

Colligative properties

PHYSICAL CHEMISTRY, 2nd course
Degree in Pharmacy

2018-2019

COLLIGATIVE PROPERTIES

- **Definition of Colligative Property**
- **Vapour Pressure Lowering**
- **Freezing Point Depression (Cryoscopy)**
- **Boiling Point Elevation (Ebullioscopy)**
- **Osmotic Pressure**
- **Colligative Properties of Electrolyte Solutions**
- **Applications**
- **Summary and Conclusions**

COLLIGATIVE PROPERTIES

- The word comes from the Latin "*colligatus*" that means together.
- These properties :
 - depend ONLY on the number of solute particles.
 - do NOT depend on the nature of the solute.

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COLLIGATIVE PROPERTIES

- The solutions have certain characteristics:
 - the solute is NOT VOLATILE.
 - the solid solute does NOT dissolve into the solid solvent.

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COLLIGATIVE PROPERTIES

- These properties are studied in :
 - A. **NON- ELECTROLYTE** Solutions.
 - B. **ELECTROLYTE** Solutions.

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COLLIGATIVE PROPERTIES

- VAPOUR PRESSURE LOWERING
- FREEZING POINT DEPRESSION (CRYOSCOPY)
- BOILING POINT ELEVATION (EBULLIOSCOPY)
- OSMOTIC PRESSURE

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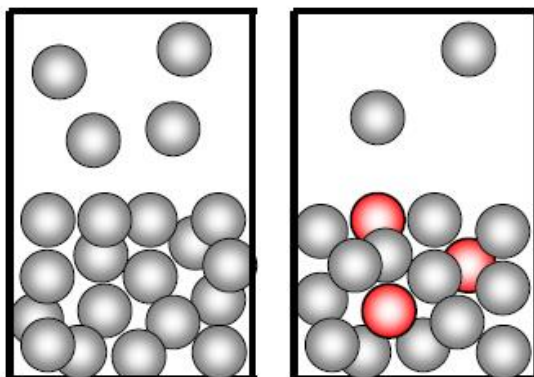
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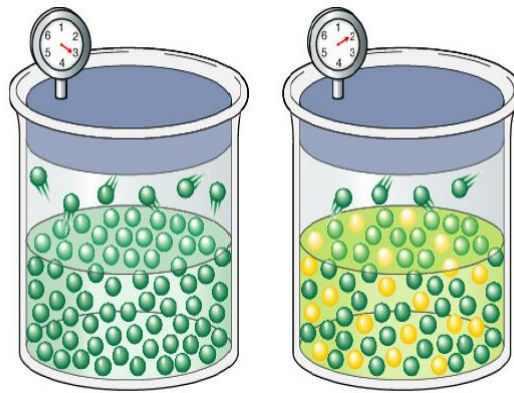
VAPOUR PRESSURE LOWERING

- The presence of the solute reduce the solvent tendency to go to the vapour phase.



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VAPOUR PRESSURE LOWERING



Pure solvent

Solution of a non-volatile solute

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VAPOUR PRESSURE LOWERING

Ideal and ideal dilute solutions

$$\Delta P = P_A^* - P = P_A^* - P_A$$

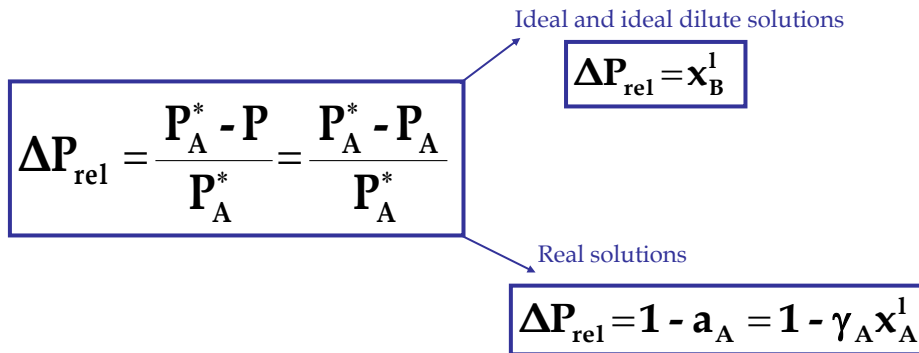
$$\Delta P = x_B^1 P_A^*$$

Real solutions

$$\Delta P = P_A^* (1 - a_A) = P_A^* (1 - \gamma_A x_A^1)$$

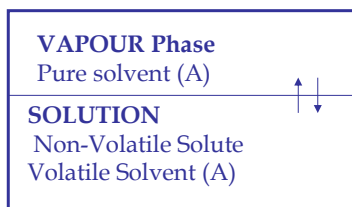
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VAPOUR PRESSURE LOWERING



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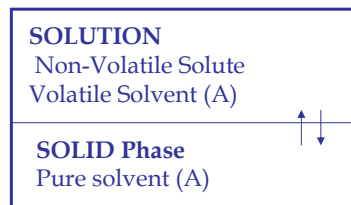
COLLIGATIVE PROPERTIES



