Inducción electromagnética

F II

Ley de Faraday. Campo eléctrico inducido

Fuerza electromotriz de movimiento

Autoinducción e inducción mútua

Energía magnética.

Circuitos L-R, L-C y L-C-R

Corriente de desplazamiento

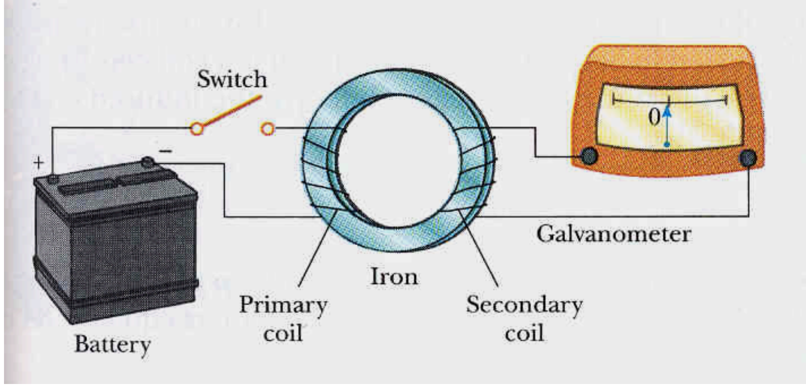
Ecuaciones de Maxwell

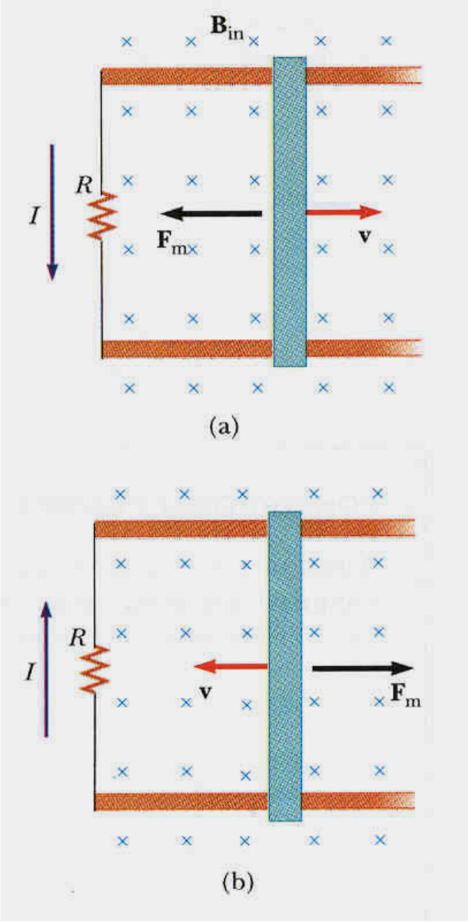
Corriente alterna

Inducción electromagnética

Experimentos de inducción

F II





Ley de Faraday

F II

Circuito de área A, rígido o deformable

Campo B, variable con el tiempo o uniforme

Flujo magnético $$\Phi_B = \Phi_B(t) = \int_A \mathbf{B} \cdot \mathbf{n}\, da$$

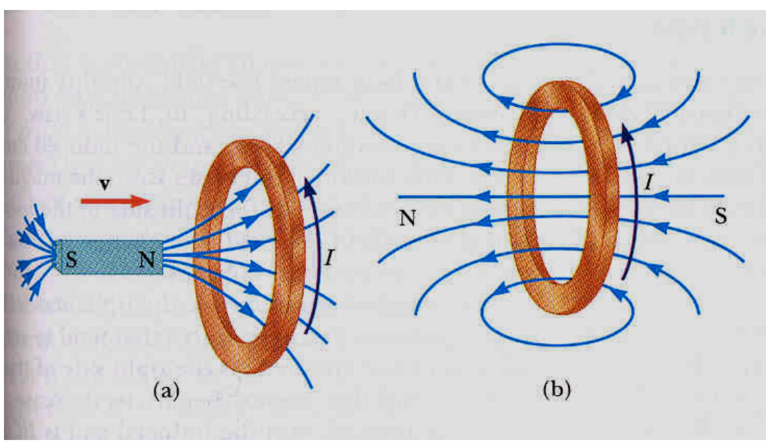
F.e.m. inducida $$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

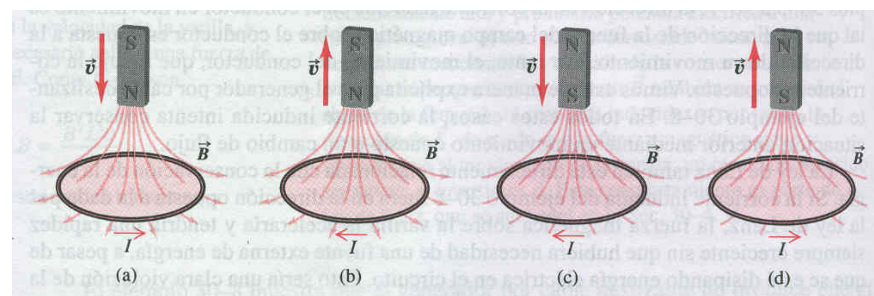
Ley de Lenz

F II

Significado del signo en ley de Faraday

Oposición a la variación del flujo



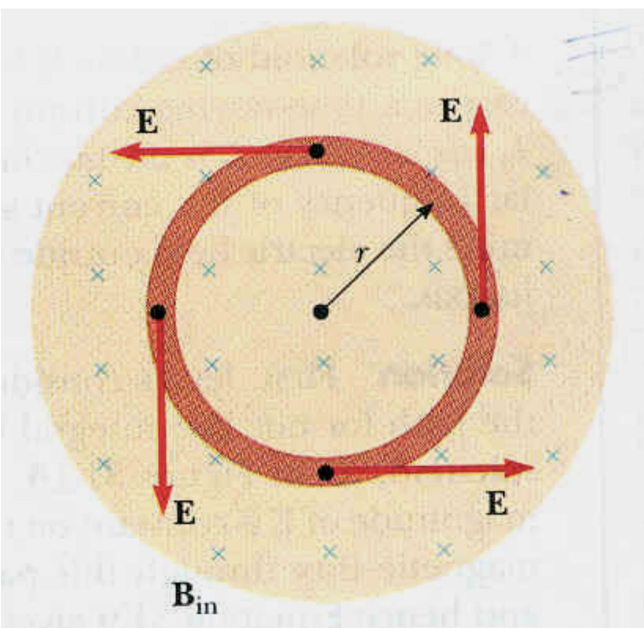


Campo eléctrico inducido

Campo magnético variable con el tiempo

F II

Campo eléctrico inducido sin circuito



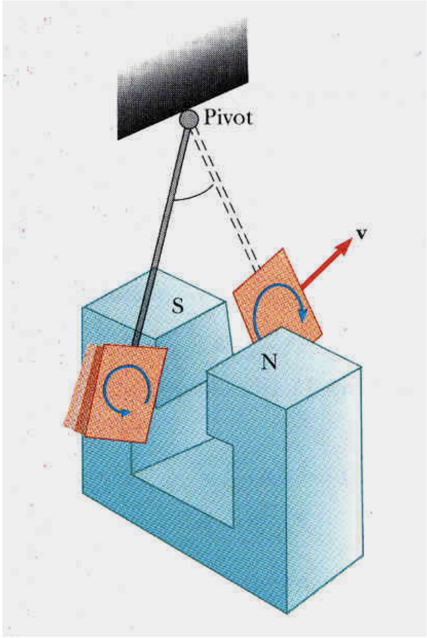
$$\mathcal{E} = \int_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_A \mathbf{B}\cdot\mathbf{n}\,da$$

