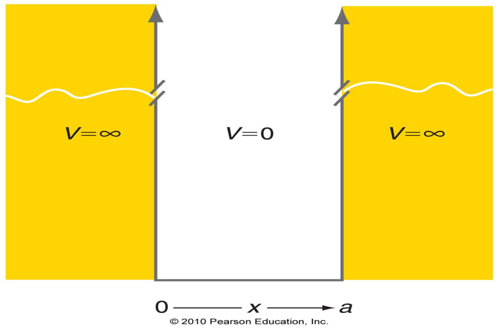
Tema 3.-Aplicación de la Mecánica Cuántica a Sistemas Sencillos

3.1. La Partícula Libre

3.2.La partícula en una caja mono-, bi- y tridimensional de paredes infinitas

3.3. Barreras y Efecto Túnel

Partícula en una caja unidimensional con barreras infinitas



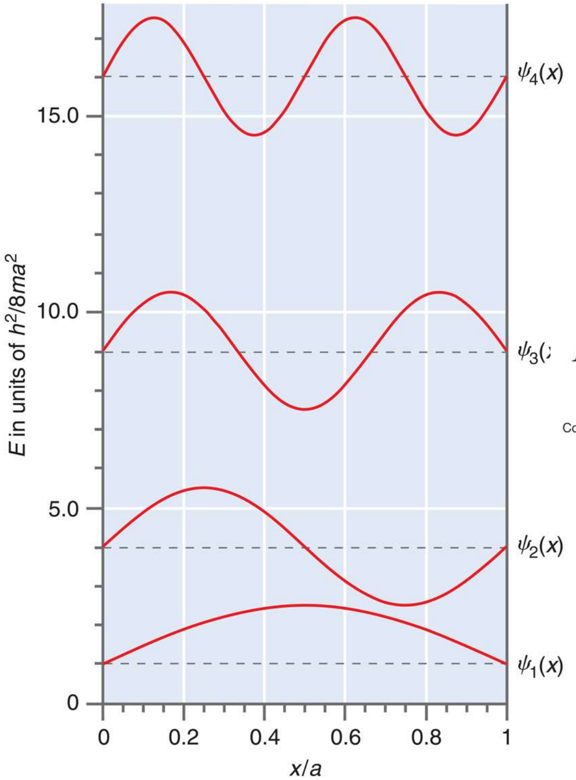
$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}[V(x) - E]\psi(x)$$