Non-volatile solute

↓

The vapour curve does not change



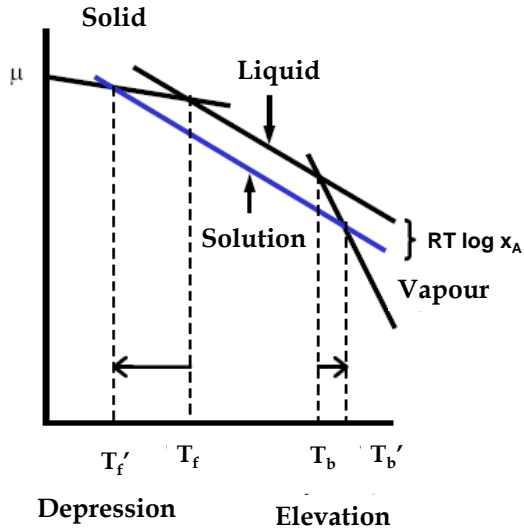
The solute does not dissolve in the solid solvent

↓

The solid curve does not change

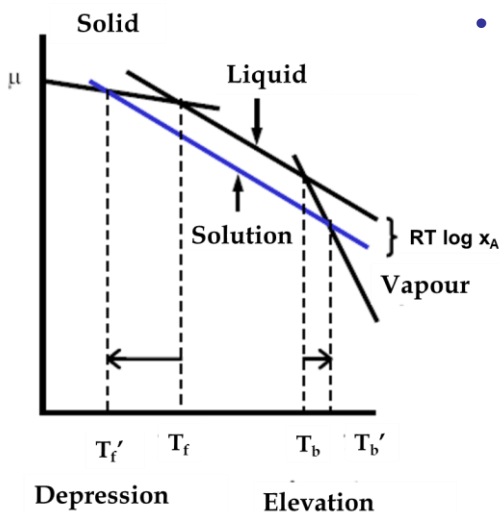
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COLLIGATIVE PROPERTIES



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COLLIGATIVE PROPERTIES



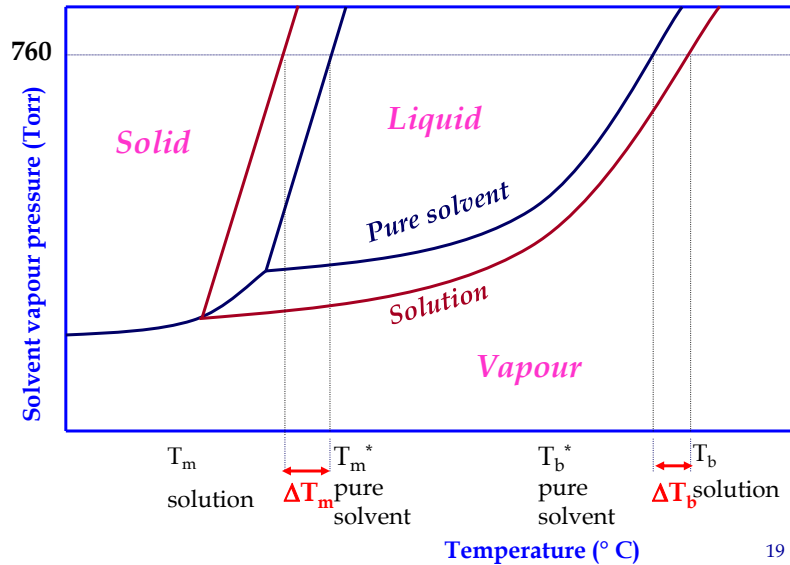
- The change in μ affects the freezing point and the boiling point

$$\left(\frac{\partial \mu}{\partial T} \right)_P = \left(\frac{\partial \bar{G}}{\partial T} \right)_P = -\bar{S}$$

$$\bar{S}_{\text{vapour}} \gg \bar{S}_{\text{liquid}} > \bar{S}_{\text{solid}}$$

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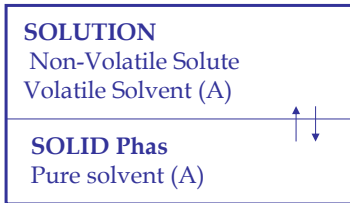
COLLIGATIVE PROPERTIES



COLLIGATIVE PROPERTIES

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FREEZING POINT DEPRESSION



This property appears when there is phase equilibrium



solution and the pure solvent in solid state



pure A solidifies from the solution when it is cooled

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$\mu_A^{*(s)} = \mu_A^{\text{sol}}$$

$$\mu_A^{*(s)} = \mu_A^{o(l)} + R T \log x_A^1$$

$$- R T \log x_A^1 = \mu_A^{o(l)} - \mu_A^{*(s)}$$

$$- R T \log x_A^1 = \mu_A^{*(l)} - \mu_A^{*(s)}$$

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$-R T \log x_A^1 = \mu_A^{*(l)} - \mu_A^{*(s)}$$

$$-R T \log x_A^1 = \overline{\Delta G_{m,A}}$$

$$\log x_A^1 = \frac{-\overline{\Delta G_{m,A}}}{R T}$$

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = \left(\frac{\partial \frac{-\overline{\Delta G_{m,A}}}{R T}}{\partial T} \right)_P$$

