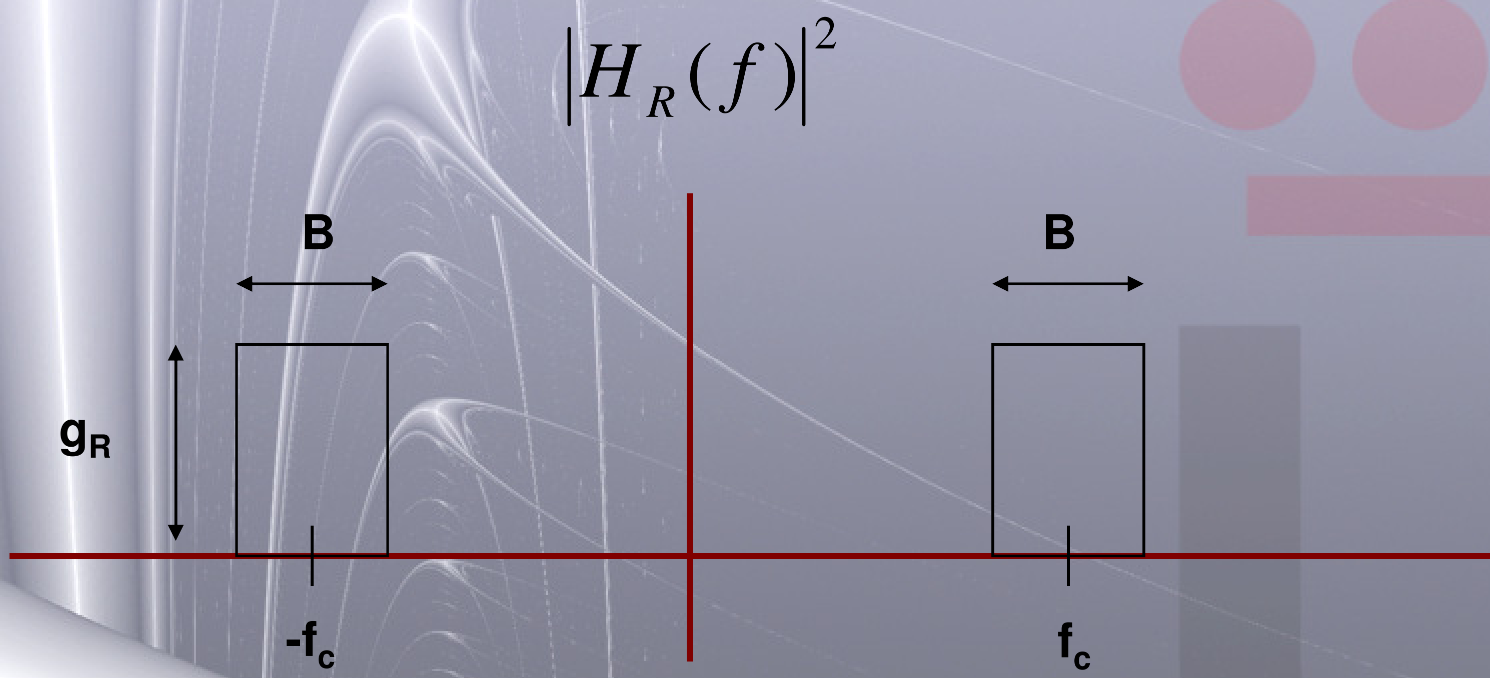
What is the received power?

And the Singal to Noise Ratio (SNR?

