

Ondas electromagnéticas

Introducción. Ecuaciones de Maxwell

Ondas electromagnéticas planas senoidales

Energía y momento

Ondas electromagnéticas en la materia.

El espectro electromagnético

Generación de ondas electromagnéticas

Ecuaciones de Maxwell

Formulación integral

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_A \mathbf{B} \cdot \mathbf{n} da$$

$$\oint_S \mathbf{E} \cdot \mathbf{n} da = \frac{q}{\epsilon_0}$$

$$\oint_S \mathbf{B} \cdot \mathbf{n} da = 0$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I + I_D) = \mu_0 \int_A \left(\mathbf{j} + \epsilon_0 \frac{d\mathbf{E}}{dt} \right) \cdot \mathbf{n} da$$

Onda plana y ecuaciones de Maxwell

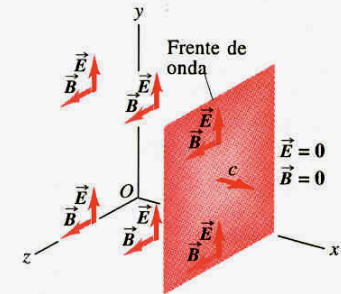
Propagación dirección x

Comprobación ley Gauss

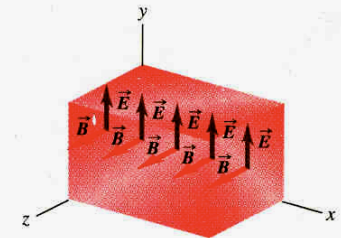
$$\oint_S \mathbf{E} \cdot \mathbf{n} da = \frac{q}{\epsilon_0}$$

$$\oint_S \mathbf{B} \cdot \mathbf{n} da = 0$$

¿ si E_x o B_x existen ?



33-2 Frente de onda electromagnética. El plano que representa al frente de onda se desplaza hacia la derecha con una rapidez c . Los campos $\vec{E}$ y $\vec{B}$ son uniformes en la región que se encuentra detrás del frente de onda, pero son cero delante de éste.



33-3 Superficie gaussiana para una onda electromagnética plana. Los flujos eléctrico y magnético totales a través de la superficie son: ambos cero.

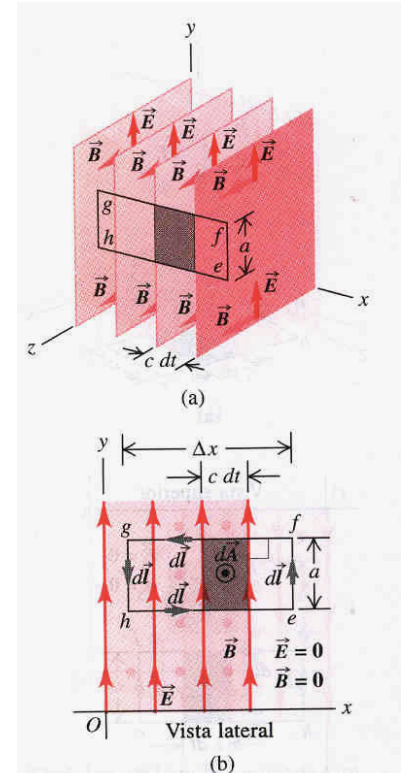
Onda plana y ecuaciones de Maxwell

Comprobación ley Faraday

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_A \mathbf{B} \cdot \mathbf{n} da$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -Ea; \frac{d\phi_B}{dt} Bac; -Ea = -Bac; c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$E = cB$$



33-4 Aplicación de la ley de Faraday a una onda plana. En el tiempo dt el frente de onda se mueve hacia la derecha una distancia $c dt$. El flujo magnético a través del rectángulo en el plano xy aumenta en una cantidad $d\Phi_B$ igual al flujo a través del rectángulo sombreado con área $ac dt$; es decir, $d\Phi_B = Bac dt$.

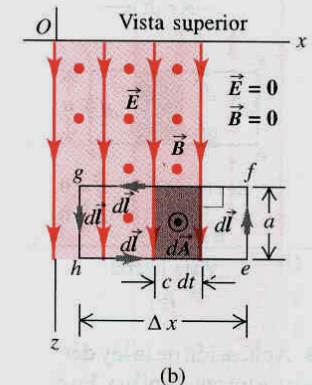
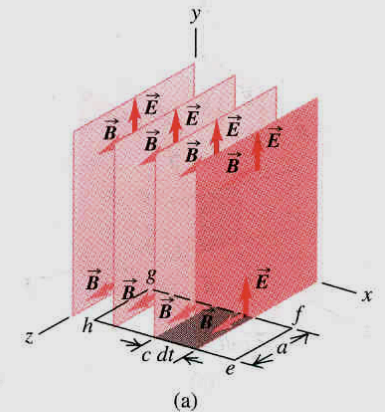
Onda plana y ecuaciones de Maxwell

Comprobación ley Ampère

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I + I_D) = \mu_0 \int_A \left(\mathbf{j} + \varepsilon_0 \frac{d\mathbf{E}}{dt} \right) \cdot \mathbf{n} da$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = Ba; \frac{d\phi_E}{dt} = Eac; Ba = \varepsilon_0 \mu_0 Eac;$$

$$B = \varepsilon_0 \mu_0 c E \Rightarrow c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$



33-5 Aplicación de la ley de Ampère a una onda plana. En el tiempo dt el flujo eléctrico a través del rectángulo en el plano xz aumenta en una cantidad igual a E por el área $ac \, dt$ del rectángulo sombreado; es decir $d\Phi_E = Eac \, dt$. Por tanto, $d\Phi_E/dt = Eac$.