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = - \frac{\left(\frac{\partial \overline{\Delta G_{m,A}}}{\partial T} \right)_P R T - \left(\frac{\partial R T}{\partial T} \right)_P \overline{\Delta G_{m,A}}}{R^2 T^2}$$

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = - \frac{\left(\frac{\partial \overline{\Delta G}_{m,A}}{\partial T} \right)_P R T - \left(\frac{\partial R T}{\partial T} \right)_P \overline{\Delta G}_{m,A}}{R^2 T^2}$$

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = - \frac{-\overline{\Delta S}_{m,A} T - \overline{\Delta G}_{m,A}}{R T^2}$$

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = \frac{\overline{\Delta H}_{m,A}}{R T^2}$$

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$\int_{\log 1}^{\log x_A^1} d \log x_A^1 = \int_{T_{m,A}^*}^{T_m} \frac{\overline{\Delta H}_{m,A}}{R T^2} dT$$

$$\log x_A^1 = \frac{-\overline{\Delta H}_{m,A}}{R} \left(\frac{1}{T_m} - \frac{1}{T_{m,A}^*} \right)$$

$$\log x_A^1 = \frac{-\overline{\Delta H}_{m,A}}{R} \left(\frac{T_{m,A}^* - T_m}{T_{m,A}^* T_m} \right)$$

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FREEZING POINT DEPRESSION

• IDEAL SOLUTION

$$\log x_A^1 = \frac{-\overline{\Delta H}_{m,A}}{R} \left(\frac{T_{m,A}^* - T_m}{T_{m,A}^* T_m} \right)$$

$$\Delta T_m = T_{m,A}^* - T_m$$

$$\log x_A^1 = \frac{-\overline{\Delta H}_{m,A}}{R} \left(\frac{\Delta T_m}{T_{m,A}^* T_m} \right)$$

$$\log x_A^1 \cong \frac{-\overline{\Delta H}_{fus,A}}{R} \left(\frac{\Delta T_{fus}}{T_{fus,A}^{*2}} \right)$$

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FREEZING POINT DEPRESSION

• IDEAL DILUTE SOLUTIONS

$$\log(1-x) = -x + \frac{1}{2!}x^2 - \frac{2}{3!}x^3 + \frac{6}{4!}x^4 - \dots$$

$$\log(1-x_B^1) \cong -x_B^1$$

$$x_B^1 \cong \frac{\overline{\Delta H}_{m,A}}{R} \left(\frac{\Delta T_m}{T_{m,A}^{*2}} \right)$$

$$x_B^1 \cong m_B M_A 10^{-3}$$

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FREEZING POINT DEPRESSION

• IDEAL DILUTE SOLUTION

$$\Delta T_m = \frac{M_A 10^{-3} R T_{m,A}^{*2}}{\Delta \overline{H}_{m,A}} m_B$$

$$\Delta T_m = K_c m_B$$

$$K_c = \frac{M_A R T_{m,A}^{*2}}{\Delta \overline{H}_{m,A} 10^3}$$

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FREEZING POINT DEPRESSION

• REAL SOLUTION

$$\mu_A^{*(s)} = \mu_A^{\text{sol}}$$

$$\mu_A^{*(s)} = \mu_A^{o(l)} + R T \log a_A$$

$$- R T \log a_A = \mu_A^{o(l)} - \mu_A^{*(s)}$$

$$- R T \log a_A = \mu_A^{*(l)} - \mu_A^{*(s)}$$

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FREEZING POINT DEPRESSION

• REAL SOLUTION

$$-R T \log a_A = \mu_A^{*(l)} - \mu_A^{*(s)}$$

$$-R T \log a_A = \overline{\Delta G_{m,A}}$$

$$\left(\frac{\partial \log a_A}{\partial T} \right)_P = \frac{\overline{\Delta H_{m,A}}}{R T^2}$$

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FREEZING POINT DEPRESSION

• REAL SOLUTION

$$\int_{\log 1}^{\log a_A} d \log a_A = \int_{T_{m,A}^*}^{T_m} \frac{\overline{\Delta H_{m,A}}}{R T^2} dT$$

$$\log a_A = \frac{-\overline{\Delta H_{m,A}}}{R} \left(\frac{\Delta T_m}{T_{m,A}^* T_m} \right)$$

$$\log a_A = \log \gamma_A x_A^1 \cong \frac{-\overline{\Delta H_{m,A}}}{R} \left(\frac{\Delta T_m}{T_{m,A}^{*2}} \right)$$

