10 – Introduction to Gust / Turbulence Dynamic Response

Part 1: Discrete Gust

Vibraciones y Aeroelasticidad
Dpto. de Vehículos Aeroespaciales
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DELTA AIRLINES FLIGHT 191
134 of 163 OCCUPANTS KILLED & 1 CAR DRIVER KILLED

Lockheed L-1011 TriStar

National Oceanic and Atmospheric Administration

View Thumbnails

THE FIRST ACCIDENT THAT TRIGGERED THE DEVELOPMENT OF RADAR WARNING SYSTEMS

DALLAS/FORT WORTH | Delta Air Lines Flight 191

Upgrade: Downdraft detection

As Delta Flight 191, a Lockheed L-1011, approached for landing at Dallas/Fort Worth airport, a thunderstorm lurked near the runway. Lightning flashed around the plane at 800 ft., and the jetliner encountered a microburst wind shear--a strong downdraft and abrupt shift in the wind that caused the plane to lose 54 knots of airspeed in a few seconds. Sinking rapidly, the L-1011 hit the ground about a mile short of the runway and bounced across a highway, crushing a vehicle and killing the driver. The plane then veered left and crashed into two huge airport water tanks. On board, 134 of 163 people were killed. The crash triggered a seven-year NASA/FAA research effort, which led directly to the on-board forward-looking radar wind-shear detectors that became standard equipment on airliners in the mid-1990s. Only one wind-shear-related accident has occurred since.

Factor	% of Events
Adverse weather	40 %
Adverse wind (all conditions)	33 %
Windshear	4 %

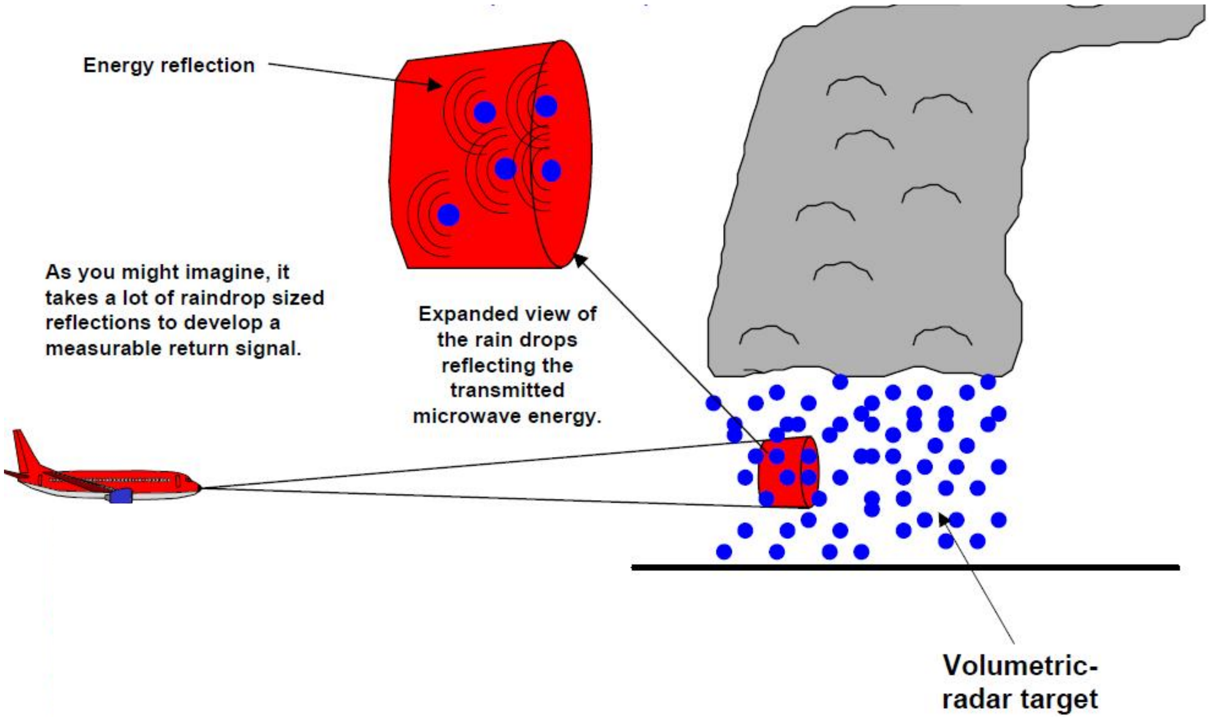
(Source: Flight Safety Foundation - Flight Safety Digest - Vol. 17/Vol. 18 - 1998-1999)