Ecuación de onda

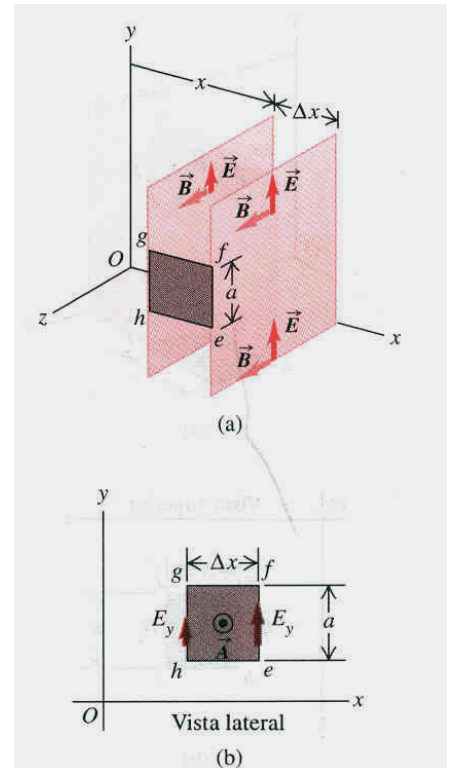
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -E_y(x, t)a + E_y(x + \Delta x, t)a = a[E_y(x + \Delta x, t) - E_y(x, t)]$$

$$\frac{d\phi_B}{dt} = \frac{\partial B_z(x, t)}{\partial t} a \Delta x$$

$$a[E_y(x + \Delta x, t) - E_y(x, t)] = -\frac{\partial B_z}{\partial t} a \Delta x$$

$$\frac{E_y(x + \Delta x, t) - E_y(x, t)}{\Delta x} = -\frac{\partial B_z}{\partial t}$$

$$\frac{\partial E_y(x, t)}{\partial x} = -\frac{\partial B_z(x, t)}{\partial t}$$



33-6 Ley de Faraday aplicada a un rectángulo de altura a y ancho Δx paralelo al plano xy .

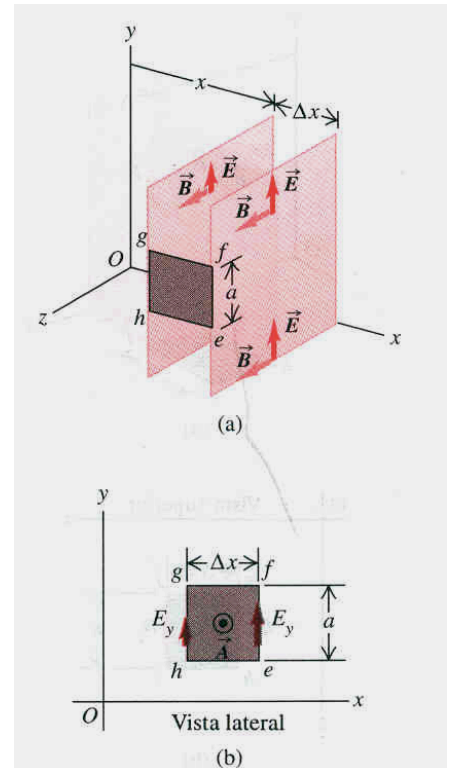
Ecuación de onda

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = -B_z(x + \Delta x, t)a + B_z(x, t)a$$

$$\frac{d\phi_E}{dt} = \frac{\partial E_y(x, t)}{\partial t} a \Delta x$$

$$a[-B_z(x + \Delta x, t) + B_z(x, t)] = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t} a \Delta x$$

$$-\frac{\partial B_z(x, t)}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t}$$



33-6 Ley de Faraday aplicada a un rectángulo de altura a y ancho Δx paralelo al plano xy .

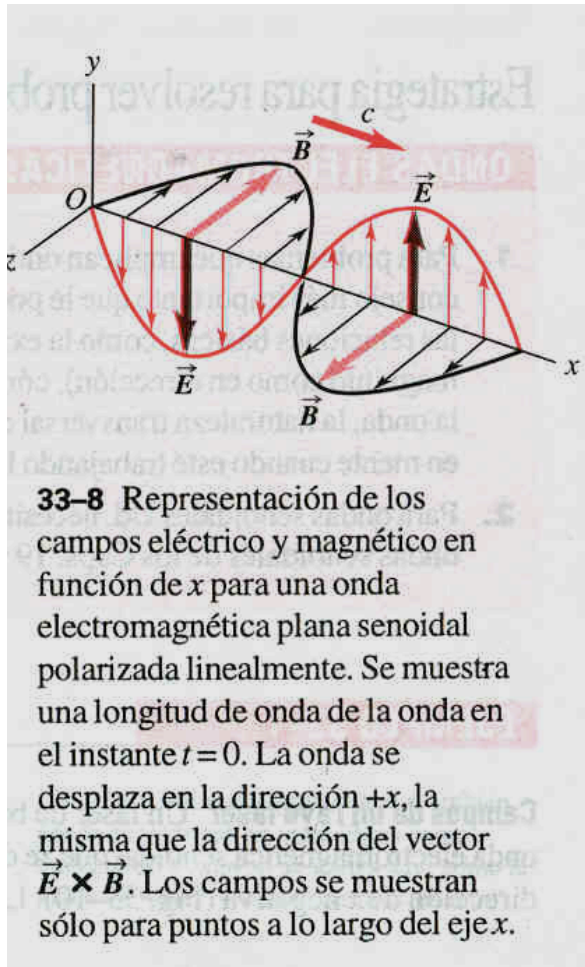
Ecuación de onda

$$\frac{\partial E_y(x,t)}{\partial x} = - \frac{\partial B_z(x,t)}{\partial t} \qquad - \frac{\partial B_z(x,t)}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = - \frac{\partial B_z(x,t)}{\partial x \partial t} \qquad - \frac{\partial^2 B_z(x,t)}{\partial x \partial t} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2} \Rightarrow \epsilon_0 \mu_0 = \frac{1}{c^2}$$

