

Seminar

Information Theory: Huffman codes

PART 1

Given a discrete r.v. $X \in \{x_1, x_2, x_3, x_4, x_5, x_6\}$ with the following probabilities:

$$P(X=x_1)=0.04$$

$$P(X=x_2)=0.3$$

$$P(X=x_3)=0.1$$

$$P(X=x_4)=0.1$$

$$P(X=x_5)=0.06$$

$$P(X=x_6)=0.4$$

- 1) Compute its entropy
- 2) Find a binary Huffman code for X. When two symbols (or group of symbols) have the same probability select the order randomly
- 3) Compute the average length of the resulting code for Y
- 4) Repeat steps 2 and 3 to obtain a different code. The average length must be the same even if the set of lengths of the codes are different.

PART 2

Given a discrete r.v. $Y \in \{y_1, y_2, y_3\}$ with the following probabilities:

$$P(Y=y_1)=0.2$$

$$P(Y=y_2)=0.7$$

$$P(Y=y_3)=0.1$$

- 5) Compute $H(Y)$
- 6) Find a binary Huffman code for Y. When two symbols (or group of symbols) have the same probability select the order randomly
- 7) Compute the average length of the resulting code for Y
- 8) Repeat questions 4, 5 and 6 with a new random variable Y' created from all possible couples, with repetitions, from Y.

Note: For instance a possible new symbol of Y' is $Y'_1=Y_1Y_2$ with probability

$$P(Y'_1)=P(Y_1)*P(Y_2)=0.2*0.7$$

Note2: To check if the new code is better than the original one the average length must be divided by 2.