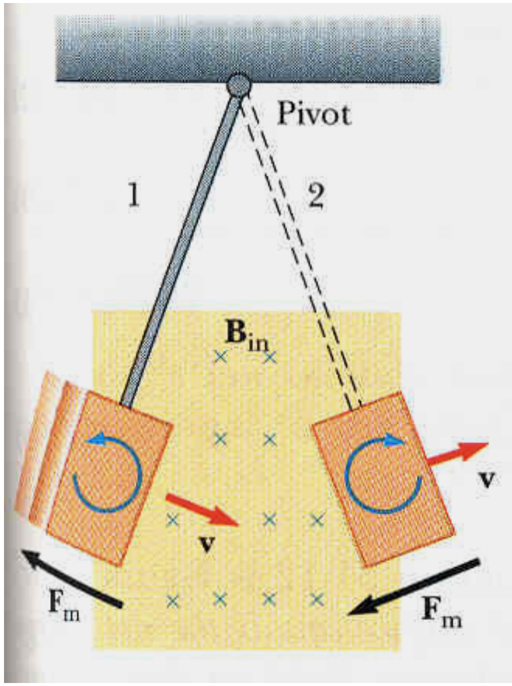
$$2\pi r E = -\frac{d}{dt}\left(B(t)\pi r^2\right); \quad E = -\frac{r}{2}\frac{dB}{dt}$$

Corrientes parásitas

F II **Campo magnético variable con el tiempo**

Corriente inducida en conductores

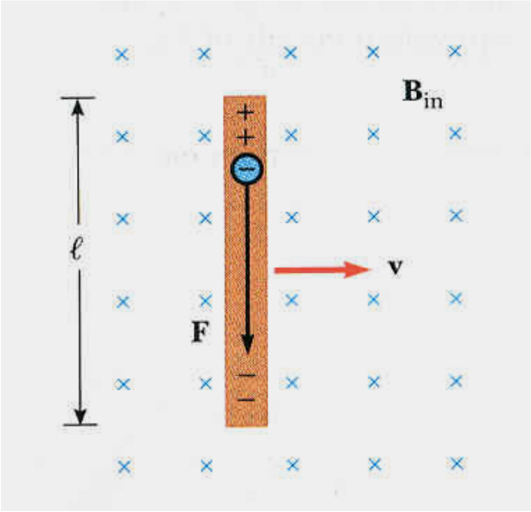




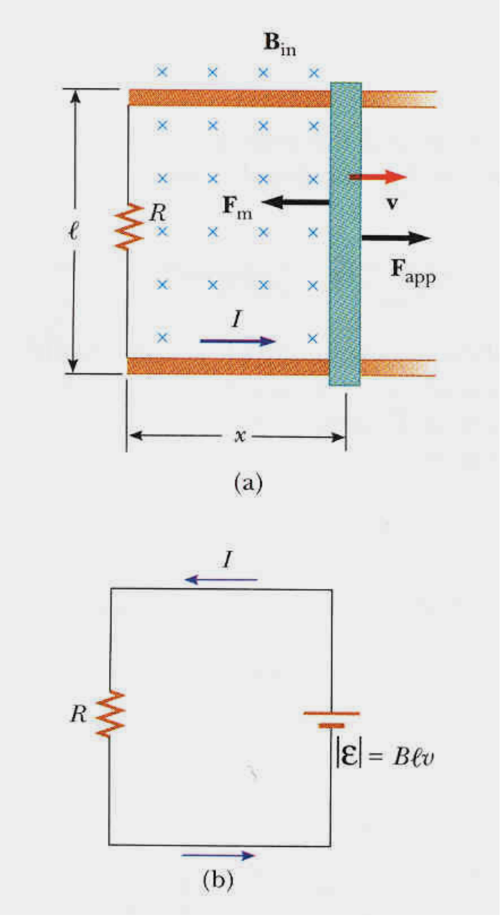
Fuerza electromotriz de movimiento

F II

$$\mathcal{E} = -\frac{d}{dt}\left(BLx(t)\right) = BLv$$

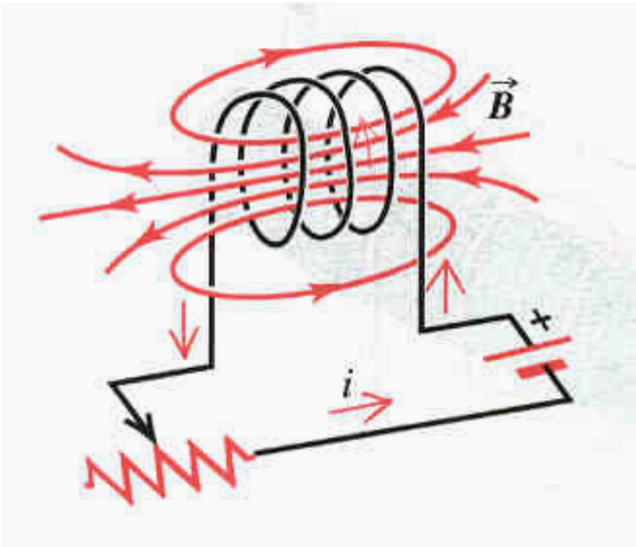


$$E = vB;\ V_{ab} = EL = vBL$$



Autoinducción

F II

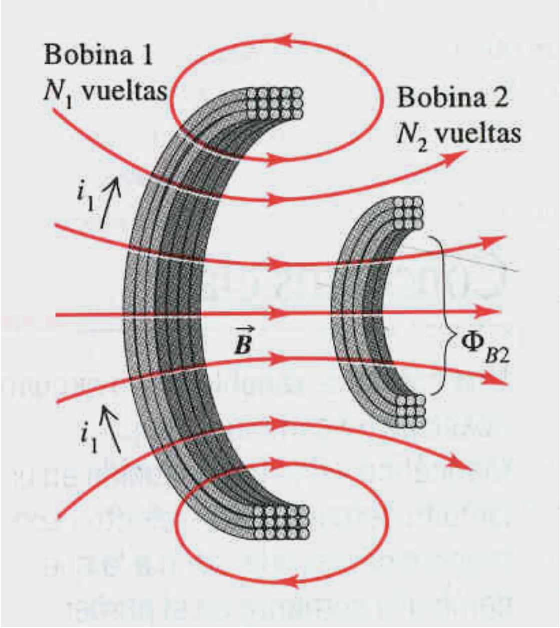


$$L = \frac{N\Phi_B}{i};\ N\frac{d\Phi_B}{dt} = L\frac{di}{dt};\ \mathcal{E} = -L\frac{di}{dt}$$

Inducción mutua

F II

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{B2}}{dt};\ N_2\Phi_{B2} = M_{21} i_1;\ \mathcal{E}_2 = -M_{21}\frac{di_1}{dt}$$



$$\mathcal{E}_1 = -N_1 \frac{d\Phi_{B1}}{dt};\ N_1\Phi_{B1} = M_{12} i_2;\ \mathcal{E}_1 = -M_{12}\frac{di_2}{dt}$$

$$M_{12} = M_{21} = M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}$$

Cálculo inductancias

F II

$$B = \mu_0 \frac{N}{l} I,\ \Phi = \mu_0 \frac{N^2 A}{l} I;\ L = \mu_0 \frac{N^2 A}{l}$$

$$2\pi R B_1 = \mu_0 N_1 I_1;\ 2\pi R B_2 = \mu_0 N_2 I_2;$$

$$B_1 = \frac{\mu_0 N_1 I_1}{2\pi R};\ B_2 = \frac{\mu_0 N_2 I_2}{2\pi R};$$

$$\Phi_{1,2} = \frac{\mu_0 N_1 N_2 A I_2}{2\pi R};\ \Phi_{2,1} = \frac{\mu_0 N_2 N_1 A I_1}{2\pi R};$$

$$M_{1,2} = \frac{\Phi_{1,2}}{I_2} = M_{2,1} = \frac{\Phi_{2,1}}{I_1} = M = \frac{\mu_0 N_1 N_2 A}{2\pi R}$$

