

SURNAME:	NAME:	DNI:	GRADE:
COURSE: Random Signals (Señales Aleatorias)	DATE: 14/06/2018	GROUP:	

Final exam training
Length: 3 hours

HOJA 1/2

FINAL EXAM TRAINING

The following learning outcomes will be assessed in this exam:

LO1 Analytically solve problems related to **probability theory and random variables**

LO2 Analytically solve problems related to **random processes**

LO3 Analytically solve problems related to **optimal filtering, detection and prediction of random signals**

LO4 Analytically solve problems related to **information theory**

LO1 LO2

1. Given two biomedical signals modeled as random processes $X(t) = A\cos(\omega_0 t + \Phi/2)$ and $Y(t) = AB\cos(\omega_0 t + \Phi/2)$, where A and ω_0 are real constants and Φ and B are **independent** random variables ($\Phi \sim U(0, 4\pi)$, $B \sim U(-1, 0)$).

- Check if $X(t)$ and $Y(t)$ are wide-sense stationary.
- Check if $X(t)$ and $Y(t)$ are jointly wide-sense stationary.
- Check if $X(t)$ is ergodic in the mean and the autocorrelation.
- Given $Z(t) = X(t) + N_2(t)$, being $N_2(t) = N_1(t) * h(t)$ with $S_{N_1 N_1}(\omega) = N_0/2$ and $H(\omega) = FT\{h(t)\} = 1, |\omega| < 1$. Compute the SNR of $Z(t)$

LO1 LO4

2. Given a random variable X with the following symbols and probabilities:

	X0	X1	X2	X3	X4
P(X)	0.1	0.15	0.65	0.05	0.05

- Obtain a Huffman code and analyze the quality of the coding in terms of the mean code length with respect to the entropy.

LO1 LO4

3. Given a family of communication channels defined by:

$$P(Y=0 | X=0) = a$$

$$P(Y=1 | X=0) = 1-a$$

$$P(Y=1 | X=1) = b$$

$$P(Y=2 | X=1) = 1-b$$

- Compute the channel capacity if $a=b=1$.
- Compute the channel capacity if $a=b=1/2$.

LO2 LO3

4. Write the first 4 values of of vector $\overline{w}(n)$ of a 4-coefficient LMS filter, given the initial vector is $\overline{w}(0) = [1 \ 1 \ 1 \ 1]^T$ and $\mu = 0.25$. The primary input and the reference inputs are:

$$\vec{x} = [x(0) \ x(1) \ \dots]^T =$$

$$[0.1, 1.3, 1.8, 3.1, 2.1, 2.7, -0.3, -1.2, -1.7, -3.2, -2.1, -0.8, 0.2, 0.7, 2.2, 3.1, \dots]^T$$

$$\vec{r} = [-0.1, -0.3, 0.2, -0.1, -0.1, 0.3, 0.3, -0.2, 0.3, 0.2, 0.1, 0.2, -0.2, -0.3, -0.2, -0.1, \dots]^T$$

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Consider that all $x(n)=0$ and $r(n)=0$ if $n<0$.

LO2 LO3

5. Given a random process $X(t)$ with cross-correlation

$$R_{XX}(-2) = 0.24649$$

$$R_{XX}(-1) = 0.24671$$

$$R_{XX}(0) = 0.27136$$

$$R_{XX}(1) = 0.24671$$

$$R_{XX}(2) = 0.24649$$

Use a 2-coefficient linear predictor to find $S_{xx}(0)/S_{xx}(0.5)$