# Received Signal and Front-End Filter



# Receiver Front-End Filter



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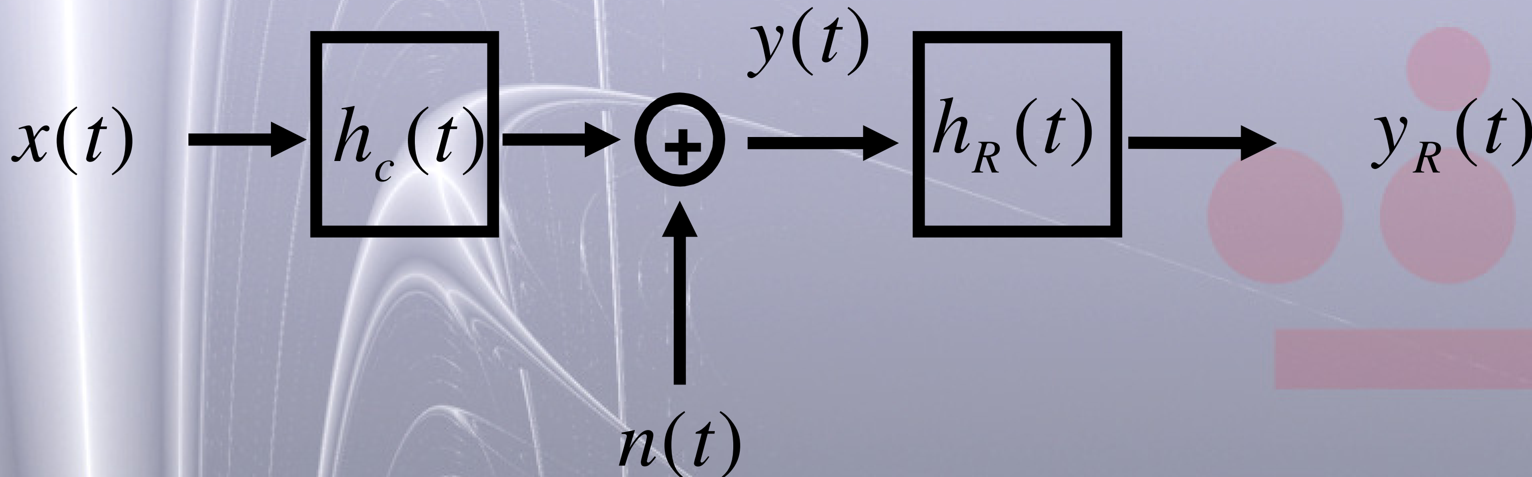
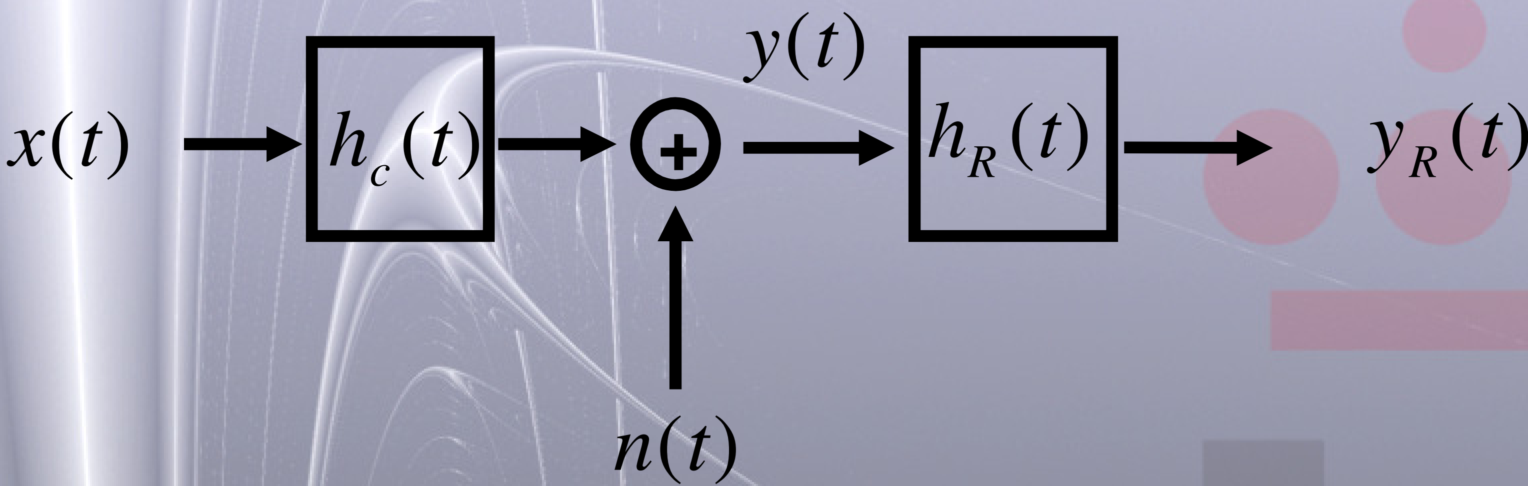
# Model of LTI Channel plus AWGN




$$R_{yy}(\tau) = \left( R_{xx}(\tau) * R_{h_C h_C}(\tau) + \frac{N_0}{2}\delta(\tau) \right) * R_{h_R h_R}(\tau)$$

$$S_{yy}(f) = \left( S_{xx}(f) \left| H_C(f) \right|^2 + \frac{N_0}{2} \right) \left| H_R(f) \right|^2$$