Energía magnética

F II

$$P(t) = -\mathcal{E} i(t) = Li\frac{di}{dt};\ U = \int_0^t P dt = \int_0^I Li di = \frac{1}{2}LI^2$$

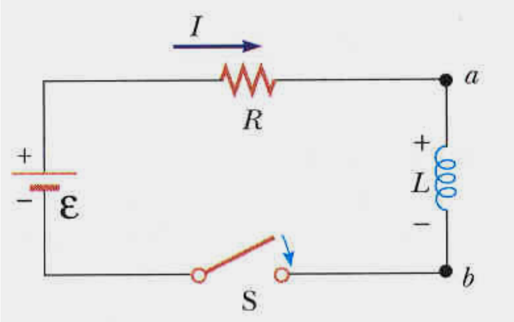
Energía localizada en la corriente

$$U = \frac{1}{2}\mu_0\frac{N^2 A}{l}I^2 = \frac{1}{2}\left(\mu_0\frac{N}{l}I\right)^2\left(\frac{lA}{\mu_0}\right) = \frac{1}{2}\frac{B^2}{\mu_0}lA$$

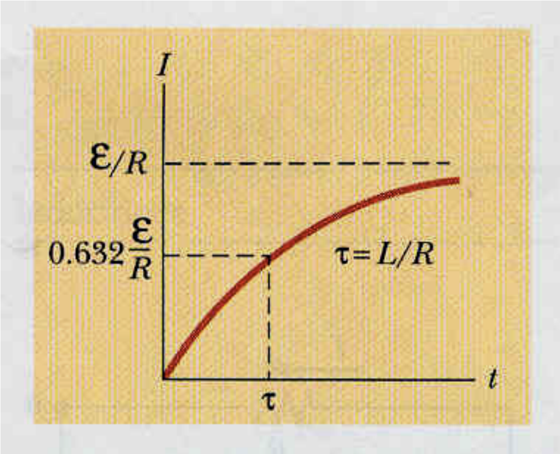
Energía localizada en el campo

Circuito R-L

F II



$$\mathcal{E} - L\frac{di}{dt} = Ri;\ i = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{R}{L}t}\right);\ \tau = \frac{L}{R}$$



$$Ri + L\frac{di}{dt} = 0;\ i = \frac{\mathcal{E}}{R}\left(e^{-\frac{R}{L}t}\right) = I_0 e^{-\frac{t}{\tau}};\ \tau = \frac{L}{R}$$

Circuito L-C

F II

$$-L\frac{di}{dt}-\frac{q}{C}=0,\;\; \frac{d^2q}{dt^2}+\frac{1}{\mathrm{LC}}q=0$$