Ondas senoidales



$$E(x, t) = E_0 \sin(kx - \omega t);$$

$$\mathbf{E}(x, t) = E_0 \mathbf{u}_y \sin(kx - \omega t)$$

$$B(x, t) = B_0 \sin(kx - \omega t);$$

$$\mathbf{B}(x, t) = B_0 \mathbf{u}_z \sin(kx - \omega t)$$

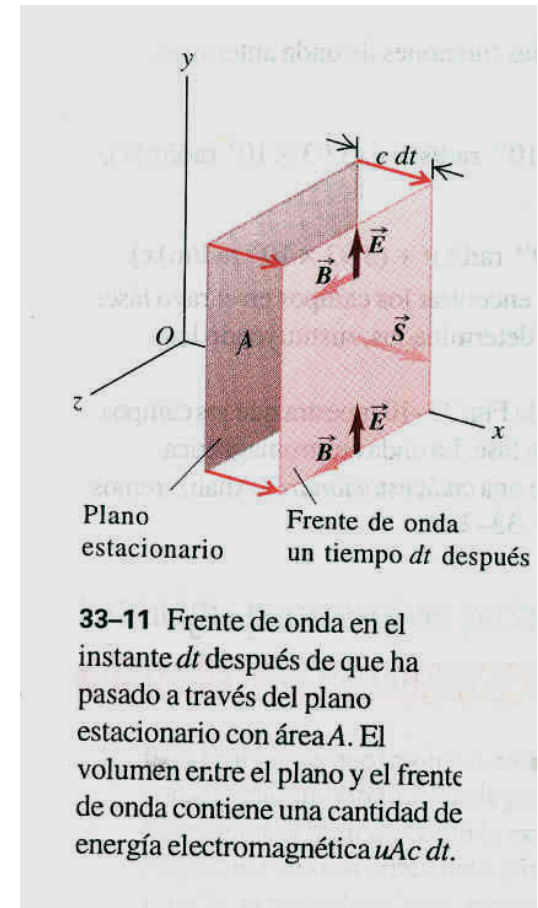
Energía

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2; B = \frac{E}{c} = \sqrt{\epsilon_0 \mu_0} E$$

$$u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} (\sqrt{\epsilon_0 \mu_0} E)^2 = \epsilon_0 E^2$$

$$dU = u dV = (\epsilon_0 E^2) (A c dt)$$

$$S = \frac{1}{A} \frac{dU}{dt} = \epsilon_0 c E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \frac{EB}{\mu_0} = \frac{cB^2}{\mu_0}$$



Vector de Poynting

Intensidad y potencia

$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$

$$P = \int_A \mathbf{S} \cdot \mathbf{n} \, dA$$

Valores instantáneos y medio

$$S(x, t) = \frac{E(x, t)B(x, t)}{\mu_0} = \frac{E_0 B_0}{\mu_0} \sin^2(kx - \omega t)$$

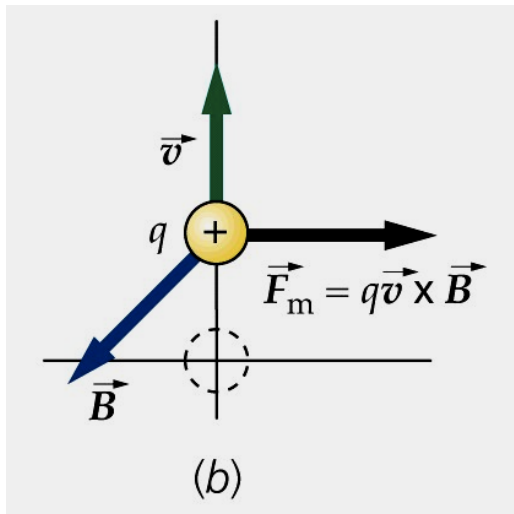
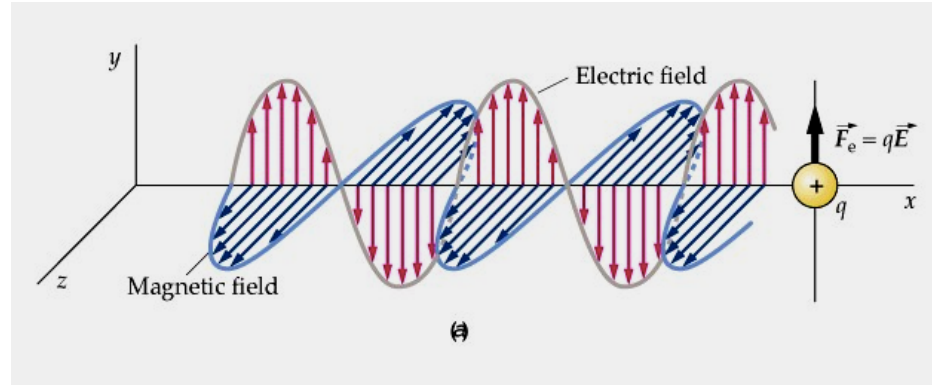
$$\langle S \rangle = I = \frac{E_0 B_0}{2\mu_0} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_0^2$$

Intensidad solar terrestre 1.34 kW/m^2

UCM

F II

Momento



$$v_y = at = \frac{qE}{m}t$$

$$E_c = \frac{1}{2}mv_y^2 = \frac{1}{2} \frac{mq^2E^2t_1^2}{m^2} = \frac{1}{2} \frac{q^2E^2}{m}t_1^2$$