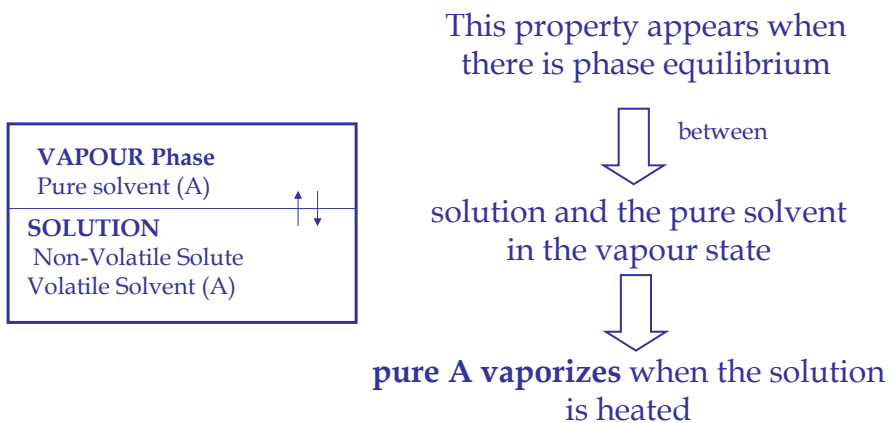
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BOILING POINT ELEVATION



BOILING POINT ELEVATION

- IDEAL SOLUTION

$$\mu_A^{*(v)} = \mu_A^{\text{sol}}$$

$$\mu_A^{*(v)} = \mu_A^{o(l)} + R T \log x_A^1$$

$$\mu_A^{*(v)} - \mu_A^{o(l)} = R T \log x_A^1$$

$$\mu_A^{*(v)} - \mu_A^{*(l)} = R T \log x_A^1$$

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BOILING POINT ELEVATION

- IDEAL SOLUTION

$$\mu_A^{*(v)} - \mu_A^{*(l)} = R T \log x_A^1$$

$$\overline{\Delta G}_{\text{vap},A} = R T \log x_A^1$$

$$\left(\frac{\partial \log x_A^1}{\partial T} \right)_P = - \frac{\overline{\Delta H}_{\text{vap},A}}{R T^2}$$

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BOILING POINT ELEVATION

• IDEAL SOLUTION

$$\int_{\log 1}^{\log x_A^1} d \log x_A^1 = \int_{T_{b,A}^*}^{T_b} - \frac{\overline{\Delta H}_{\text{vap},A}}{R T^2} dT$$

$$\log x_A^1 = \frac{-\overline{\Delta H}_{\text{vap},A}}{R} \left(\frac{\Delta T_b}{T_{b,A}^* T_b} \right) \quad \Delta T_b = T_b - T_{b,A}^*$$

$$\log x_A^1 \cong \frac{-\overline{\Delta H}_{\text{vap},A}}{R} \left(\frac{\Delta T_b}{T_{b,A}^{*2}} \right)$$

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BOILING POINT ELEVATION

• IDEAL DILUTE SOLUTION

$$\log(1-x) = -x + \frac{1}{2!}x^2 - \frac{2}{3!}x^3 + \frac{6}{4!}x^4 - \dots$$

$$\log(1-x_B^1) \cong -x_B^1$$

$$x_B^1 \cong \frac{\overline{\Delta H}_{\text{vap},A}}{R} \left(\frac{\Delta T_b}{T_{b,A}^{*2}} \right)$$

$$x_B^1 \cong m_B M_A 10^{-3}$$

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BOILING POINT ELEVATION

• IDEAL DILUTE SOLUTION

$$\Delta T_b = \frac{M_A 10^{-3} R T_{b,A}^{*2}}{\Delta \bar{H}_{\text{vap},A}} m_B$$

$$\Delta T_b = K_b m_B$$

$$K_b = \frac{M_A R T_{b,A}^{*2}}{\Delta \bar{H}_{\text{vap},A} 10^3}$$

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BOILING POINT ELEVATION

• REAL SOLUTION

$$\mu_A^{*(v)} = \mu_A^{\text{sol}}$$

$$\mu_A^{*(v)} = \mu_A^{o(l)} + R T \log a_A$$

$$\mu_A^{*(v)} - \mu_A^{o(l)} = R T \log a_A$$

$$\mu_A^{*(v)} - \mu_A^{*(l)} = R T \log a_A$$

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BOILING POINT ELEVATION

• REAL SOLUTION

$$\mu_A^{*(v)} - \mu_A^{*(l)} = R T \log a_A$$

$$\overline{\Delta G}_{\text{vap},A} = R T \log a_A$$

$$\left(\frac{\partial \log a_A}{\partial T} \right)_P = - \frac{\overline{\Delta H}_{\text{vap},A}}{R T^2}$$

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BOILING POINT ELEVATION

• REAL SOLUTION

$$\int_{\log 1}^{\log a_A} d \log a_A = \int_{T_{b,A}^*}^{T_b} - \frac{\overline{\Delta H}_{\text{vap},A}}{R T^2} dT$$

$$\log a_A = \frac{-\overline{\Delta H}_{\text{vap},A}}{R} \left(\frac{\Delta T_b}{T_{b,A}^* T_b} \right)$$

$$\log a_A = \log \gamma_A x_A^1 \cong \frac{-\overline{\Delta H}_{\text{vap},A}}{R} \left(\frac{\Delta T_b}{T_{b,A}^{*2}} \right)$$