$$q=Q\cos(\omega t+\varphi);\;\; \omega=\sqrt{\frac{1}{LC}}$$

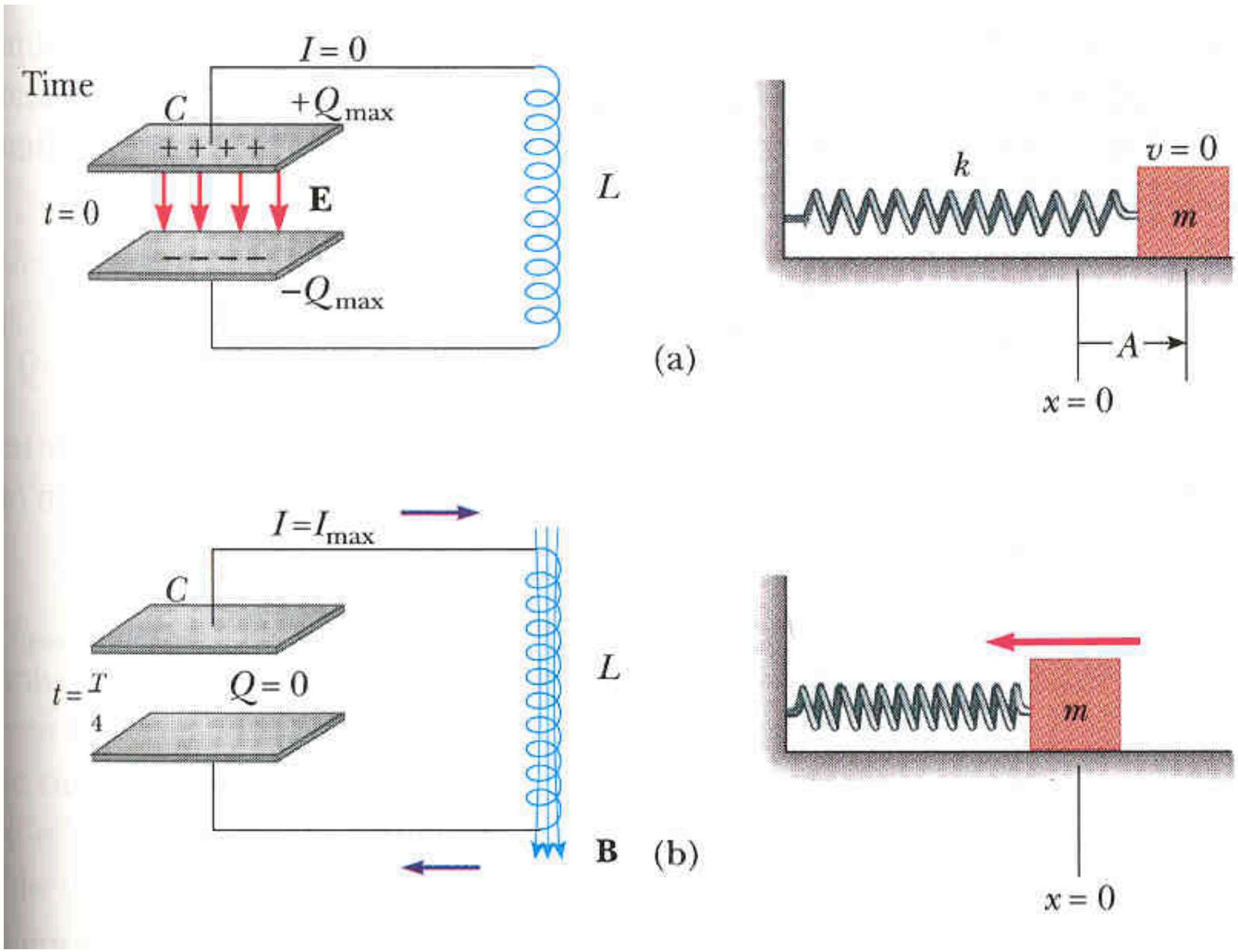
$$i=-\omega Q sen(\omega t+\varphi)$$

$$\frac{1}{2}Li^2+\frac{1}{2}\frac{q^2}{C}=\frac{1}{2}\frac{Q^2}{C}$$

$$i=\pm\sqrt{\frac{1}{LC}}\sqrt{Q^2-q^2}$$

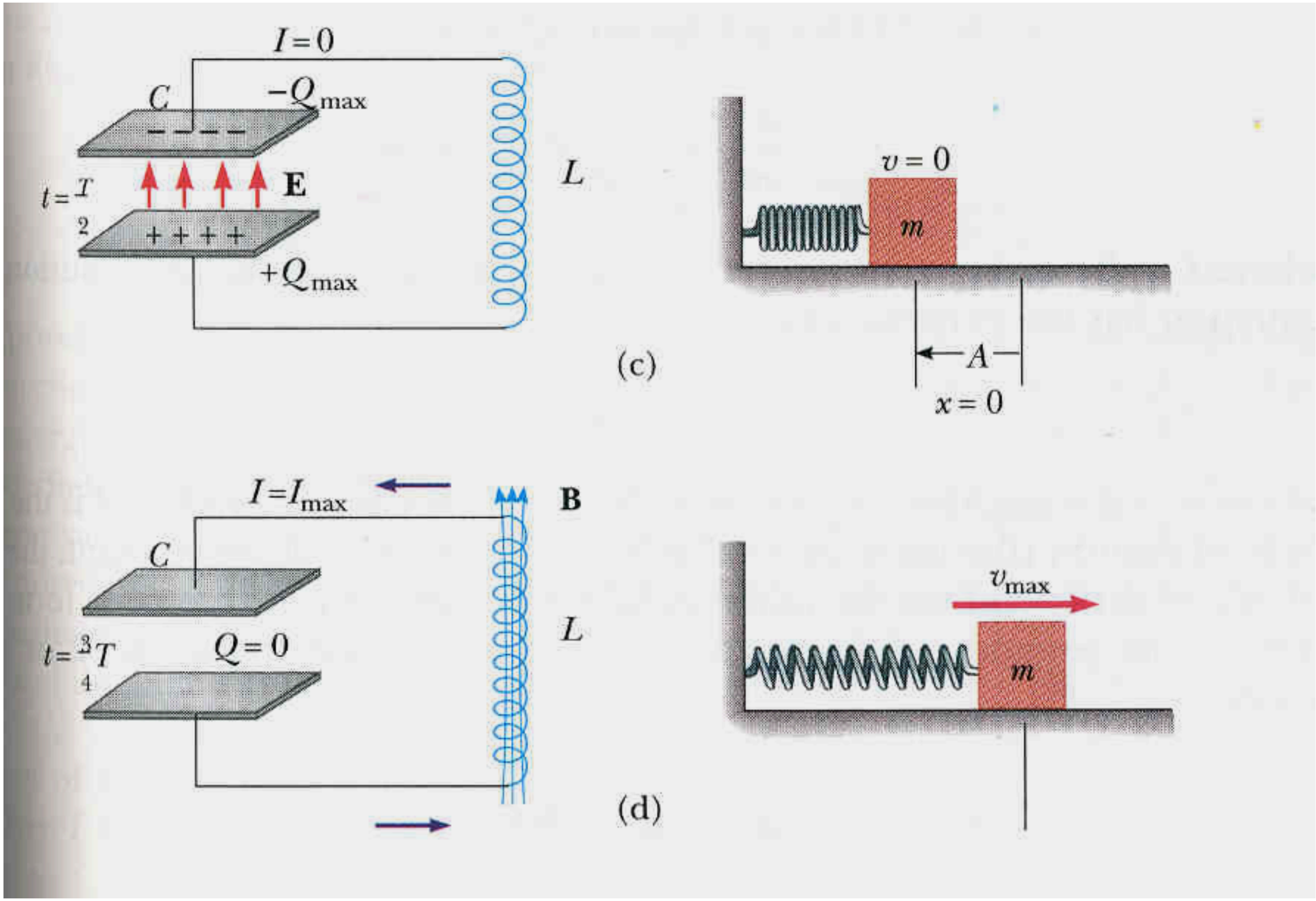
Circuito L-C

F II



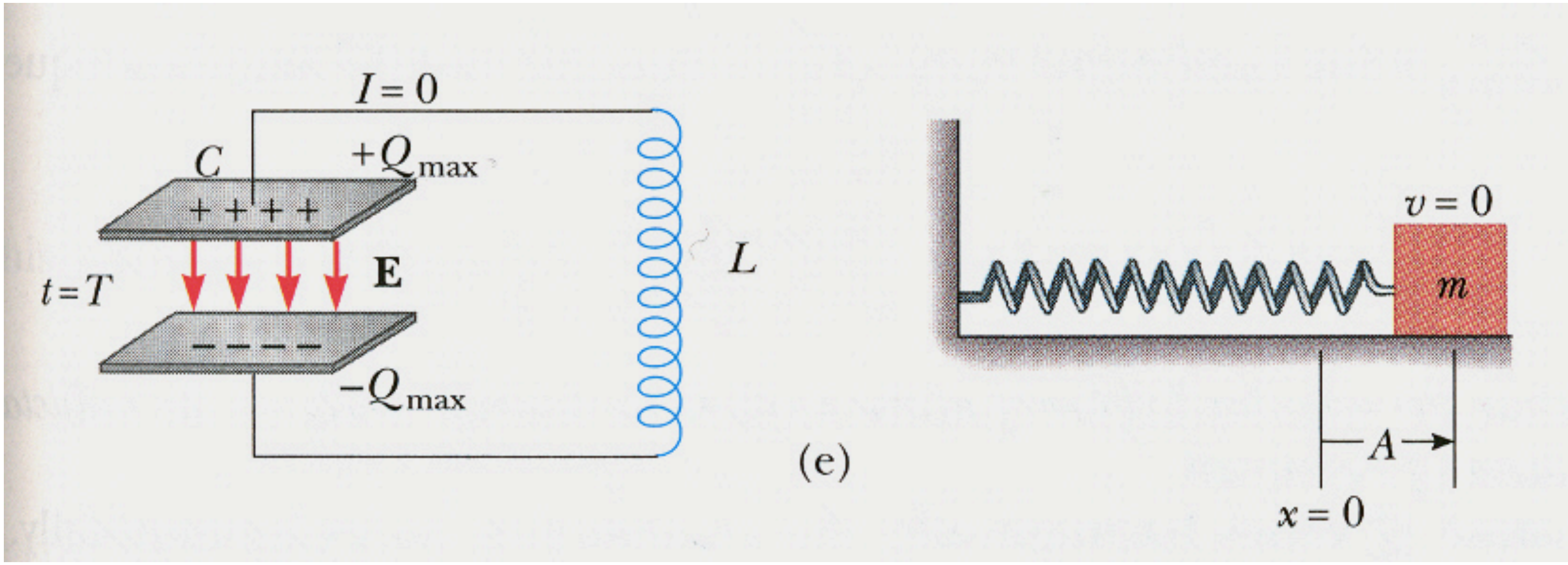
Circuito L-C

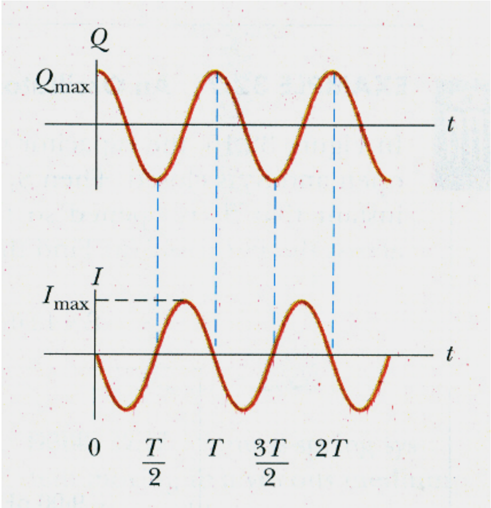
F II

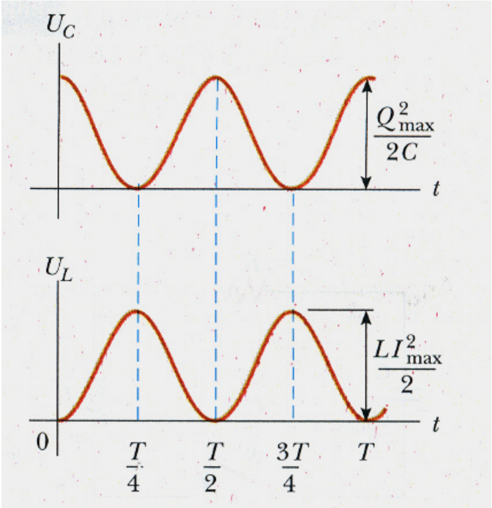


Circuito L-C

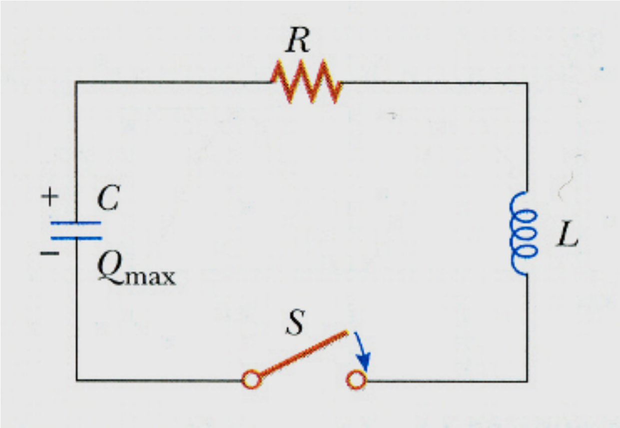
F II

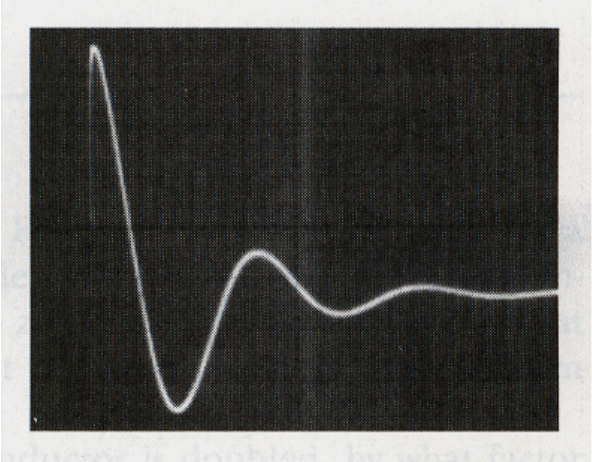






Circuito L-C-R





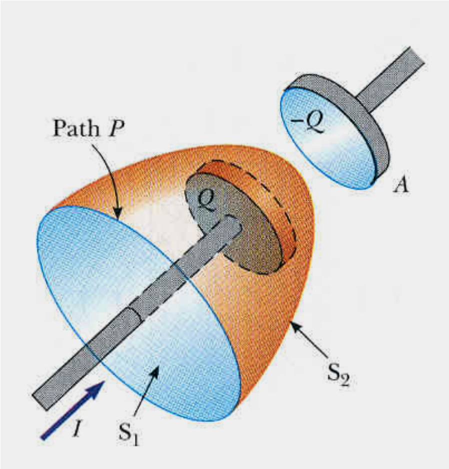
$$-iR - L\frac{di}{dt} - \frac{q}{C} = 0, \quad \frac{d^2q}{dt^2} + \frac{\mathrm{R}}{\mathrm{L}}\frac{dq}{dt} + \frac{1}{\mathrm{LC}}q = 0$$

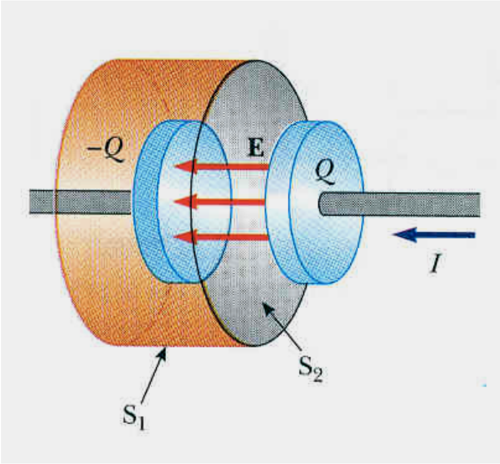
$$q = Ae^{-\left(\frac{R}{2L}\right)t}\cos(\omega' t + \varphi); \quad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Corriente de desplazamiento

Carga condensador / ley Ampère

F II