$$\mathbf{F}_m = q\mathbf{v} \times \mathbf{B} = \frac{q^2EB}{m}t\mathbf{u}_x$$

$$p_x = \int_0^{t_1} F_x dt = \frac{1}{2} \frac{q^2EB}{m}t_1^2$$

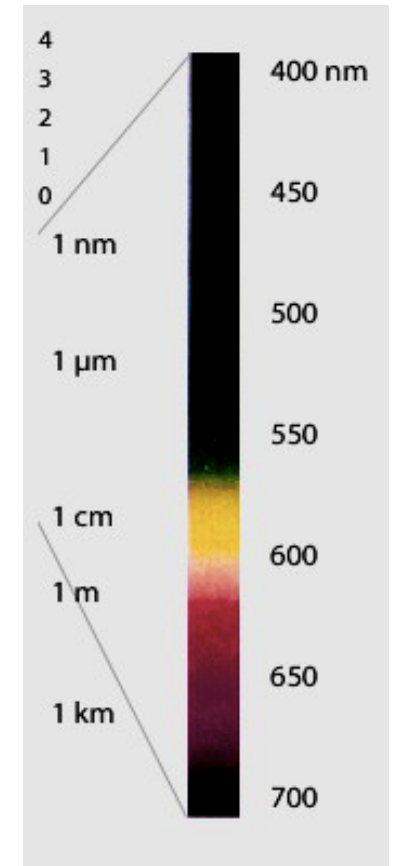
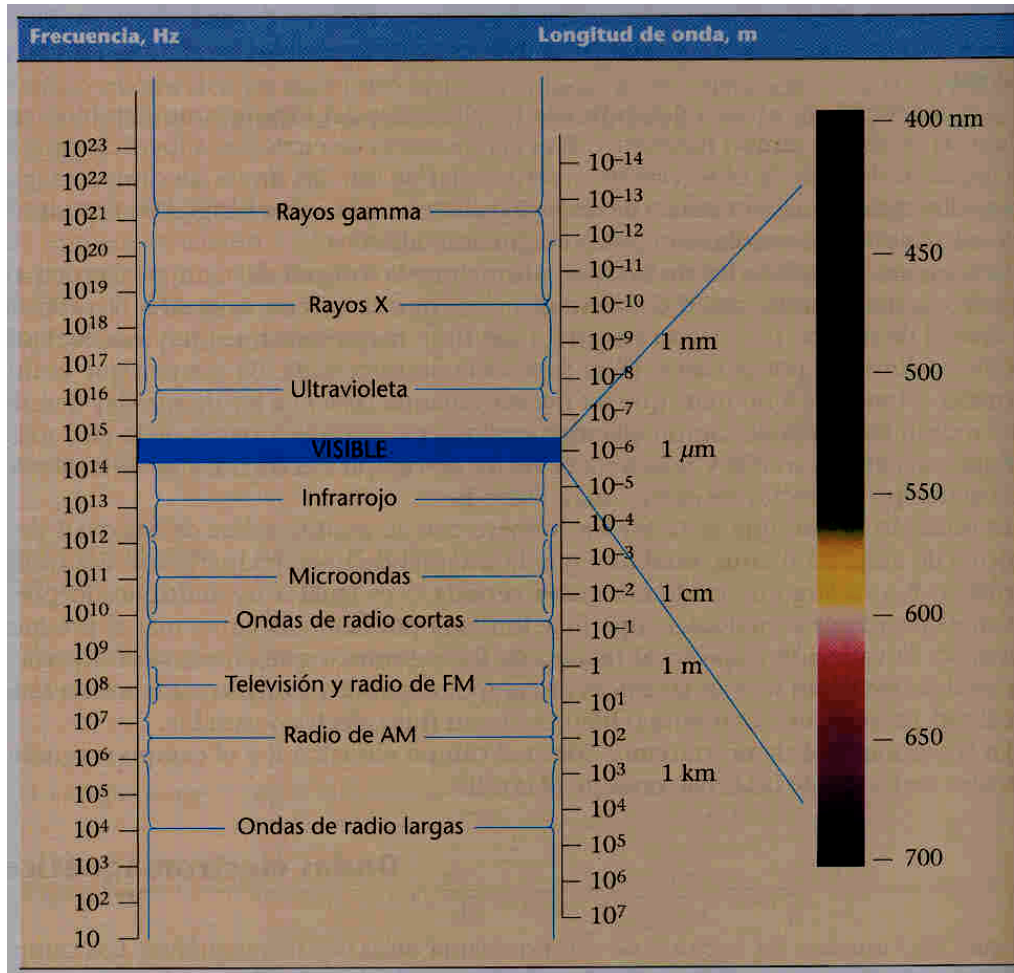
$$p = \frac{U}{c}$$