$$\psi(x) = A \sin kx + B \cos kx$$

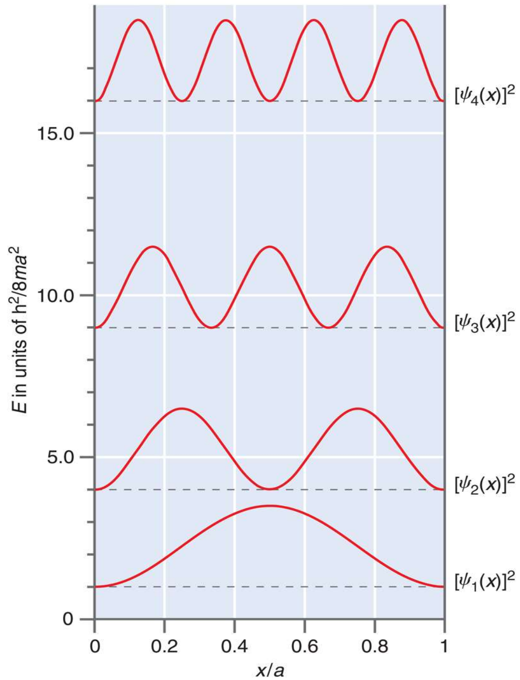
Condiciones de contorno

$$\psi(0) = \psi(a) = 0$$

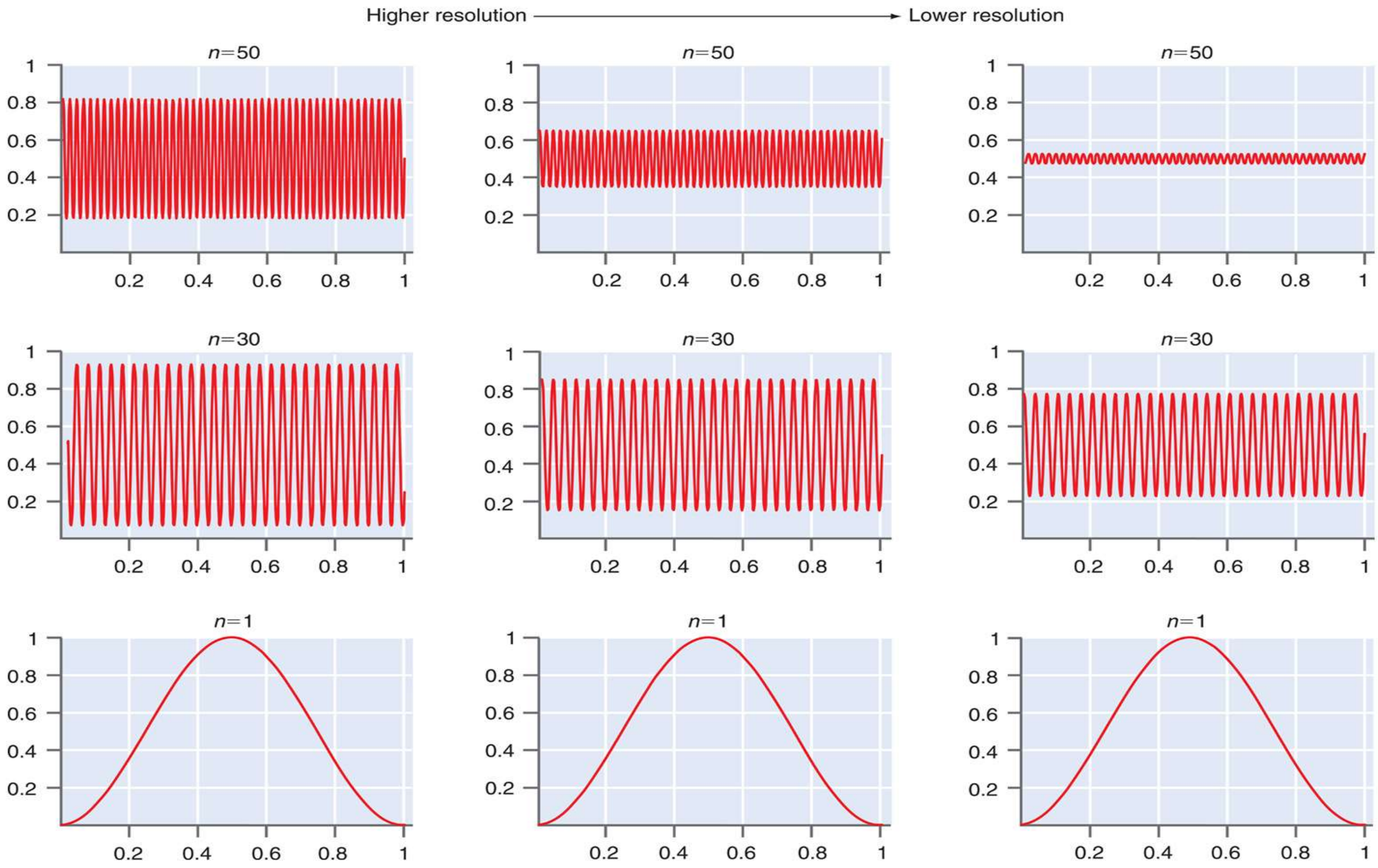


$$\psi_n(x) = A \sin\left(\frac{n\pi x}{a}\right), \quad \text{for } n = 1, 2, 3, 4, \ldots$$

$$E_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2 = \frac{h^2 n^2}{8ma^2}, \quad \text{for } n = 1, 2, 3, \ldots$$



Principio de Correspondencia de Born: el límite del comportamiento clásico



Partícula en una caja tridimensional

$$V(x, y, z) = 0 \quad \text{for } 0 < x < a; \quad 0 < y < b; \quad 0 < z < c$$
$$= \infty \quad \text{otherwise}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x, y, z) = E\psi(x, y, z)$$

$$\psi(x, y, z) = X(x)Y(y)Z(z)$$

Factorización de la función de ondas

$$-\frac{\hbar^2}{2m}\frac{d^2X(x)}{dx^2} = E_xX(x); \quad -\frac{\hbar^2}{2m}\frac{d^2Y(y)}{dy^2} = E_yY(y); \quad -\frac{\hbar^2}{2m}\frac{d^2Z(z)}{dz^2} = E_zZ(z)$$

$$\psi_{n_xn_yn_z}(x, y, z) = N\sin\frac{n_x\pi x}{a}\sin\frac{n_y\pi y}{b}\sin\frac{n_z\pi z}{c}$$