$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}; \quad \mathbf{j}_D = \varepsilon_0 \frac{d\mathbf{E}}{dt} = \frac{d\mathbf{D}}{dt} \quad \oint_C B \cdot dl = \mu_0 \left(I + I_D \right) = \mu_0 \oint_C \left(\mathbf{j} + \frac{d\mathbf{D}}{dt} \right) \cdot \mathbf{n} da$$

Ecuaciones de Maxwell I

F II **En el vacío**

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\oint_A \mathbf{B}\cdot\mathbf{n}da \qquad \oint_S \mathbf{E}\cdot\mathbf{n}da = \frac{q}{\varepsilon_0}$$

$$\oint_S \mathbf{B}\cdot\mathbf{n}\,da = 0 \qquad \oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0\left(I + I_D\right) = \mu_0\oint_C\left(\mathbf{j} + \varepsilon_0\frac{d\mathbf{E}}{dt}\right)\cdot\mathbf{n}da$$

Ecuaciones de Maxwell II

En medios materiales

F II

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\oint_A \mathbf{B}\cdot\mathbf{n}da$$

$$\oint_S \mathbf{D}\cdot\mathbf{n}da = q$$

$$\oint_S \mathbf{B}\cdot\mathbf{n}\, da = 0$$

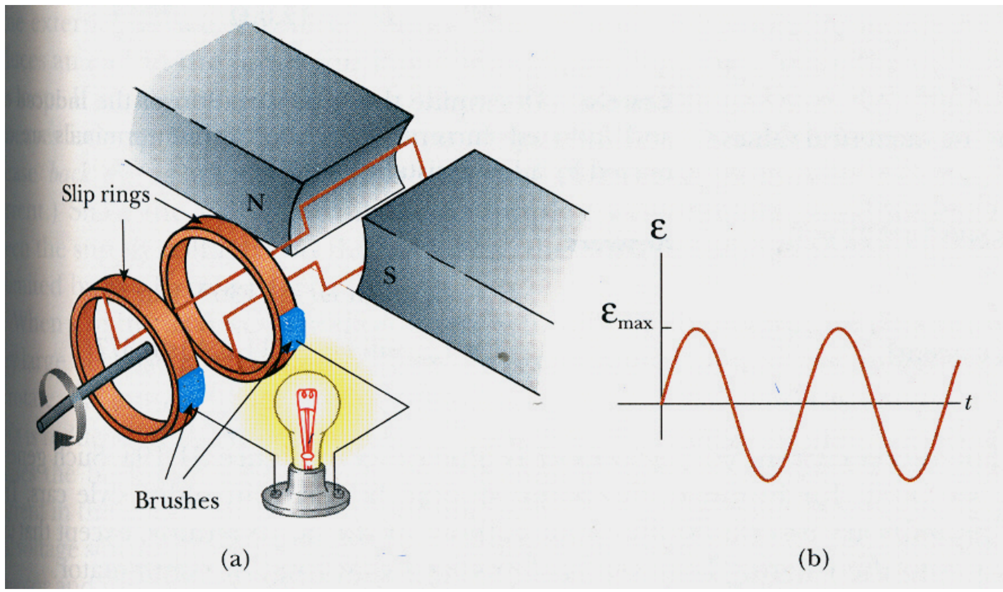
$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \oint_C\left(\mathbf{j}+\frac{d\mathbf{D}}{dt}\right)\cdot\mathbf{n}da$$

$$\mathbf{j} = \sigma\mathbf{E};\ \ \mathbf{D} = \varepsilon\mathbf{E};\ \ \mathbf{B} = \mu\mathbf{H}$$