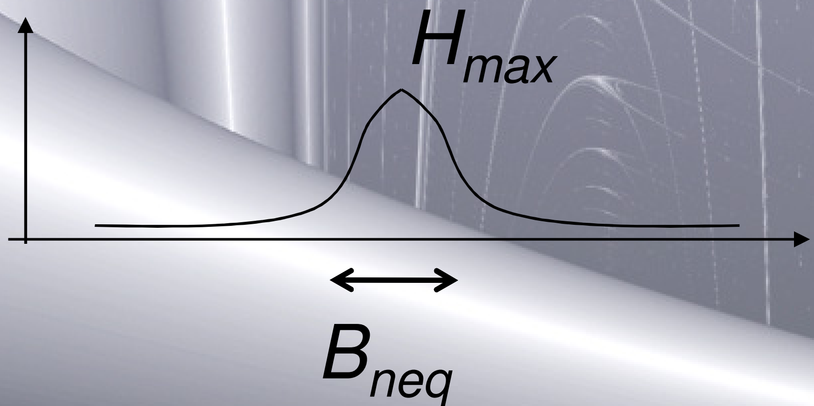
# Noise Power at the Receiver (Rx)



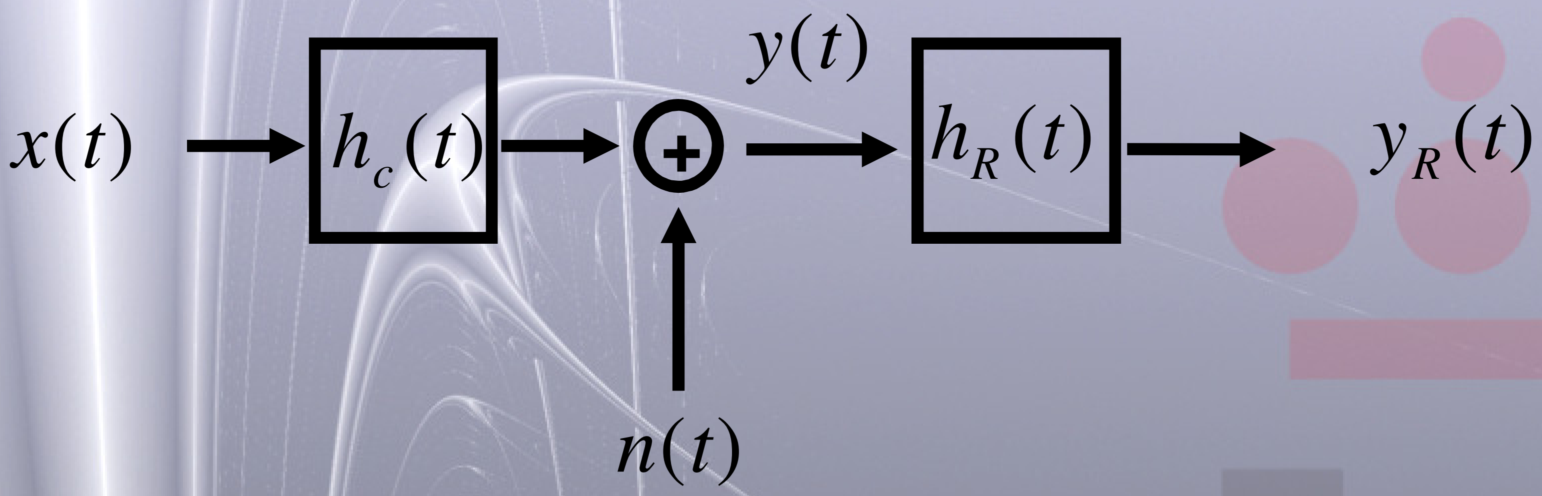

$$S_{nn}(f) = \frac{N_0}{2}\left|H_R(f)\right|^2$$

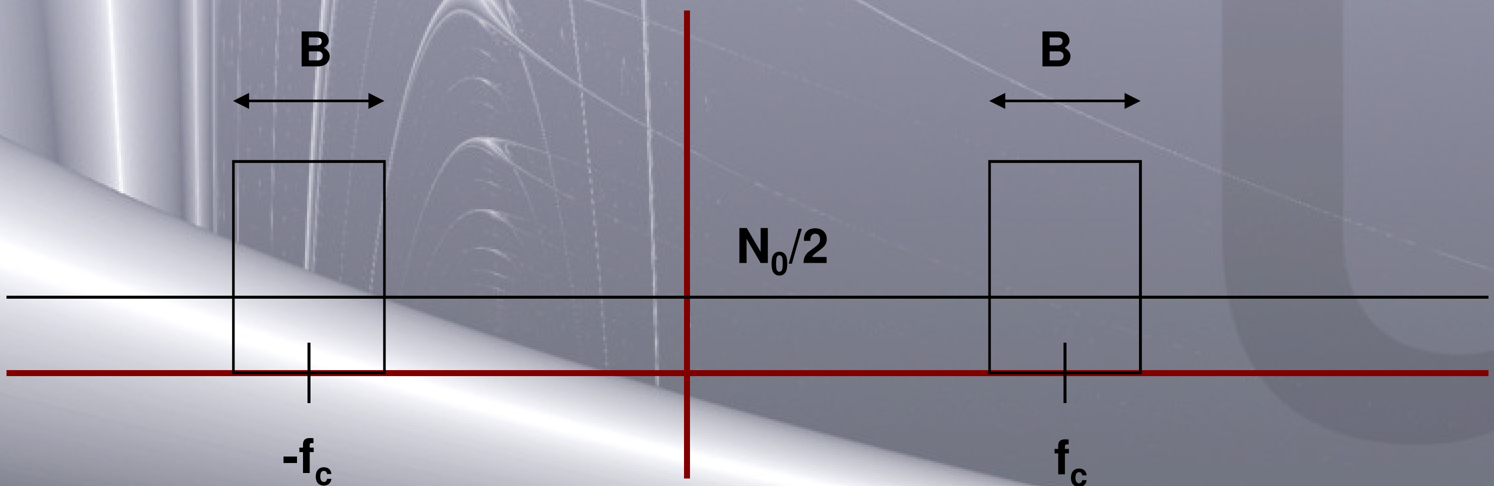
$$P_{nn} = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_R(f)\right|^2 df$$