Presión de radiación

$$p_r = \frac{I}{c} = \frac{E_0^2}{2\mu_0 c^2} = \frac{B_0^2}{2\mu_0}$$

Presión solar terrestre $4.5 \times 10^6 \text{ N/m}^2$

Espectro electromagnético



Antena dipolar

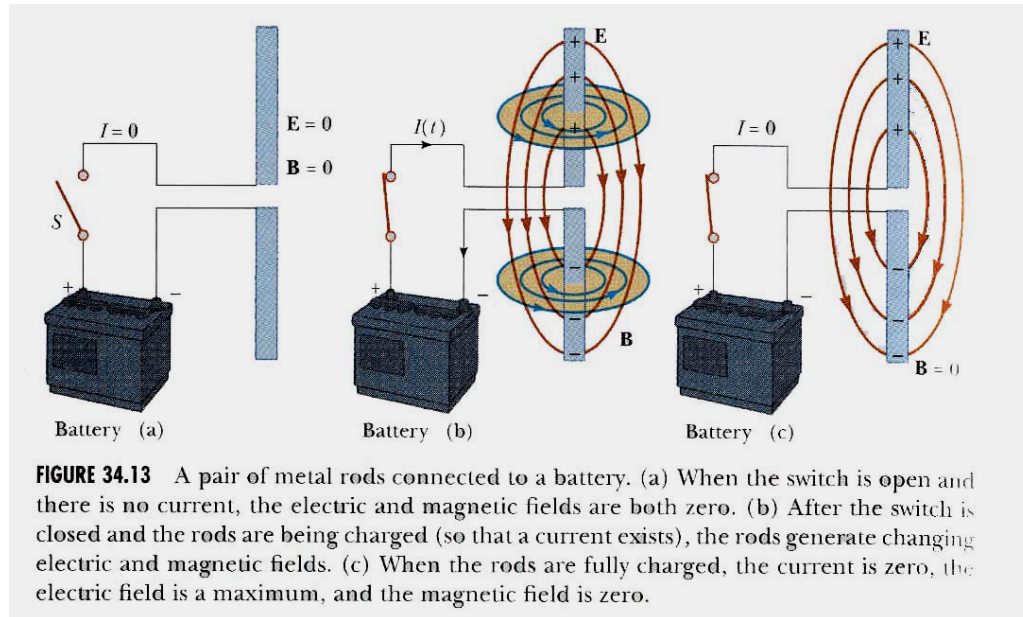


FIGURE 34.13 A pair of metal rods connected to a battery. (a) When the switch is open and there is no current, the electric and magnetic fields are both zero. (b) After the switch is closed and the rods are being charged (so that a current exists), the rods generate changing electric and magnetic fields. (c) When the rods are fully charged, the current is zero, the electric field is a maximum, and the magnetic field is zero.

$$E = \frac{pk^2}{4\pi\epsilon_0} \frac{\sin\theta}{r} \sin(kr - \omega t)$$

$$B = \frac{pk^2}{4\pi\epsilon_0 c} \frac{\sin\theta}{r} \sin(kr - \omega t)$$

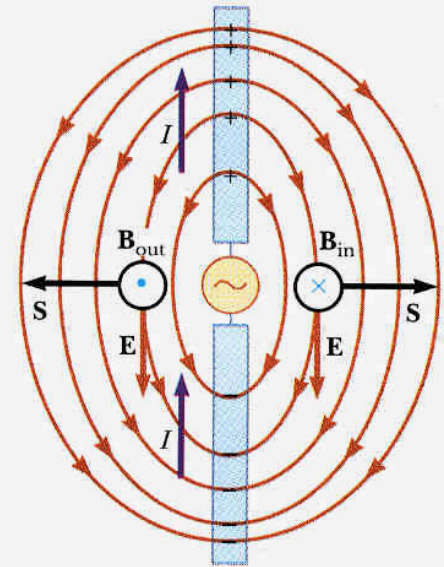
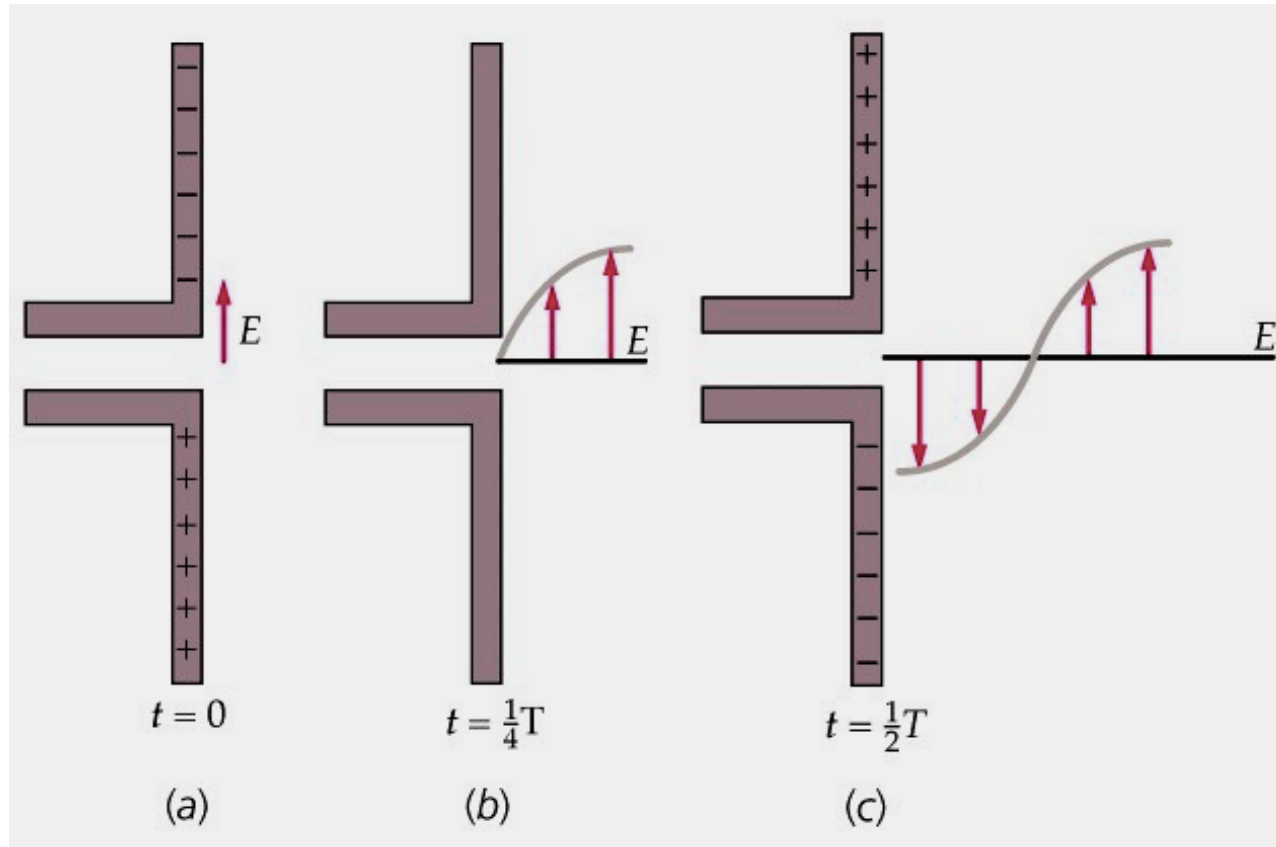