Generación de corriente alterna

F II

Campo uniforme+bobina giratoria

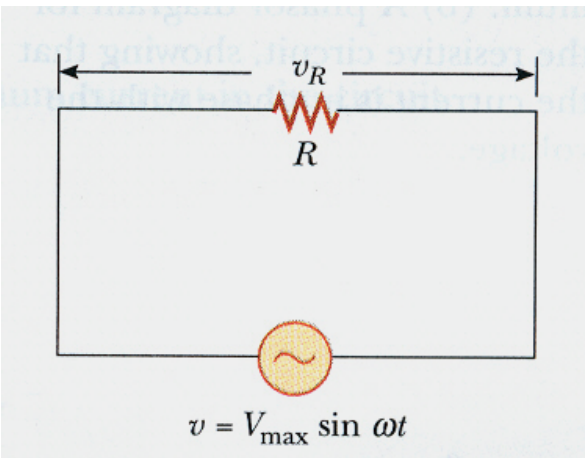


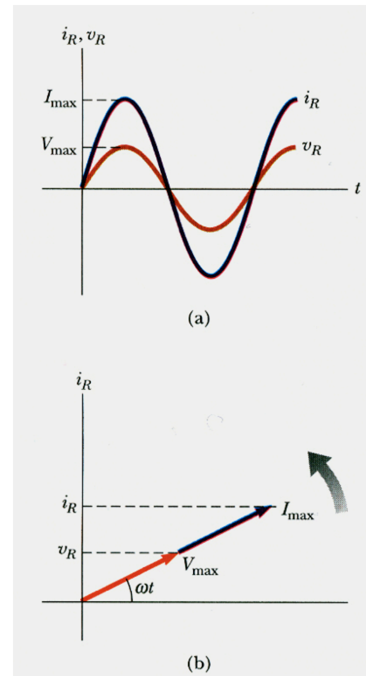
$$\Phi_B = BA\cos\varphi = BA\cos\omega t; \ \mathcal{E} = -\frac{d\Phi_B}{dt} = \omega BA sen\,\omega t$$

Corriente alterna I

F II

Resistencia pura R





$$v = v_R = V_0 sen\, \omega t = R i_R;$$

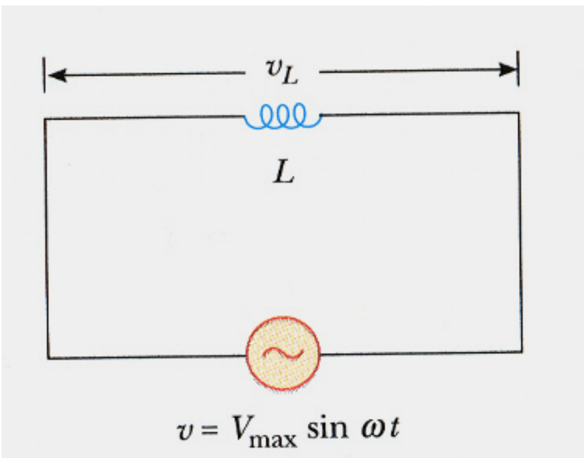
$$i_R = \frac{v}{R} = \frac{V_0}{R} sen\, \omega t = \mathrm{I}_0 sen\, \omega t; I_0 = \frac{V_0}{R}$$

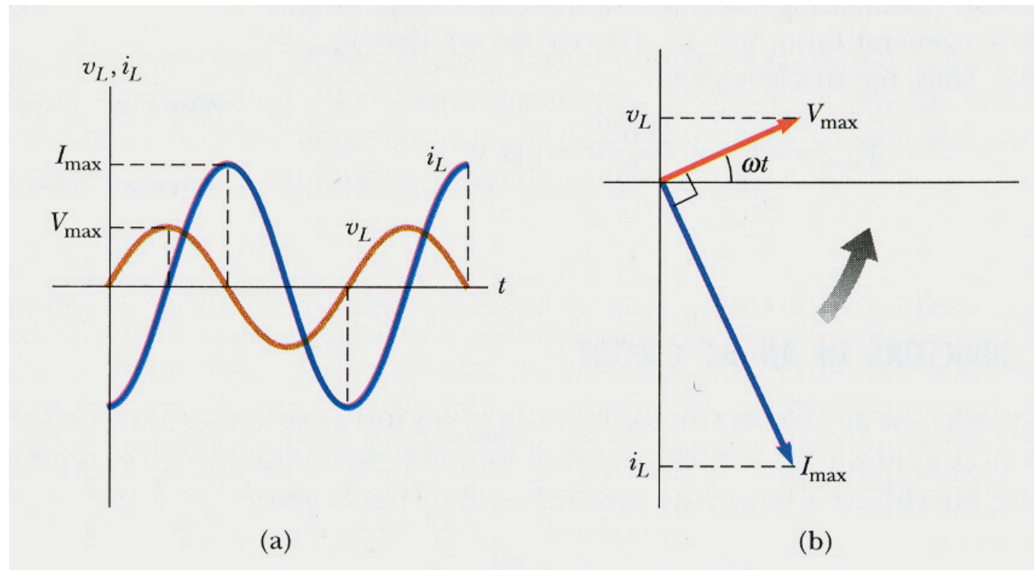
$$v_R = I_0 R\, sen\, \omega t$$

Corriente alterna II

Bobina ideal de autoinducción L

F II



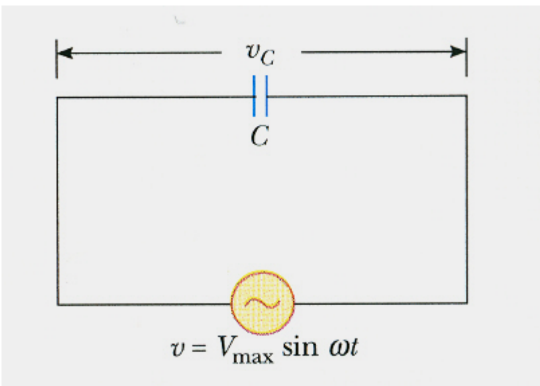


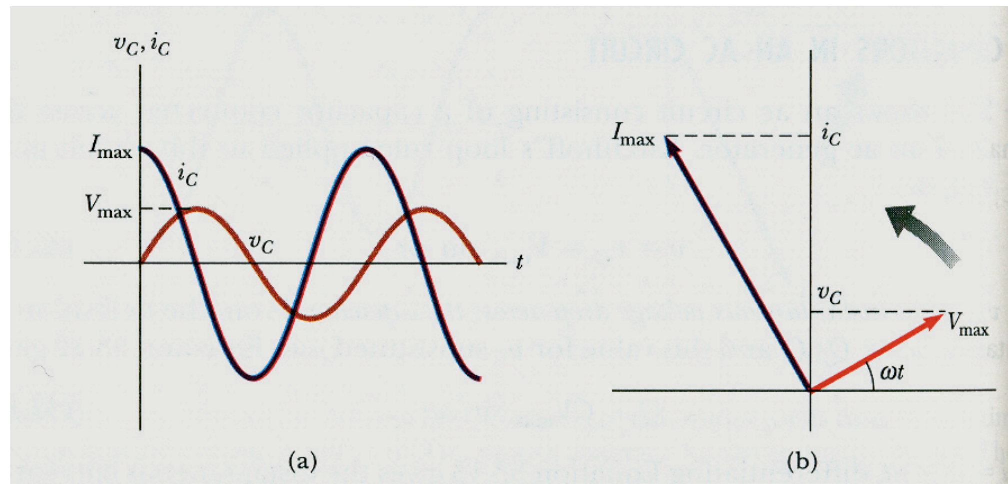
$$v - L\frac{di}{dt} = 0;\ L\frac{di}{dt} = \mathrm{V}_0 \operatorname{sen} \omega t;\ \mathrm{i} = \frac{\mathrm{V}_0}{\omega \mathrm{L}} sen\left(\omega t - \frac{\pi}{2}\right); I_0 = \frac{V_0}{\omega L} = \frac{V_0}{X_L}$$