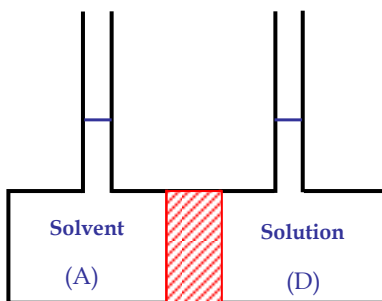
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OSMOTIC PRESSURE

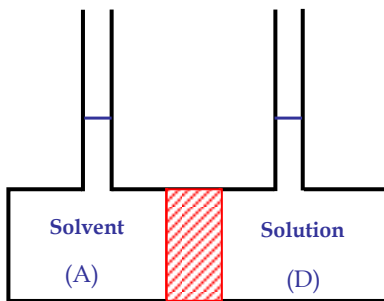


This property appears when there is phase equilibrium



solution and the pure solvent in liquid phase

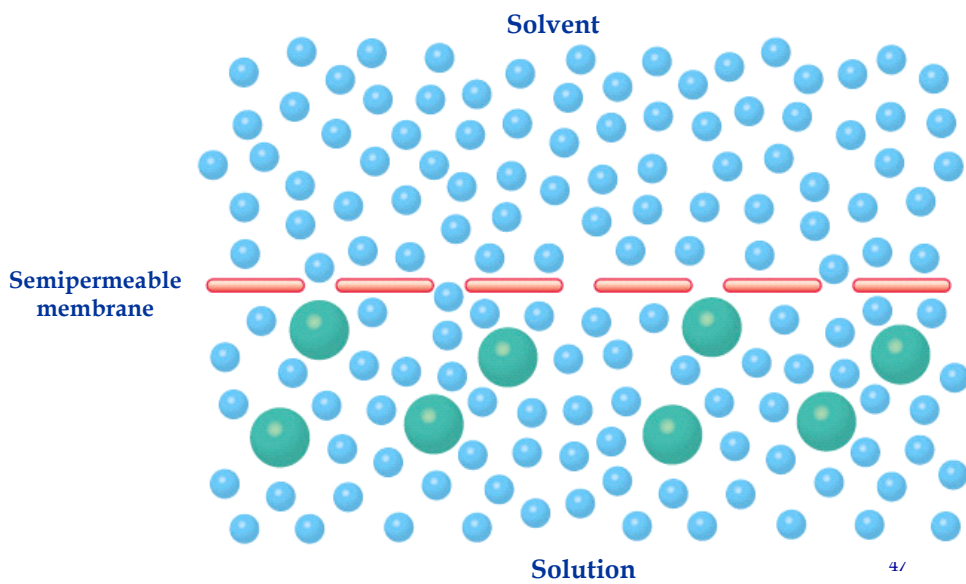
OSMOTIC PRESSURE



- rigid $\rightarrow V = \text{constant}$
- diathermy $\rightarrow T_A = T_D$
- semipermeable \rightarrow only allows A to pass through

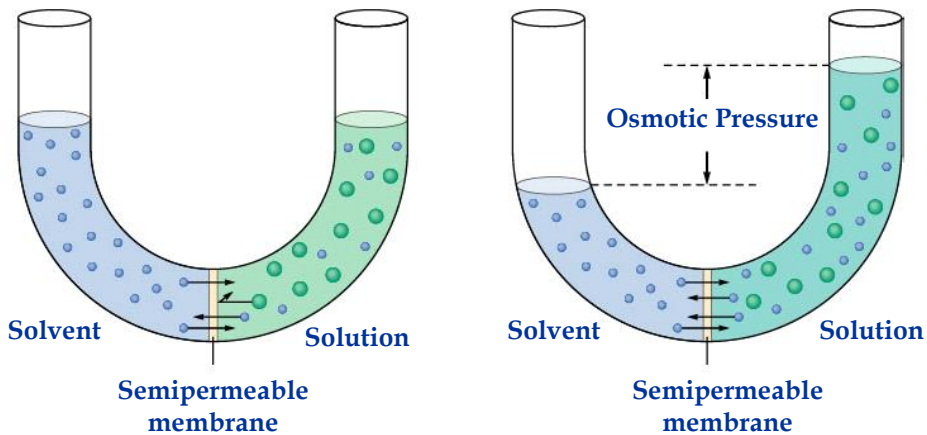
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OSMOTIC PRESSURE