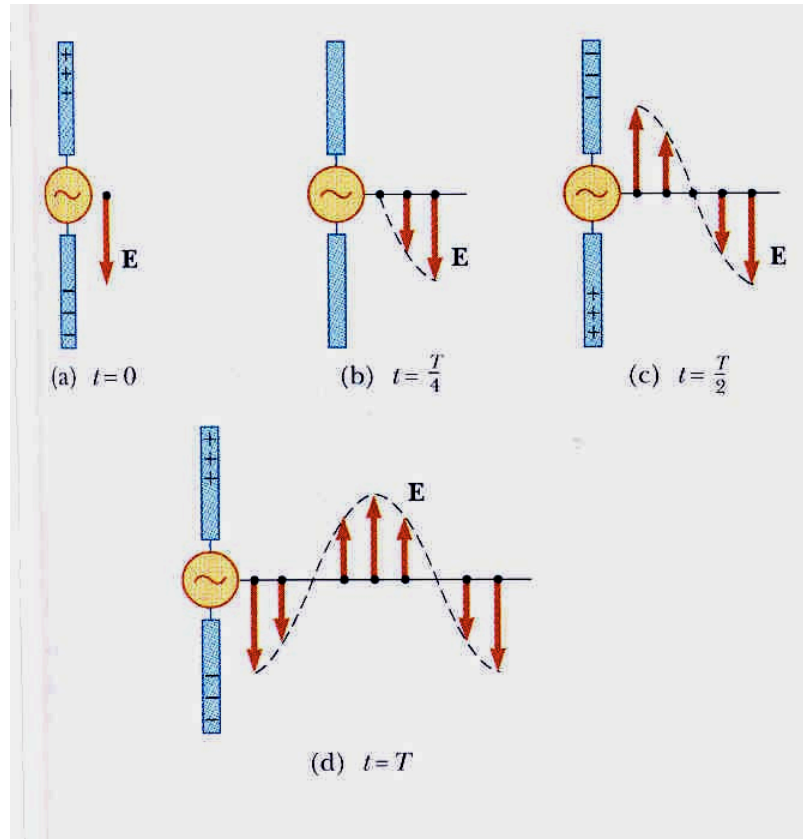
FIGURE 34.14 A half-wave (dipole) antenna consists of two metal rods connected to an alternating voltage source. The diagram shows E and B at an instant when the current is upward. Note that the electric field lines resemble those of a dipole.