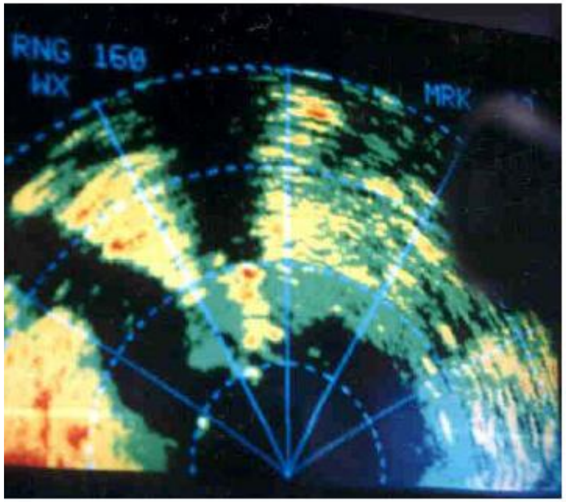
Table 1

Weather factors in Approach-and-landing Accidents

A **weather radar**, or **weather surveillance radar** (**WSR**), is a type of radar used to locate precipitation, calculate its motion, estimate its type (rain, snow, hail, etc.), and forecast its future position and intensity.

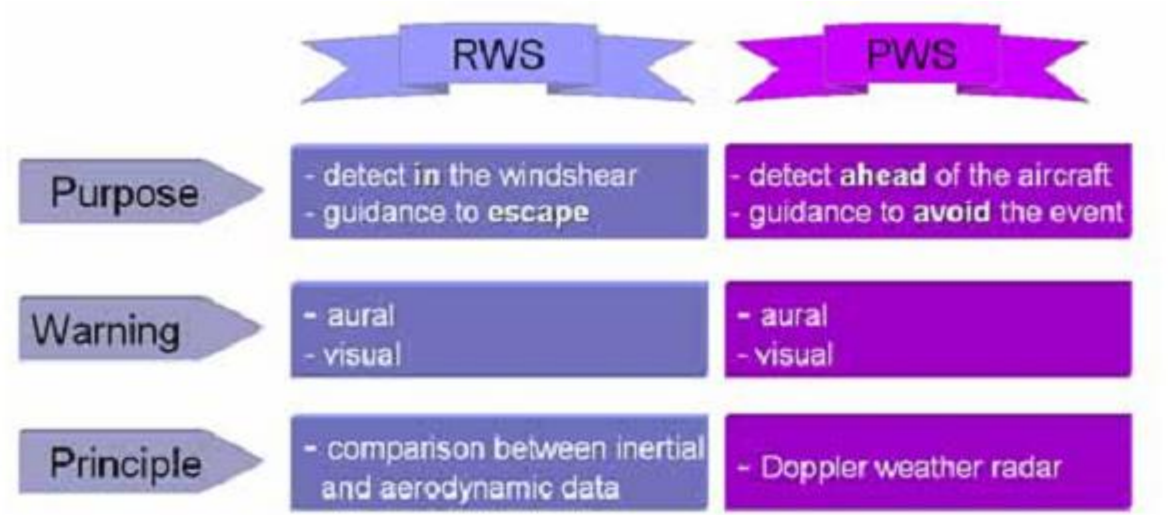
Modern weather radars are mostly pulse-Doppler radars, capable of detecting the motion of rain droplets in addition to intensity of the precipitation. Both types of data can be analyzed to determine the structure of storms and their potential to cause severe weather.





PROBLEM: ONLY IF PRECIPITATION EXCISTS (RAIN, ICE, …)

- The **Airborne wind shear detection and alert system**, fitted in an aircraft, detects and alerts the pilot both visually and aurally of a wind shear condition.
  - RWS (reactive wind shear detection system): the detection takes place when the aircraft penetrates a wind shear condition of sufficient force, which can pose a hazard to the aircraft.
  - PWS (predictive wind shear detection system) the detection takes place, if such wind shear condition is ahead of the aircraft. This system collects wind velocity data gathered by the weather radar (Doppler) to identify the existence of wind shear.