$$= N_0 B_{neq} H_{\max}^2$$



# Noise Power at the Rx



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  - Signal to Noise Ratio
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  - Distortion
  - Interference.
- Quality metrics in communication systems
  - Noise Equivalent Temperature
  - Noise Factor
  - Noise Factor of a cascade of systems

# Channel Degradation

- Sources of signal degradation
  - Distortion: channel is modeled as a LTI by its impulse response, *h(t)*
  - Noise and Interferences

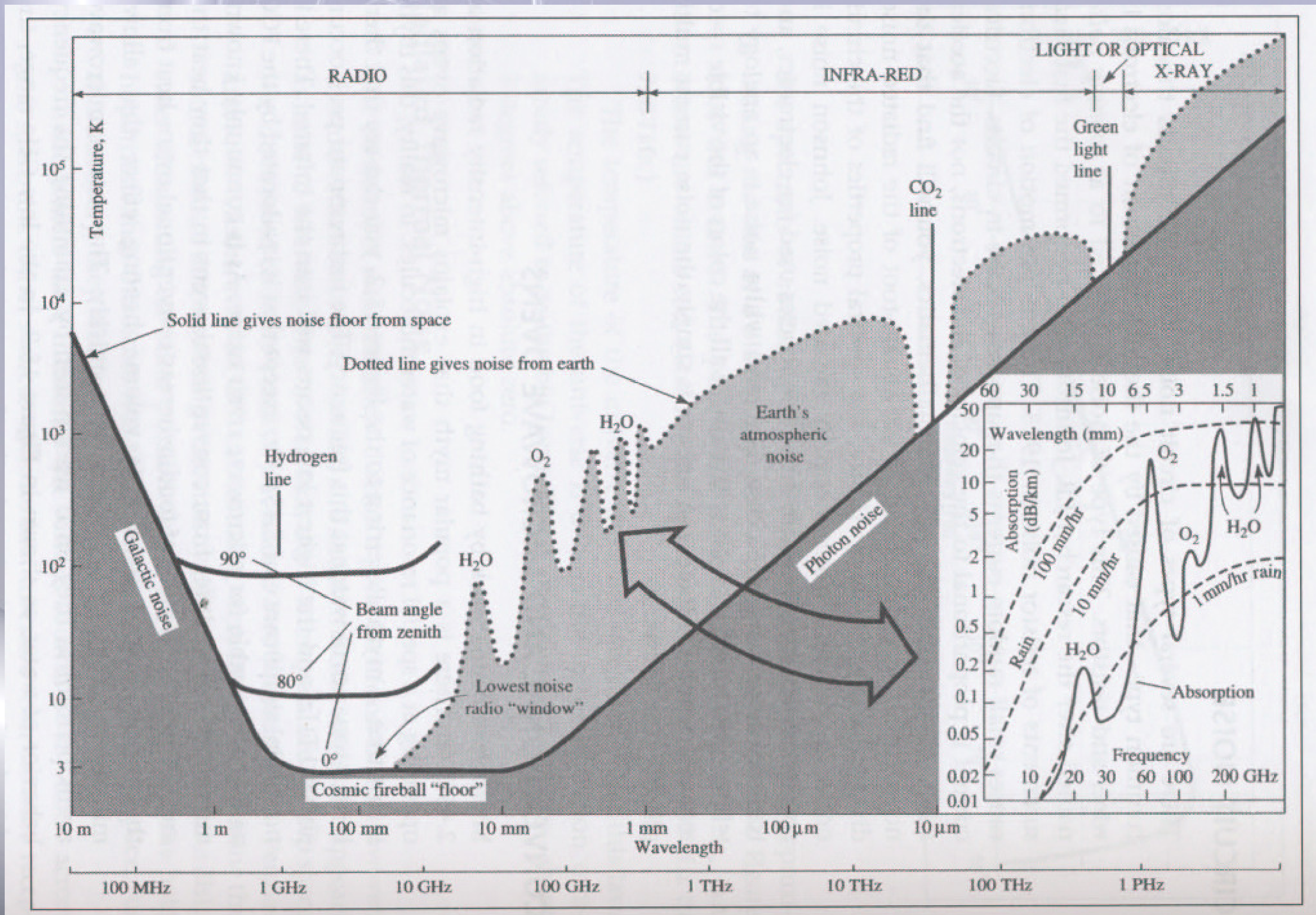
# Noise

- "Any unwanted input"
- Limits systems ability to process weak signals
- Sources:
  1. Random noise in resistors and transistors
  2. Mixer noise
  3. Undesired cross-coupling noise
  4. Power supply noise

# Sources of Noise

- External (Input at the antenna)
  - Atmospheric
  - Solar/Sky (Background thermal noise)
  - Interferers
- Receiver internal
  - Thermal (circuit noise)
  - Flicker noise (low frequency)
  - Shot noise
  - Non-linearity induced ‘noise’

# “Sky” Noise



RADIO
INFRA-RED
LIGHT OR OPTICAL
X-RAY
Temperature, K
Solid line gives noise floor from space
Dotted line gives noise from earth
Green light line
CO2 line
Earth's atmospheric noise
Hydrogen line
Galactic noise
Beam angle from zenith
90°
80°
0°
Lowest noise radio "window"
Cosmic fireball "floor"
Photon noise
H2O
O2
Wavelength
10 m
1 m
100 mm
10 mm
1 mm
100 µm
10 µm
100 MHz
1 GHz
10 GHz
100 GHz
1 THz
10 THz
100 THz
1 PHz
Wavelength (mm)
Absorption (dB/km)
100 mm/hr
10 mm/hr
1 mm/hr rain
Rain
Absorption
Frequency
10 20 30 60 100 200 GHz

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# Thermal noise

- Due to random motion of electrons
- It is ubiquitous (resistors, speakers, microphones, antennas, ...)
- It is proportional to absolute temperature
- Uncorrelated - White noise - Frequency independent up to $10^{13}$ Hz

# Thermal noise modeling

- Amplitude of the thermal noise is modeled by a Gaussian with zero mean and variance

$$\sigma_n^2 = 4kTRB$$

  - Where *k* is the Boltzman's constant equals *to k=1'381.10^-23^ Juls/ºK*, *T* is the temperature at the resistor (*ºK),* and *B* is the Bandwidth over which we measure the noise