Generación de ondas

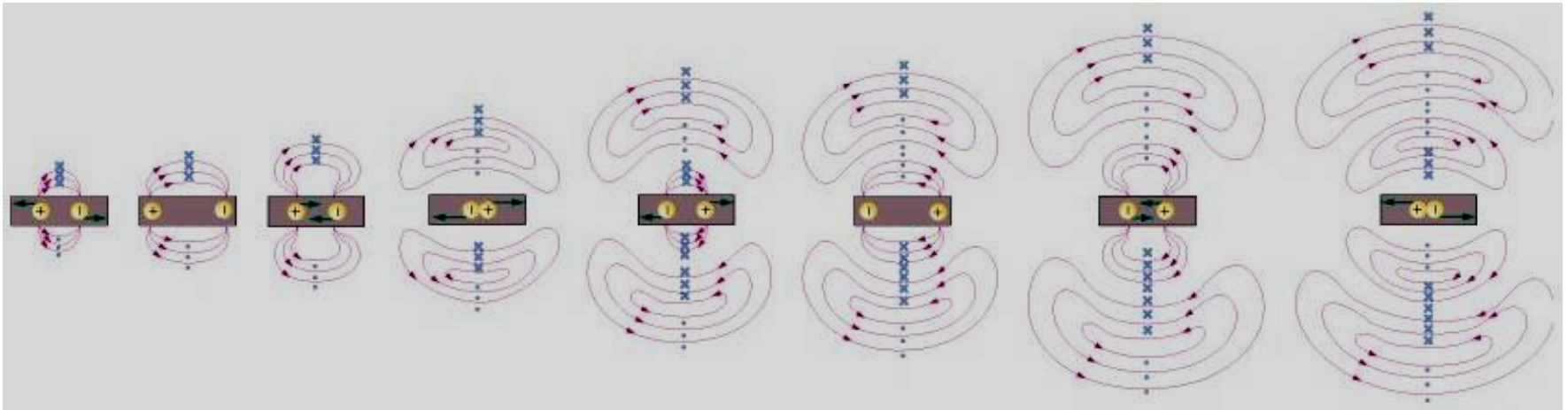


Dipolo eléctrico

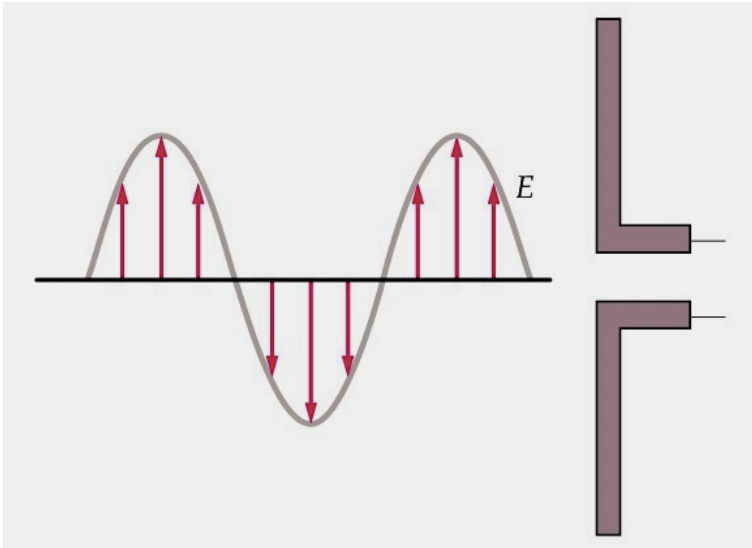
Antena dipolar eléctrica



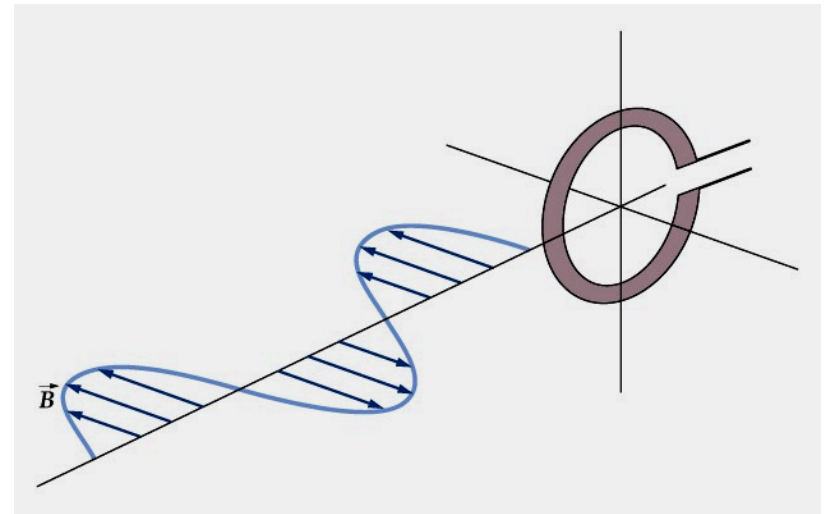
Generación de ondas



Detección de ondas



Campo eléctrico



Campo magnético

Radiación dipolar

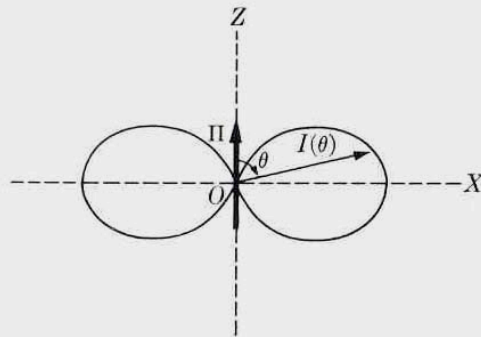


Fig. 19-10. Dependencia angular de la intensidad de la radiación electromagnética producida por un dipolo eléctrico oscilante.

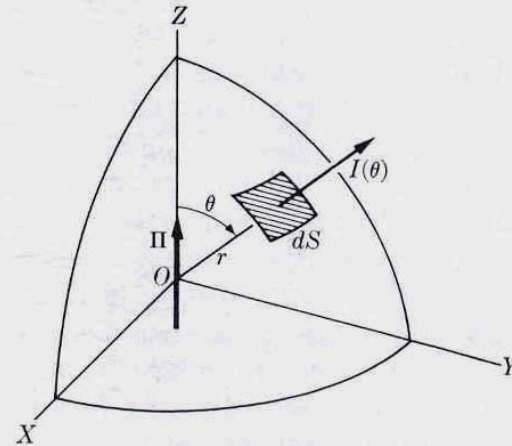
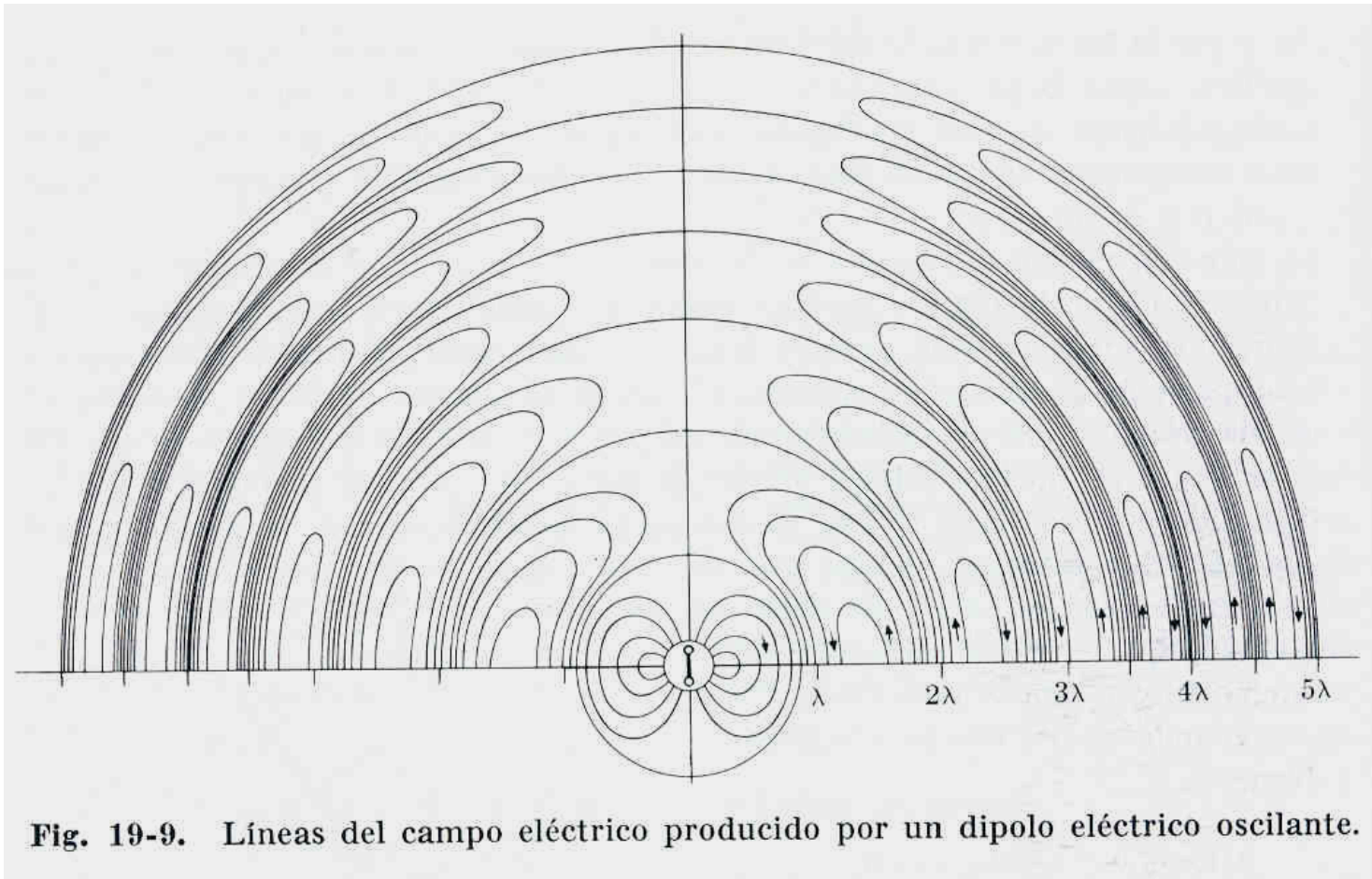