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OSMOTIC PRESSURE



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OSMOTIC PRESSURE

• IDEAL SOLUTION

$$\mu_A^{*(l)}(T, P_\alpha) = \mu_A^{(l)}(T, P_\beta)$$

$$\mu_A^{*(l)}(T, P_\alpha) = \mu_A^{*(l)}(T, P_\beta) + R T \log x_A^1$$

$$\mu_A^{*(l)}(T, P_\alpha) - \mu_A^{*(l)}(T, P_\beta) = R T \log x_A^1$$

$$\left(\frac{\partial \mu_A^*}{\partial P} \right)_T = \bar{V}_A^*$$

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OSMOTIC PRESSURE

• IDEAL SOLUTION

$$\mu_A^{*(l)}(T, P_\beta) - \mu_A^{*(l)}(T, P_\alpha) = \bar{V}_A^* \Pi$$

$$\mu_A^{*(l)}(T, P_\alpha) - \mu_A^{*(l)}(T, P_\beta) = R T \log x_A^1$$

$$\boxed{-R T \log x_A^1 = \bar{V}_A^* \Pi}$$

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OSMOTIC PRESSURE

• IDEAL DILUTE SOLUTION

$$\log(1 - x_B^1) \cong -x_B^1$$

$$x_B^1 R T \cong \bar{V}_A^* \Pi$$

$$x_B^1 \cong c_B \bar{V}_A^*$$

$$\boxed{\Pi \cong c_B R T}$$

Van't Hoff Equation

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OSMOTIC PRESSURE

• REAL SOLUTION

$$\mu_A^{*(l)}(T, P_\alpha) = \mu_A^{(l)}(T, P_\beta)$$

$$\mu_A^{*(l)}(T, P_\alpha) = \mu_A^{*(l)}(T, P_\beta) + R T \log a_A$$

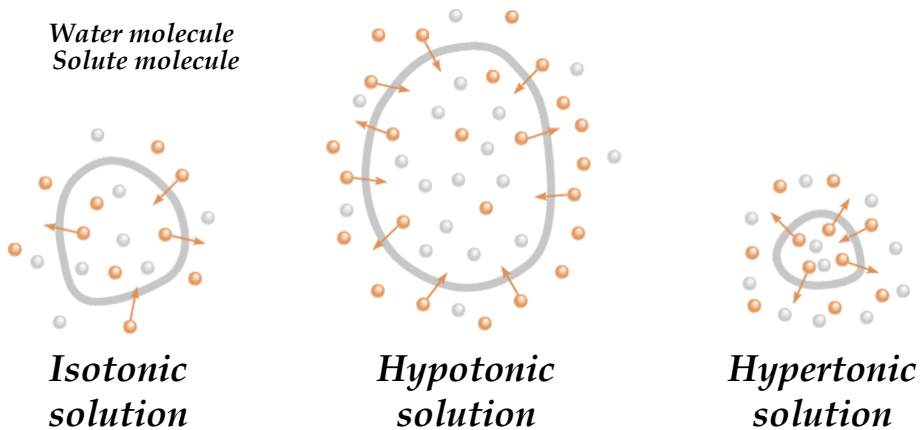
$$\mu_A^{*(l)}(T, P_\beta) - \mu_A^{*(l)}(T, P_\alpha) = \bar{V}_A^* (P_\beta - P_\alpha)$$

$$\mu_A^{*(l)}(T, P_\beta) - \mu_A^{*(l)}(T, P_\alpha) = \bar{V}_A^* \Pi$$

$$-R T \log a_A = -R T \log \gamma_A x_A^l = \bar{V}_A^* \Pi$$

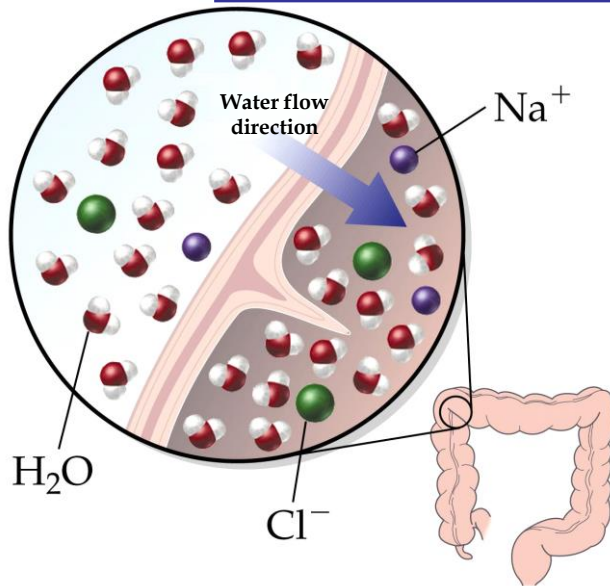
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OSMOTIC PRESSURE