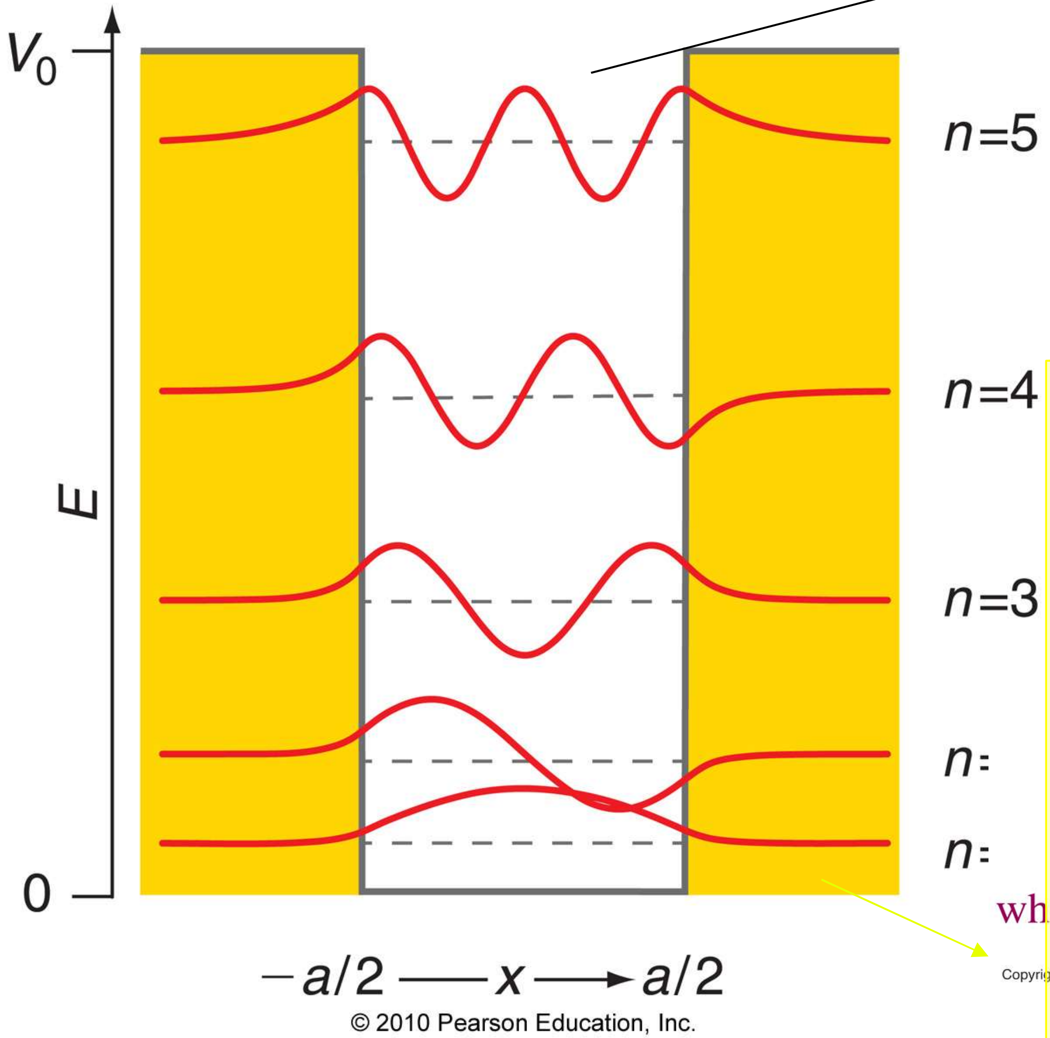
$$E = \frac{h^2}{8m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)$$

Partícula en una caja unidimensional con barreras finitas

$$V(x) = 0, \quad \text{for } -a/2 < x < a/2$$
$$V(x) = V_0, \quad \text{for } x \geq a/2, x \leq -a/2$$

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{\hbar^2}\psi(x)$$

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{a}\right), \quad \text{for } n = 1, 2, 3, 4, \ldots$$