Corriente alterna III

F II

Condensador ideal de capacidad C





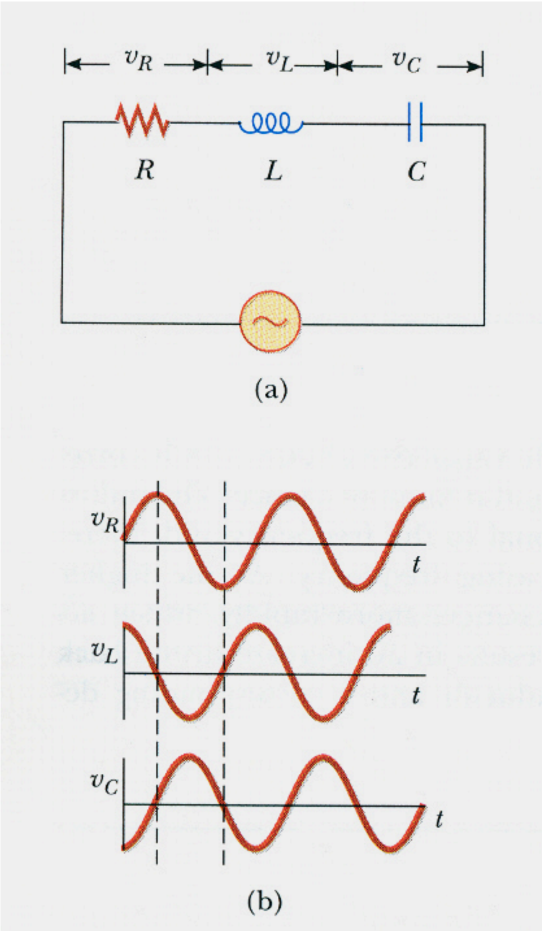
$$v = v_C = V_0 sen\omega t;\; Q = CV_0 sen\omega t; i_C = \frac{dQ}{dt} = \omega C V_0 \cos\omega t$$

$$i_C = \omega C V_0 sen\left(\omega t + \frac{\pi}{2}\right); I_0 = \omega C V_0 = \frac{V_0}{X_C}; X_C = \frac{1}{\omega C}$$

Corriente alterna IV

Circuito R-L-C

F II



$$v = V_0 sen\omega t;\ i = I_0 sen(\omega t - \phi)$$

$$v_R = I_0 R sen\omega t = V_R sen\omega t$$

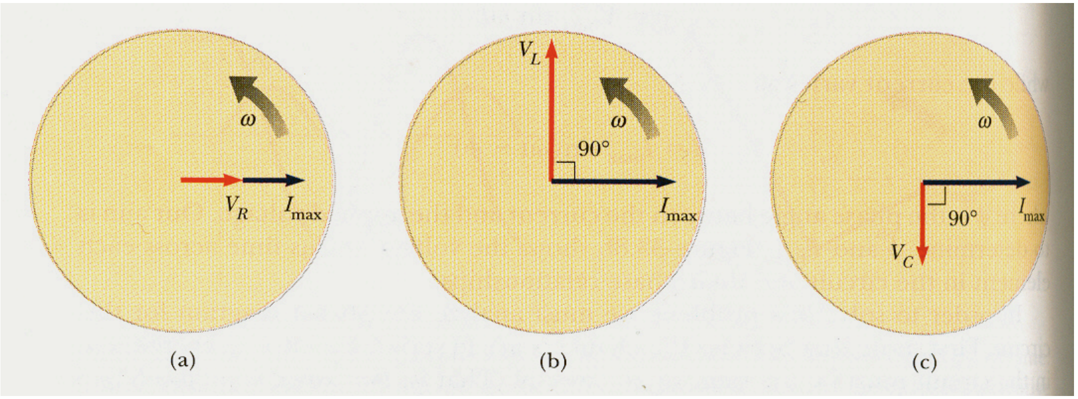
$$v_L = I_0 X_L sen\left(\omega t + \frac{\pi}{2}\right) = V_L \cos\omega t$$

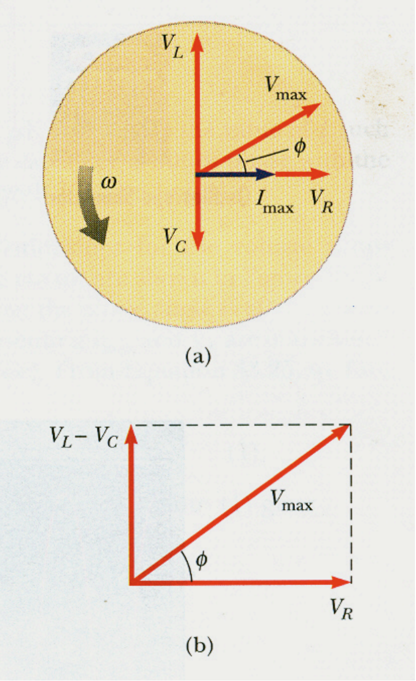
$$v_C = I_0 X_C sen\left(\omega t - \frac{\pi}{2}\right) = -V_C \cos\omega t$$

Corriente alterna V

Circuito R-L-C

F II





$$v = v_R + \mathrm{v_L} + \mathrm{v_C}\, ; V_0 = \sqrt{{V_R}^2 + \left(V_L - V_C\right)^2} = \mathrm{I}_0\sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}\, ;\; tg\phi = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}\, ;\; V_0 = ZI_0$$