  - Remember (*ºK)*= (*ºC)+273*

- Note this is the noise power that any resistor generates inside it, but it is not necessarily the power of noise getting into the rest of the receiver

# Thermal noise modeling

- The maximum power of noise inputting into the systems occurs when impedances are matched, and it corresponds to

$$\sigma_n^2 = kTB$$

- This thermal noise is uncorrelated (white)
- The Thermal Noise power, expressed on dBw, can be computed as

$$N(dBw) = K + 10\log(T) + 10\log(B)$$

  – Where *K = 10log(k) =10log(1.381.10$^{-23}$ ) = -228.6 dBw/Hz/ºK* and *T* the absolute temperature *(ºK)* while *B* is the bandwidth expressed in Hz

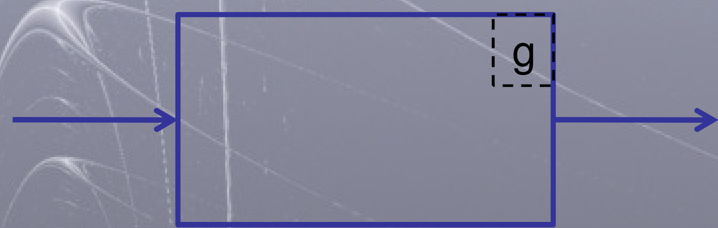
# Thermal noise modeling

- When characterizing the noise, it is important to differentiate different noise-temperatures (or sources)

*$t_0$ =290 ªK  is the* ***Reference Temperature***

*$t_{phy}$  is the physical temperature or environment temperature at which the system is*

$n_i = k t_i b$




$n_o = k t_o b = n_i g + n_{sys}$

*$t_i$  is the* ***Noise-Temperature at the Input*** *of the system*

*$g$  is the Gain of the system*

*$t_o$  is the* ***noise-temperature at the output*** *of the system*

# Thermal noise modeling

- To characterize the noise power generated by the system ($n_{sys}$ noise power), the following parameters are utilized (they are related among them):
  - Noise Equivalent Temperature ($T_{eq}$). It is a value for the temperature that accounts for the noise added by the system. If we had a noiseless system with the same noise at the input and the same gain, we would have the same noise power at the output than the actual system by increasing the input temperature by $T_{eq}$
  - Noise Factor (f): it is the ratio between equivalent noise-temperature of the actual system ($T_o + T_{eq}$) and the noise temperature in the ideal noise-free system ($T_o$) asumming the system is at reference temperature($T_o$)

# Thermal noise modeling

- So, what is the relationship between $t_{eq}$ and $n_{sys}$?