Fig. 19-11. Cálculo de la energía total que un dipolo eléctrico oscilante irradia por unidad de tiempo.

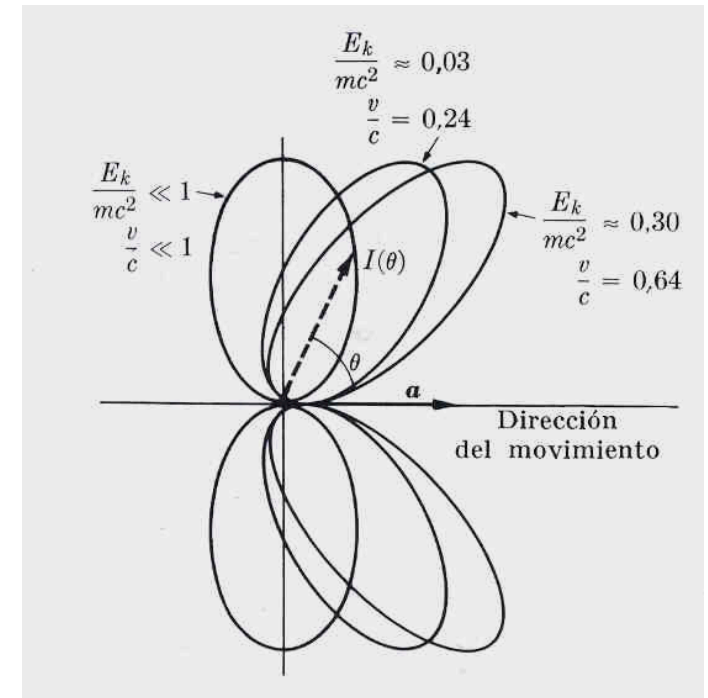
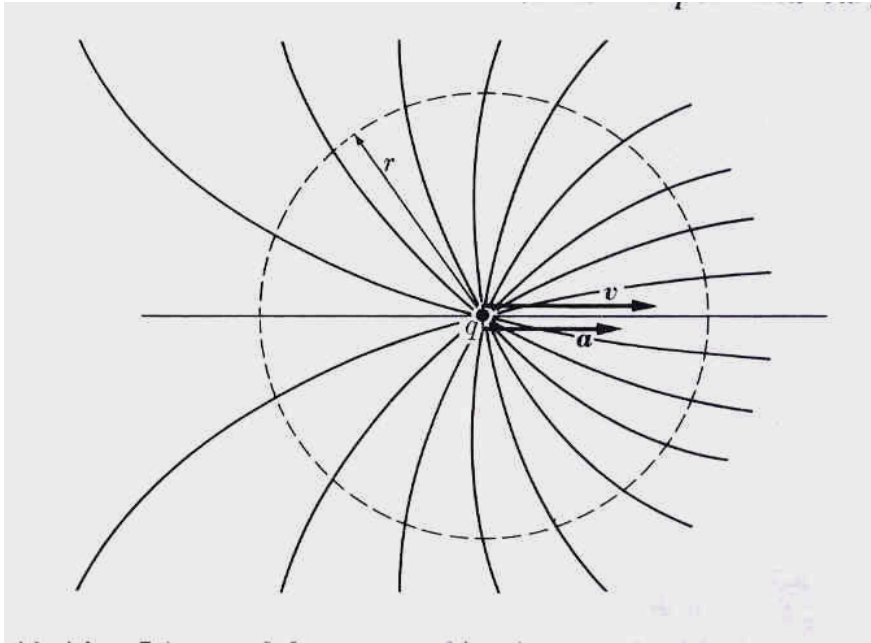
$$I(\theta) = \frac{p^2 \omega^4}{32\pi^2 c^3 \epsilon_0 r^2} \sin^2 \theta$$

$$\frac{dE}{dt} = \oint I(\theta) dS = \frac{p^2 \omega^4}{12\pi \epsilon_0 c^3}$$

Diagrama radiación dipolar



Radiación carga acelerada



$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

Ondas electromagnéticas en medios materiales

Medio material: ϵ, μ

$$E = vB; \quad B = \epsilon\mu vB; \quad v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon_r\epsilon_0}} \frac{1}{\sqrt{\mu_r\mu_0}} = \frac{c}{\sqrt{\epsilon_r\mu_r}}$$

Índice de refracción n ($\mu=\mu_0$)

$$v = \frac{1}{\sqrt{\epsilon_r}} \frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{\epsilon_r}}; \quad n = \frac{c}{v} \cong \sqrt{\epsilon_r}$$

Energía

$$u = \frac{1}{2}\epsilon E^2 + \frac{1}{2\mu} B^2 = \epsilon E^2; \quad \mathbf{S} = \frac{1}{\mu} \mathbf{E} \times \mathbf{B}; \quad S = \sqrt{\frac{\epsilon}{\mu}} E^2$$

$$I = \frac{E_0 B_0}{2\mu} = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} E_0^2 = \frac{1}{2} \epsilon v E_0^2$$

La luz