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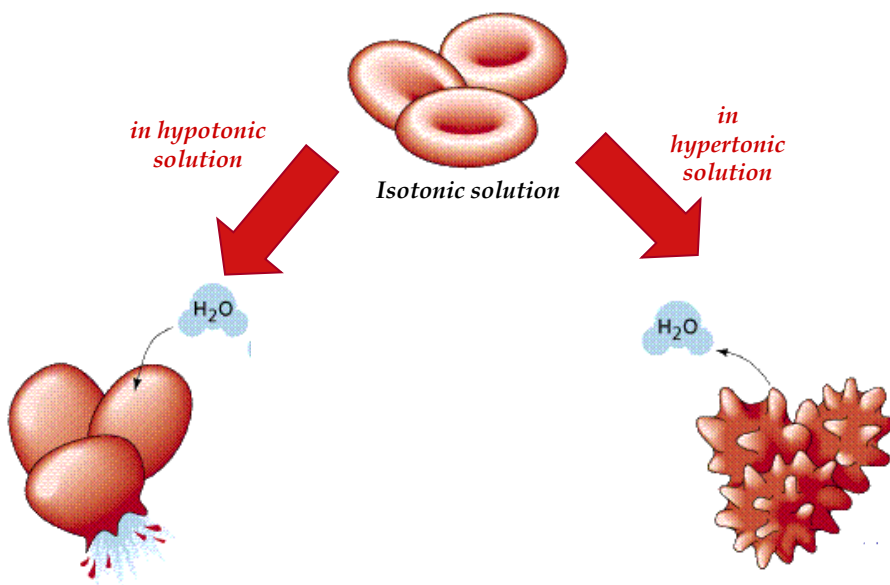
OSMOTIC PRESSURE



Drinking sea water causes dehydration.

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OSMOTIC PRESSURE

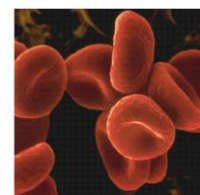


OSMOTIC PRESSURE

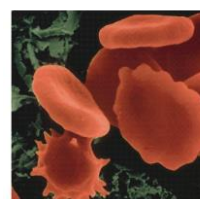
Isotonic solution



Hypotonic solution



Hypertonic solution



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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

Colligative properties are proportional to the concentration of
ALL solute particles
(ION + MOLECULES)



The dissociation of electrolytes modify the value of the
colligative properties of a solution.

One must distinguish between:

- Ideal dilute solutions of electrolytes
- Real solutions of electrolytes

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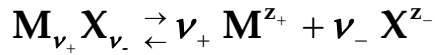
COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

A. IDEAL DILUTE SOLUTION:

- The effective concentration is greater than the real concentration.
- The Van't Hoff factor, i , needs to be introduced.

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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS



$$\begin{array}{l} t = 0 \quad m \quad 0 \quad 0 \\ t = t_{eq} \quad m(1-\alpha) \quad \nu_+ m\alpha \quad \nu_- m\alpha \end{array}$$

$$\begin{aligned} m_{\text{effect}} &= m(1-\alpha) + \nu_+ m\alpha + \nu_- m\alpha = \\ &= m(1-\alpha + \nu\alpha) = m(1 + \alpha(\nu - 1)) = m i \end{aligned}$$

$$\boxed{i = 1 + \alpha(\nu - 1)}$$

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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

➤ Vapour pressure lowering

✓ Ideal dilute solution:

$$\boxed{\Delta P = x_B i P_A^*}$$

➤ Freezing Point Depression (Cryoscopy)

✓ Ideal dilute solution:

$$\boxed{\Delta T_m = K_c m_B i}$$

➤ Boiling point elevation (ebullioscopy)

✓ Ideal dilute solution:

$$\boxed{\Delta T_b = K_b m_B i}$$

➤ Osmotic Pressure

✓ Ideal dilute solution:

$$\boxed{\Pi = R T c_B i}$$

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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

- For a electrolyte completely dissociated: $i = \nu$

- Vapour pressure lowering

- ✓ Ideal dilute solution: $\Delta P = x_B \nu P_A^*$

- Freezing Point Depression (Cryoscopy)

- ✓ Ideal dilute solution: $\Delta T_m = K_c m_B \nu$

- Boiling point elevation (ebullioscopy)

- ✓ Ideal dilute solution: $\Delta T_b = K_b m_B \nu$

- Osmotic Pressure

- ✓ Ideal dilute solution: $\Pi = R T c_B \nu$

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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

B. REAL SOLUTION:

- The effective concentration is greater than the real concentration.
- The expressions are the same that the ones for a real solution, but now the values depend on the total number of particles

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COLLIGATIVE PROPERTIES ELECTROLYTE SOLUTIONS

B. REAL SOLUTIONS:

$$\Delta P = P_A^* (1 - a_A)$$
$$\log a_A = - \frac{\Delta T_m \Delta \bar{H}_{m,A}}{R T_{m,A}^2}$$
$$\log a_A = - \frac{\Delta T_b \Delta \bar{H}_{vap,A}}{R T_{b,A}^2}$$
$$-R T \log a_A = \bar{V}_A^* \Pi$$

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APPLICATIONS

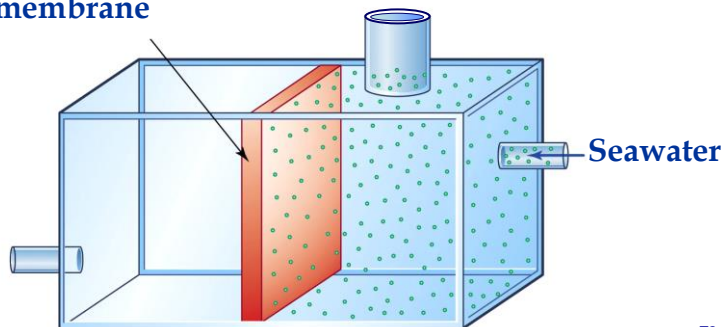
- Determination of solutes molecular weight.
- Antifreeze compounds.
- Ascent of sap.
- Mechanical stability of the plants.
- Use of salt and sugar to preserve food.
- Gherkin in vinegar.
- Reverse osmosis.