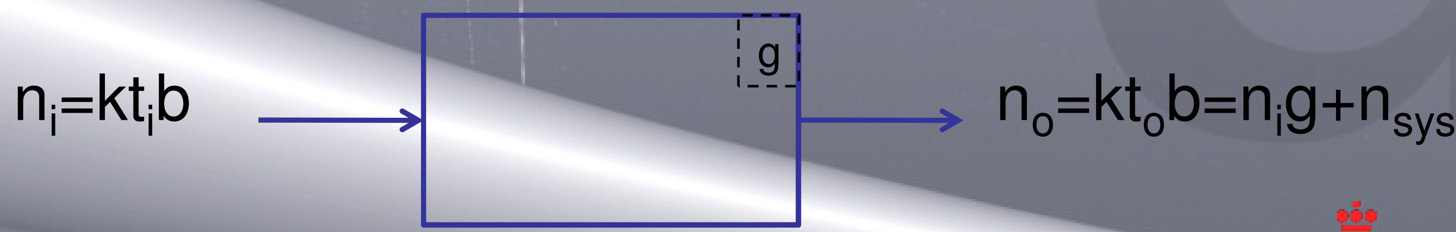
$$n_{sys} = kT_{eq}g$$

  - and therefore

$$n_o = k(T_o + T_{eq})g$$

- Thus the Noise Factor is

$$f = \frac{n_o^{actual}}{n_o^{noiseless}} = \frac{k(T_o + T_{eq})g}{kT_o g} = \frac{T_o + T_{eq}}{T_o} = 1 + \frac{T_{eq}}{T_o}$$



# Noise Factor

- Used for resistive source impedance.
- Most communication systems have a 50-Ω source impedance.
- In terms of Signal-to-Noise ratio (S/N) Noise Factor can be defined as

$$f = \frac{\left(\frac{S}{N}\right)_i}{\left(\frac{S}{N}\right)_o} = \frac{\left(\frac{S}{kT_o}\right)}{\left(\frac{gS}{k(T_o+T_{eq})g}\right)} = \frac{T_o+T_{eq}}{T_o} = 1+\frac{T_{eq}}{T_o}$$

# Noise Factor

- So f is a direct measure of the S/N ratio degradation caused by the system.
  - Noise Factor is always larger than 1 (=1 for the ideal case of noiseless)

$$f > 1$$

- The temperature of noise added by system is related to Noise Factor by

$$T_{eq} = T_o\left(f - 1\right)$$

## Noise Factor

- IEEE Standards: "The noise factor, at a specified input frequency, is defined as the ratio of (1) the total noise power per unit bandwidth available at the output port when noise temperature of the input termination is standard (290 K) to (2) that portion of (1) engendered at the input frequency by the input termination."

$$f = \frac{available - output - noise - power}{available - output - noise - due - to - source}$$