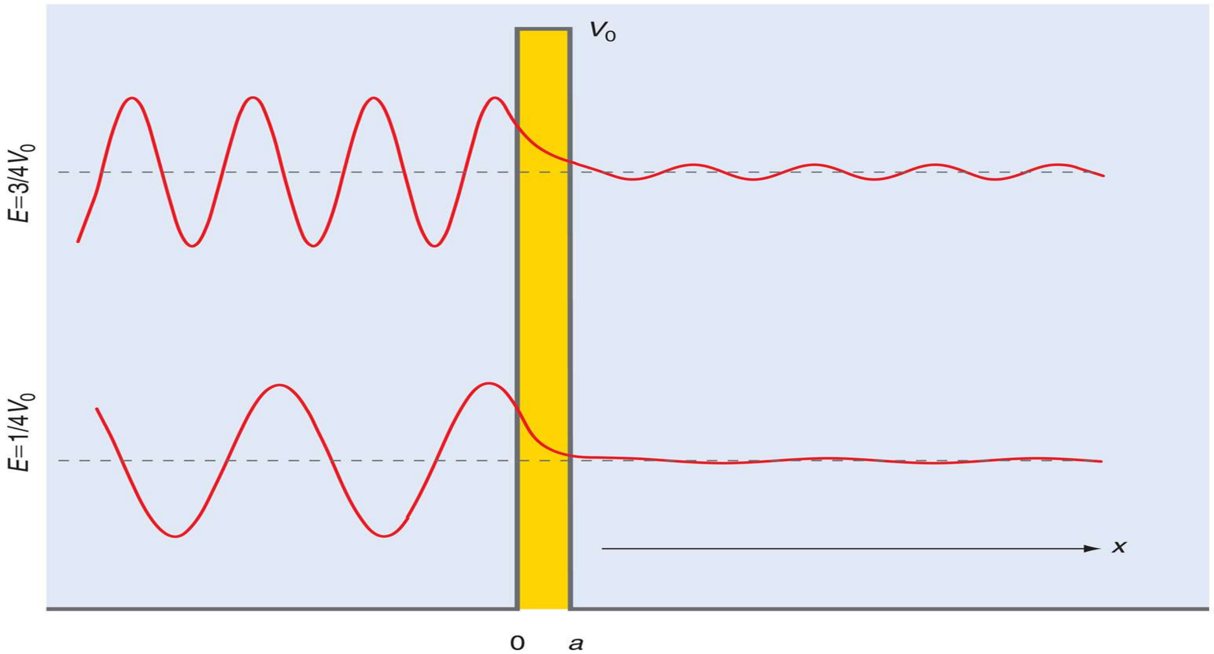
$$\frac{d^2\psi(x)}{dx^2} = \frac{2m(V_0 - E)}{\hbar^2}\psi(x)$$

$$\psi(x) = A\, e^{-\kappa x} + Be^{+\kappa x} \quad \text{for } \infty \geq x \geq a/2 \quad \text{and}$$
$$\psi(x) = A'\, e^{-\kappa x} + B' e^{+\kappa x} \quad \text{for } -\infty \leq x \leq -a/2$$

where $\kappa = \sqrt{\dfrac{2m(V_0 - E)}{\hbar^2}}$

Efecto Túnel

Barrera de altura finita



$$V(x) = 0, \quad \text{for } x < 0$$
$$V(x) = V_0, \quad \text{for } 0 \leq x \leq a$$
$$V(x) = 0, \quad \text{for } x > a$$

Microscopio de efecto túnel de barrido *(STM, Scanning Tunneling Microscope)*

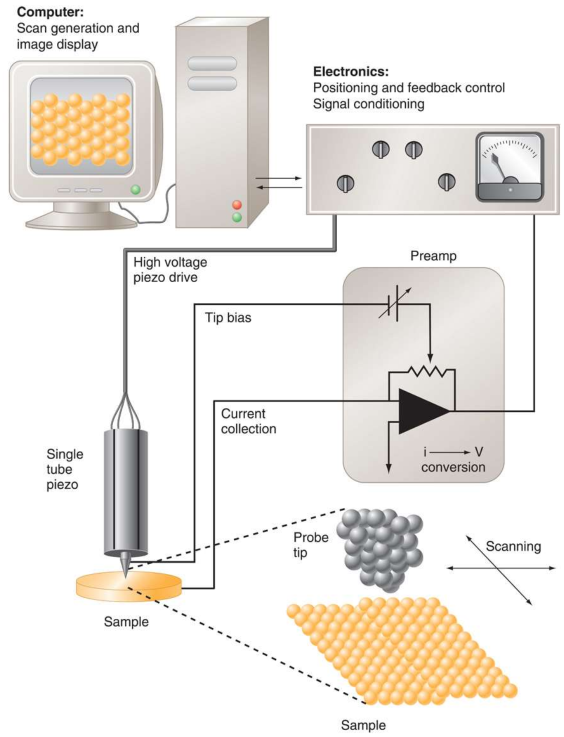
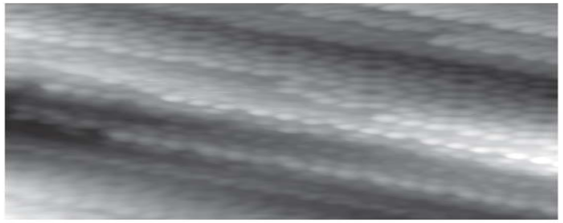


Imagen STM de la superficie (111) de Si



Perspectiva de varios planos paralelos