Escalado Capa Límite Térmica

Ecuación de la energía (EE) en régimen incompresible :

$$\underbrace{\rho c\left(u\frac{\partial T}{\partial x}}_{\boxed{\sim \rho c U_c \frac{\Delta T}{\ell}}} + \underbrace{v\frac{\partial T}{\partial y}\right)}_{\sim \rho c V_c \frac{\Delta T}{\delta_T}} = \underbrace{k\left(\cancel{\frac{\partial^2 T}{\partial x^2}}}_{\sim k\frac{\Delta T}{\ell^2}} \overset{\ll}{+} \underbrace{\frac{\partial^2 T}{\partial y^2}\right)}_{\boxed{\sim k\frac{\Delta T}{\delta_T^2}}} \overset{\ll}{+} \underbrace{\cancel{\Phi_v}}_{\sim \mu\left(\frac{U}{\delta}\right)^2}$$

c : calor específico

k : conductividad térmica

Φ : disipación viscosa

Comparando órdenes de magnitud de los términos convectivo y difusivo :

$$\rho c U_c \frac{\Delta T}{\ell} \sim k\frac{\Delta T}{\delta_T^2} \longrightarrow \left(\frac{\delta_T}{\ell}\right)^2 = \underbrace{\frac{k}{\mu c}}_{Pr^{-1}}\,\underbrace{\frac{\mu}{\rho}}_{\nu}\,\frac{1}{U_e \ell} \longrightarrow \boxed{\frac{\delta_T}{\ell} \sim Re^{-1/2}\,Pr^{-1/2}}$$

$$\xrightarrow{\frac{\delta}{\ell} \sim Re_x^{-1/2}} \boxed{\frac{\delta_T}{\delta} \sim Pr^{-1/2}}$$

NOTA

$Pr = \dfrac{\nu}{\alpha}$; $\alpha = \dfrac{k}{\rho c}$

α : difusitividad térmica

Reteniendo únicamente los términos más importantes :

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}$$

$y = 0$: $\begin{cases} \dfrac{\partial T}{\partial y} = -\dfrac{q_p}{k} & \text{(0 si PARED ADIABÁTICA)} \\ \text{ó} & \\ T = T_p & \text{TEMPERATURA DE PARED ESPECIFICADA} \end{cases}$

$y \to \infty$: $T \to T_e$

$x = x_0$: $T(x_0, y) = T_0(y)$

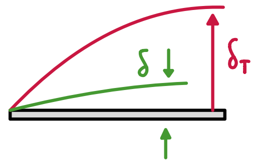
Vamos a hacer un análisis asintótico para estimar órdenes de magnitud para diferentes valores del número de Prandtl :

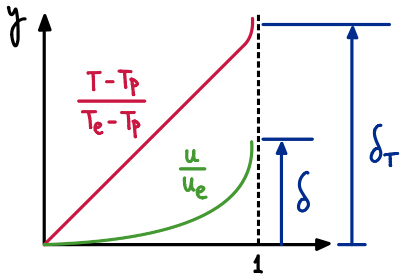
$Pr \sim 1$

$\delta_T \sim \delta$ (TÍPICO EN GASES)

$Pr \ll 1$

$\delta_T \gg \delta$





TÍPICO EN METALES LÍQUIDOS $(\uparrow\uparrow\uparrow \alpha)$

Conducen bien el calor por tener electrones en banda de conducción.

Prácticamente toda la CLT ve flujo de Euler:

$$\underbrace{u}_{u_e} \frac{\partial T}{\partial x} + \cancel{v \frac{\partial T}{\partial y}} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}$$

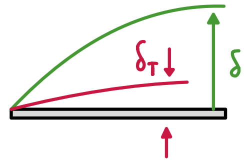
$$\underbrace{u_e \frac{\partial T}{\partial x}}_{\sim u_e \frac{\Delta T}{\ell}} = \underbrace{\frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}}_{\sim \frac{\nu}{Pr}\frac{\Delta T}{\delta_T^2}}$$

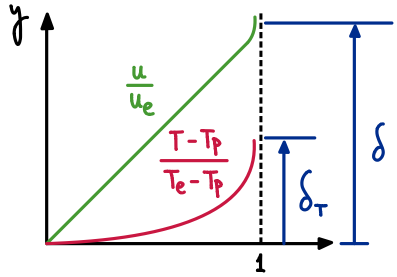
$$\boxed{\left.\frac{\delta_T}{\ell}\right|_{Pr \ll 1} \sim Re^{-1/2} Pr^{-1/2}}$$

$$\boxed{\left.\frac{\delta_T}{\delta}\right|_{Pr \ll 1} \sim Pr^{-1/2}}$$

$Pr \gg 1$

$\delta_T \ll \delta$





TÍPICO EN LUBRICACIÓN CON ACEITES

TAYLOR: $u = \left.\frac{\partial u}{\partial y}\right|_{y=0} y = \frac{\tau_p}{\mu} y \sim \frac{\Delta u_e}{\delta}\delta_T \sim u_e \frac{\delta_T}{\delta}$

$$\underbrace{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}_{\sim u_e \frac{\delta_T}{\ell}\frac{\Delta T}{\ell}} = \underbrace{\frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}}_{\sim \frac{\nu}{Pr}\frac{\Delta T}{\delta_T^2}}$$

Comparando:

$$\left(\frac{\delta_T}{\ell}\right)^3 \sim \underbrace{\frac{\nu}{\ell u_e}}_{Re^{-1}} \frac{1}{Pr} \underbrace{\frac{\delta}{\ell}}_{\sim Re^{-1/2}} \sim Re^{-3/2} Pr^{-1}$$

$$\boxed{\left.\frac{\delta_T}{\ell}\right|_{Pr \gg 1} \sim Re^{-1/2} Pr^{-1/3}}$$

$$\boxed{\left.\frac{\delta_T}{\delta}\right|_{Pr \gg 1} \sim Pr^{-1/3}}$$

EL ESCALADO GENERAL DEL PRINCIPIO NO ES VÁLIDO PARA $Pr \gg 1$

Flujo de calor en la pared: $\boxed{q_p = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}} \sim k \frac{|T_p - T_e|}{\delta_T}$

Nusselt: $\boxed{Nu = \frac{q_p \ell}{k \Delta T}} \sim \left(\frac{\delta_T}{\ell}\right)^{-1}$

Transferencia de calor en la pared: $\boxed{Q_p = \int q_p \, dx} \sim k \frac{|T_p - T_e|}{\delta_T} \ell \longrightarrow \frac{Q_p}{k(T_p - T_e)} \sim \left(\frac{\delta_T}{\ell}\right)^{-1}$

$\Delta T \sim T_p - T_e$ en distancias $y \sim \delta_T$

$$\boxed{\left. Nu \right|_{Pr \ll 1} \sim \left.\frac{Q_p}{k(T_p - T_e)}\right|_{Pr \ll 1} \sim Re^{1/2} Pr^{1/2}}$$

$$\boxed{\left. Nu \right|_{Pr \gg 1} \sim \left.\frac{Q_p}{k(T_p - T_e)}\right|_{Pr \ll 1} \sim Re^{1/2} Pr^{1/3}}$$