# Noise Figure

- When expressing Noise Factor in dB's we usually refer to it as Noise Figure

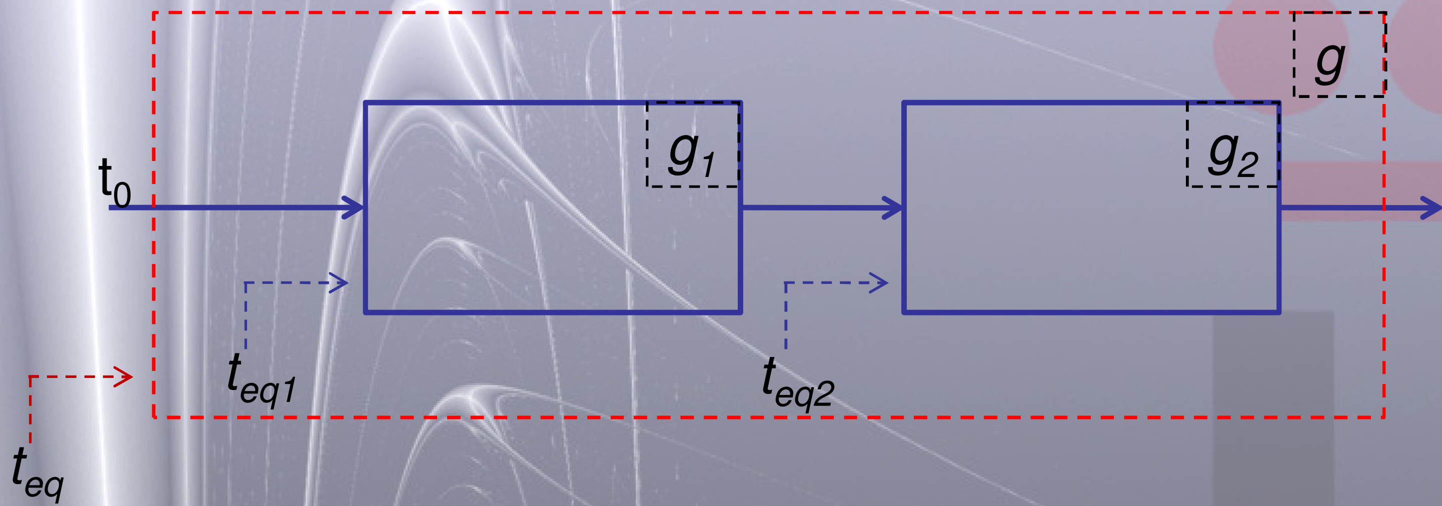
$$NF = F(dB) = 10\log(f)$$

# Noise Factor

- A particular case interesting to study is the attenuator at $T_o$ and gain $g=1/a$, being $a>1$.
  - If the physical temperature of the attenuator is To, the noise at the output is $kT_0 B$ too


and then it is straight forward to demonstrate that $f=a$

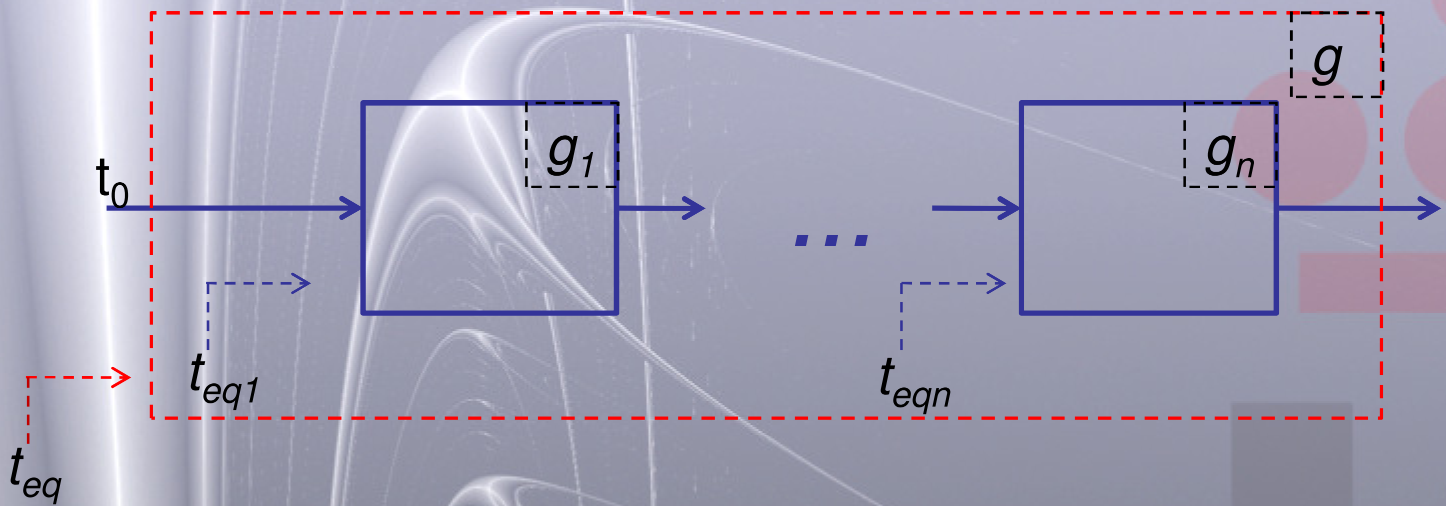
# Noise Temperature and Noise Factor of a Cascade of Two Systems