Corriente alterna VI

Circuito R-L-C

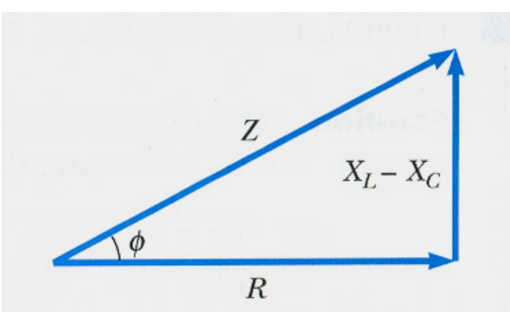
F II

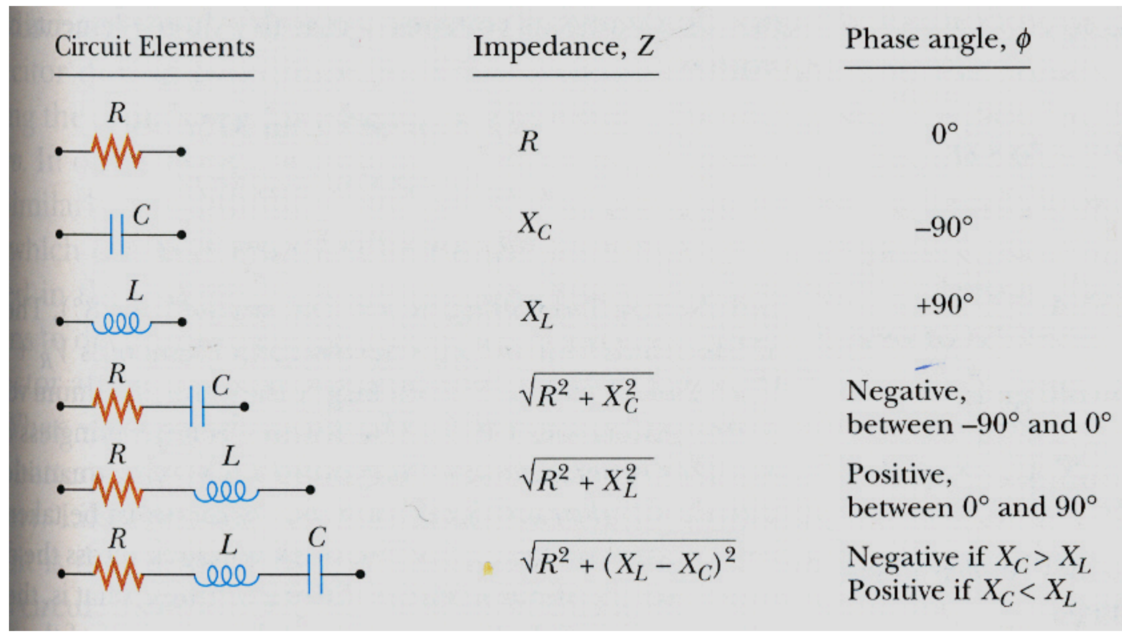
$$v = V_0 sen\omega t; \; i = I_0 sen(\omega t - \phi)$$

$$v_R = I_0 R sen\omega t = V_R sen\omega t$$

$$v_L = I_0 X_L sen\left(\omega t + \frac{\pi}{2}\right) = V_L \cos\omega t$$

$$v_C = I_0 X_C sen\left(\omega t - \frac{\pi}{2}\right) = -V_C \cos\omega t$$





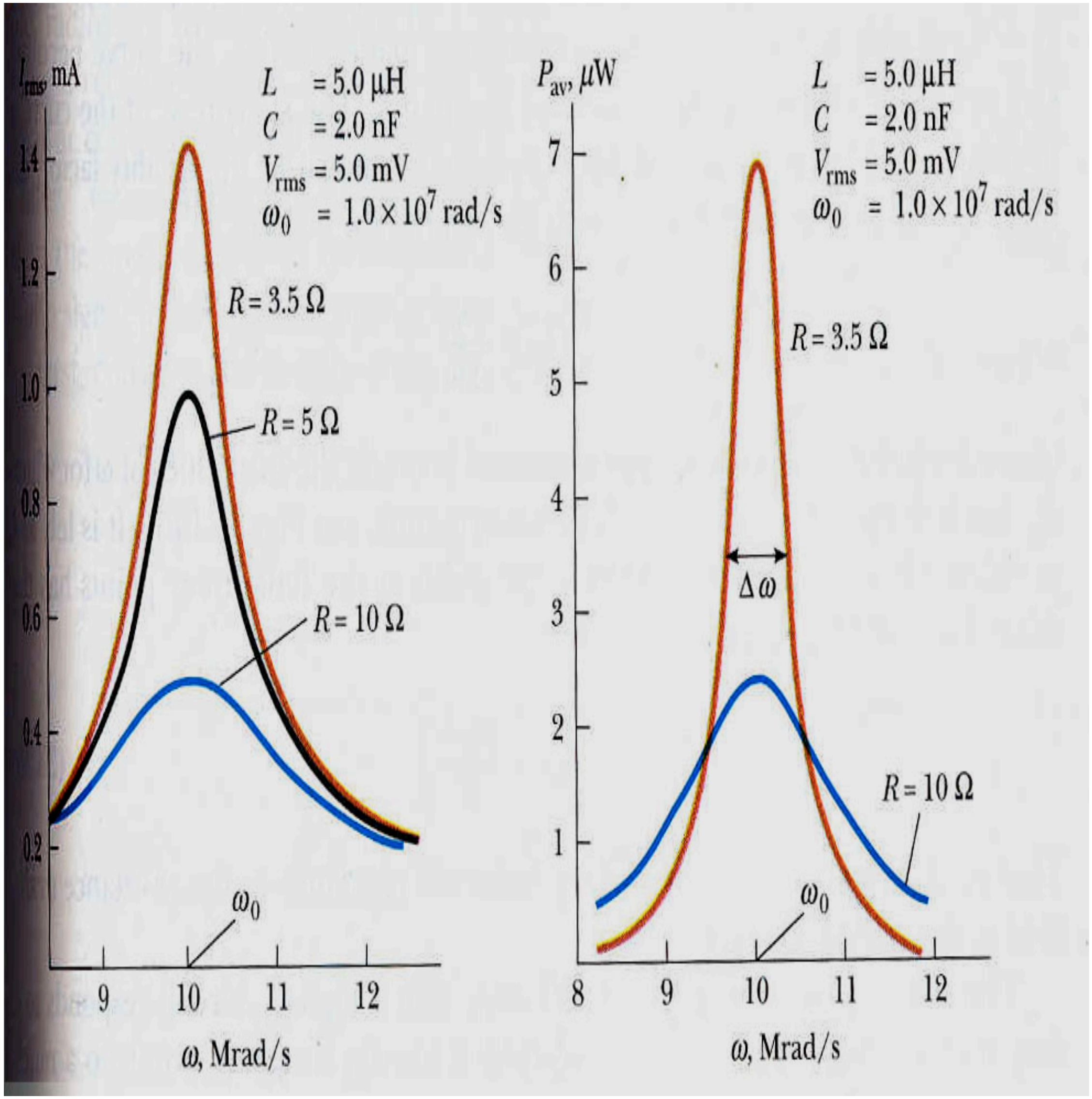
Circuit Elements	Impedance, Z	Phase angle, ϕ
R	R	0°
C	X_C	–90°
L	X_L	+90°
R C	$\sqrt{R^2 + X_C^2}$	Negative, between –90° and 0°
R L	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Resonancia

F II

Si $$\omega L = \frac{1}{\omega C}$$

$$\omega = \omega_r = \frac{1}{\sqrt{LC}}$$



Potencia circuito resistivo

F II

$$v = V_0\, sen\,\omega t;\; i = I_0\, sen\,\omega t$$

$$P_R(t) = V_0 I_0\, sen^2\,\omega t; \langle P_R \rangle = \frac{1}{T}\int_0^T P_R\, dt = \frac{V_0 I_0}{2}$$

$$V_{ef} = \frac{V_0}{\sqrt{2}} \text{ e } I_{ef} = \frac{I_0}{\sqrt{2}}$$

$$\langle P_R \rangle = R I_{ef}^{\;2} = \frac{V_{ef}^{\;2}}{\mathrm{R}}$$

Potencia circuito reactivo

F II

$$v = V_0 \, sen\omega t; \; i = \mathrm{I}_0 \, sen\,(\omega t - \varphi)$$

$$P_R(t) = V_0 I_0 \, sen\,\omega t \; sen(\omega t - \varphi)$$

$$\langle P_R \rangle = \frac{1}{T}\int_0^T P_R dt = \frac{V_0 I_0}{2}\cos\varphi =$$

$$\frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\varphi = V_{ef}\; I_{ef}\cos\varphi$$

Transformadores

F II $$V_1 = -N_1 \frac{d\Phi}{dt;}; \; V_2 = -N_2 \frac{d\Phi}{dt;} \qquad \frac{V_1}{V_2} = \frac{N_1}{N_2}$$