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OSMOTIC PRESSURE

Reverse Osmosis: Water Purification

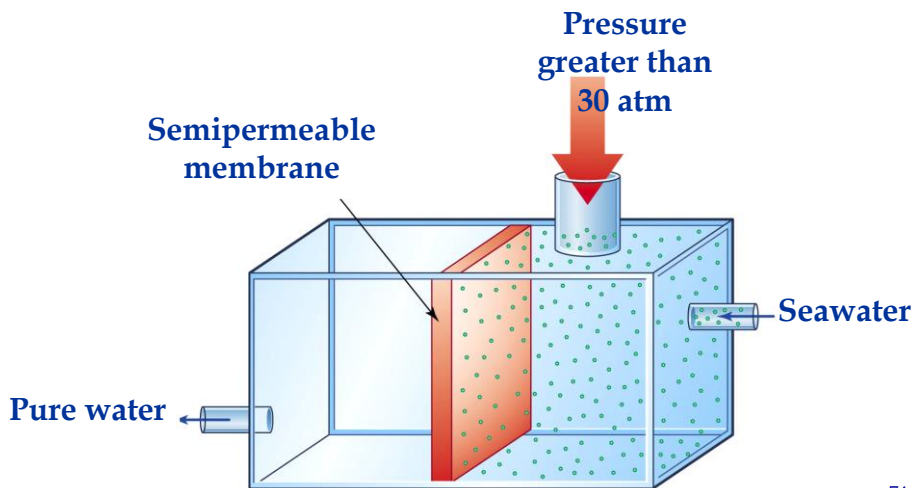
Semipermeable
membrane



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OSMOTIC PRESSURE

Reverse Osmosis: Water Purification



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SUMMARY AND CONCLUSIONS

- VAPOUR PRESSURE LOWERING:

➤ Ideal and an ideal dilute solution: $\Delta P = x_B P_A^*$

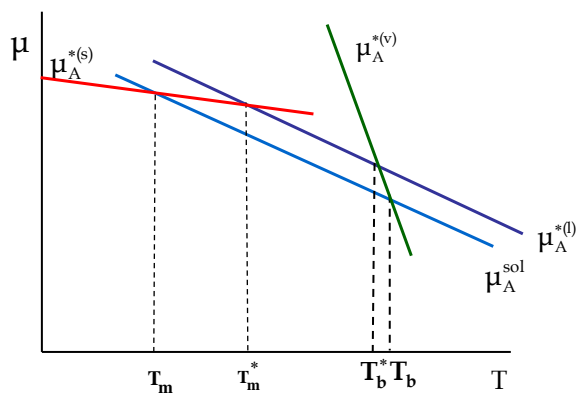
➤ Real Solution: $\Delta P = P_A^* (1 - a_A)$

➤ Electrolyte solutions: $\Delta P = x_B i P_A^*$



SUMMARY AND CONCLUSIONS

- FREEZING POINT DEPRESSION AND BOILING POINT ELEVATION



SUMMARY AND CONCLUSIONS

• FREEZING POINT DEPRESSION :

➤ Ideal solution: $\log x_A^1 = - \frac{\Delta T_m \Delta \bar{H}_{m,A}}{R T_{m,A}^2}$

➤ Ideal dilute solution: $\Delta T_m = K_c m_B$

➤ Real Solution: $\log a_A = - \frac{\Delta T_m \Delta \bar{H}_{m,A}}{R T_{m,A}^2}$

➤ Electrolyte solution: $\Delta T_m = K_c m_B i$



SUMMARY AND CONCLUSIONS

• BOILING POINT ELEVATION (EBULLIOSCOPY):

➤ Ideal solution: $\log x_A^1 = - \frac{\Delta T_b \Delta \bar{H}_{\text{vap},A}}{R T_{b,A}^2}$

➤ Ideal dilute solution: $\Delta T_b = K_b m_B$

➤ Real Solution: $\log a_A = - \frac{\Delta T_b \Delta \bar{H}_{\text{vap},A}}{R T_{b,A}^2}$

➤ Electrolyte solution: $\Delta T_b = K_b m_B i$



SUMMARY AND CONCLUSIONS

- Osmotic PRESSURE:

- Ideal solution: $-R T \log x_A^l = \bar{V}_A^* \Pi$

- Ideal dilute solution: $\Pi = c_B R T$

- Real Solution: $-R T \log a_A = \bar{V}_A^* \Pi$

- Electrolyte solution: $\Pi = R T c_B i$