$g = g_1 \cdot g_2$

$$t_{eq} = t_{eq1} + \frac{t_{eq2}}{g_1}$$

# Noise Temperature and Noise Factor of a Cascade of Several Systems




$g=g_1 \ldots g_n$

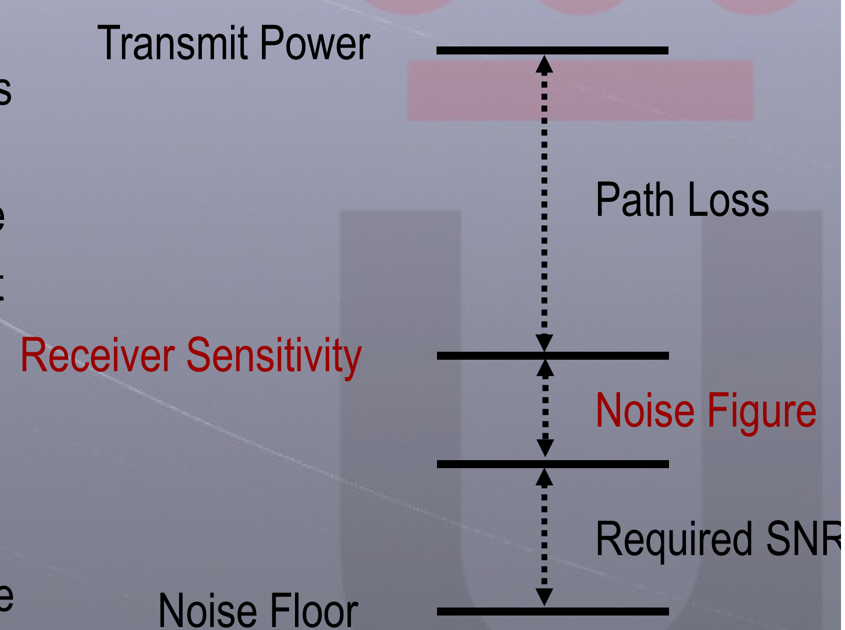
$$t_{eq} = t_{eq1} + \frac{t_{eq2}}{g_1} + \frac{t_{eq3}}{g_1 g_2} + \ldots + \frac{t_{eqn}}{g_1 g_2 \cdots g_{n-1}}$$

$$f = f_1 + \frac{f_2 - 1}{g_1} + \frac{f_3 - 1}{g_1 g_2} + \ldots + \frac{f_n - 1}{g_1 g_2 \cdots g_{n-1}}$$

We should pay most attention to the reduce the noise of the first system
(Why???)

# Required Receiver Sensitivity – A Qualitative View

- What is the required receiver NF to achieve a certain level of sensitivity?
  - To find Receiver Noise Factor
    - Transmit Power is usually fixed by other factors (regulations, battery, interference,…)
    - Path loss depends of the channel and distance
    - Required SNR – depends on BER requirement and modulation scheme
    - Noise floor – thermal noise or circuit noise is determined by To and the Noise Factor
    - Minimum Received Signal – also known as Receiver Sensitivity – fixes the maximum Noise Factor of the Receiver



# Required Receiver Sensitivity – A Qualitative View

- Receiver "noise level" directly limits sensitivity
- Receiver sensitivity = minimum input power that the receiver can detect
- Noise figure of cascaded stages
  - Noise figure of RF receivers from antenna to ADC output
  - Noise figure of passive networks
  - Noise figure of ADC

# Shot Noise

- Shot noise is due to the random arrivals of electron packets at the potential barrier of forward biased P/N junctions.
- It is always associated the a dc current flow in diodes and transistors.
- It is frequency independent (white noise) well into the GHz region.

## Shot Noise Modeling

- The noise amplitude is represented by the rms value:

$$i_n = \sqrt{2qI_D\Delta f}$$

$$\text{where}\ \ q = 1.6\times10^{-19}\ \text{C}$$

- The rms noise current for a diode current of 1 mA is about 20 pA/Hz$^{1/2}$.
- The amplitude distribution is Gaussian with $\mu = I_D$ and $\sigma = i_n$.
- A parallel current source ($i_n$) can be added to a diode to account for the shot noise.

# Flicker Noise

- Flicker noise is due to contamination and crystal defects.
- It is found in all active devices.
- It is inversely proportional to frequency (also called 1/f noise) .
- DC current in carbon resistors cause flicker noise.
- Metal film resistors have no flicker noise.

# Flicker Noise Modeling

- The noise amplitude is represented by the rms value:

$$i_n = \sqrt{K_1 \frac{I^a}{f^b} \Delta f}$$

$$\text{where } a \cong 0.5 \text{ to } 2 \text{ and } b \cong 1$$

- The constant $K_1$ is device dependent and must be determined experimentally.
- The amplitude distribution is non-Gaussian.
- It is often the dominating noise factor in the low-frequency region.
- It can be described in more details with fractal theory.

## Summary of Concepts in this Chapter

- We have established models for the degradation suffer by the signal at the receiver end
- We have defined a linear model (convolution) for the signal distortion
- We have defined two parameters (Noise Temperature and Noise Factor) to quantify the impact of the noise in the receiver
- We can calculate SNR at the receiver using above parameters