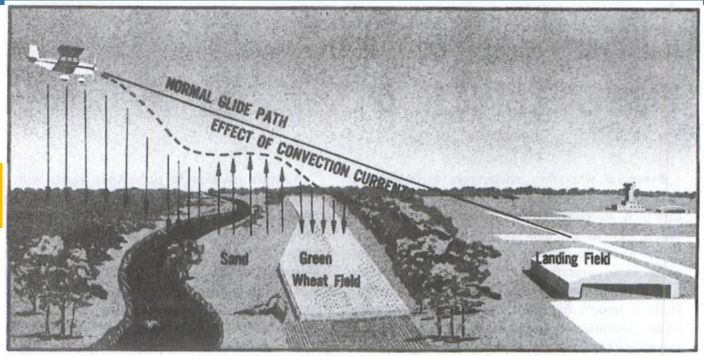
PROBLEM:

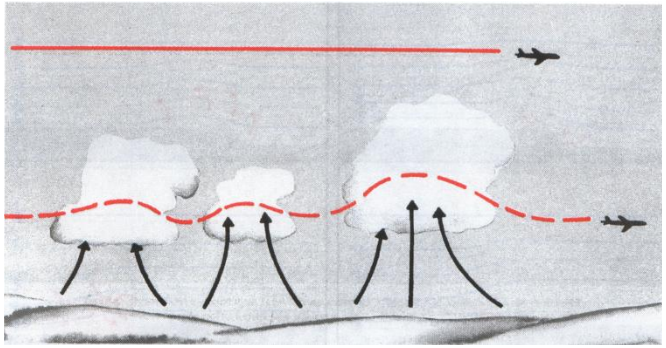
Maybe too late?

PROBLEM:

Not working in dry conditions

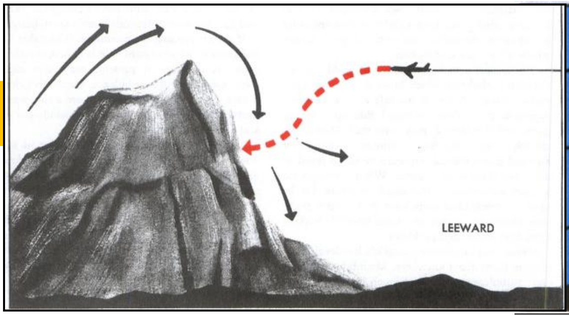
CONVECTIVE CURRENTS

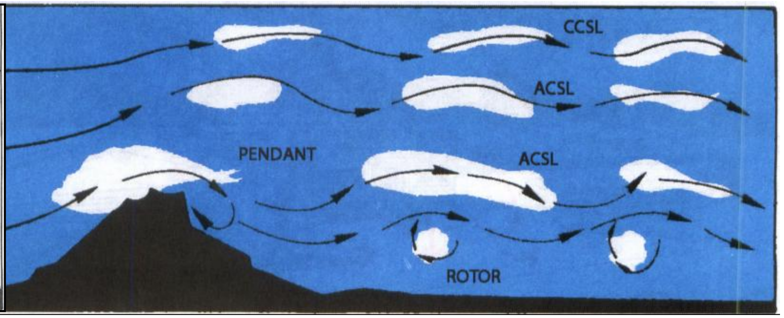




MECHANICAL TURBULENCE

Boeing 707 Fuji



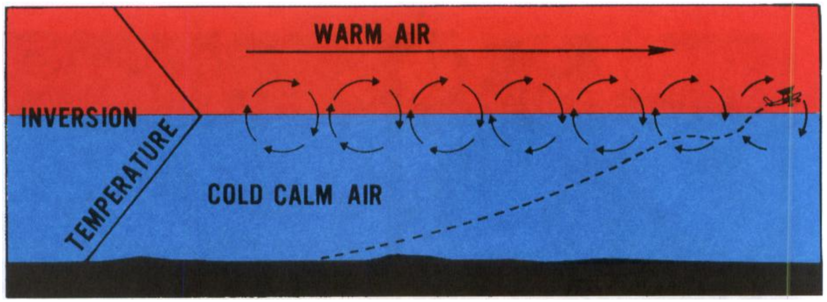


WIND SHEAR

… with a low-level temp inversion

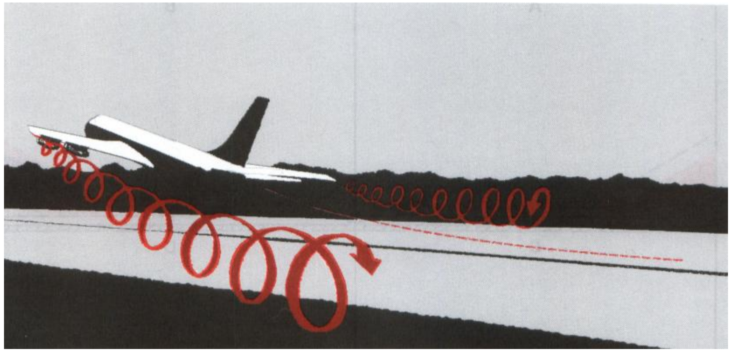
… in a frontal zone

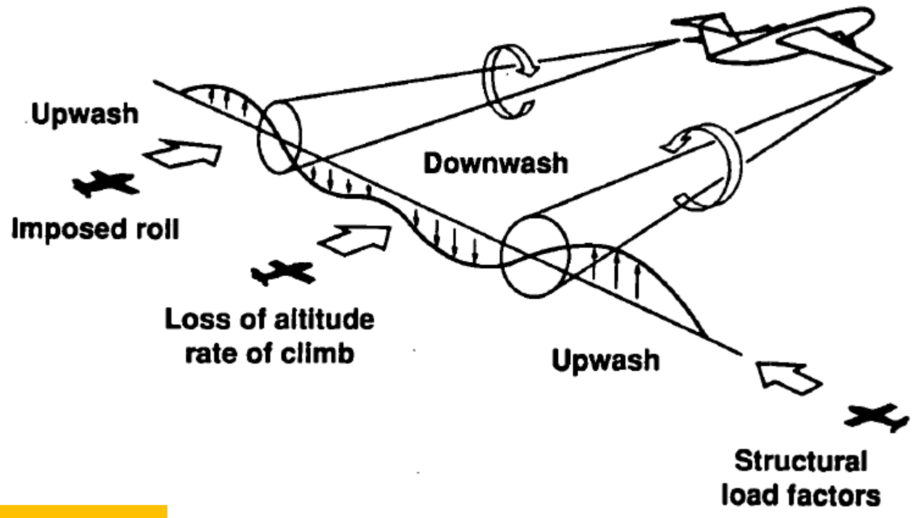
… High-level CAT (jet stream)



WHERE TO EXPECT HIGH LEVELS OF TURBULENCE (2/3)
RECOMMENDATION: LECTURE OF "WEATHER FOR PILOTS" OF FAA PUBLICATIONS

WAKE TURBULENCE

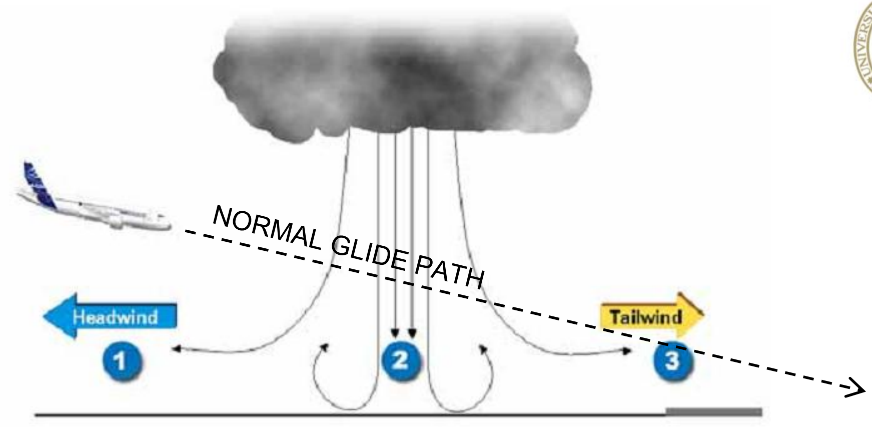




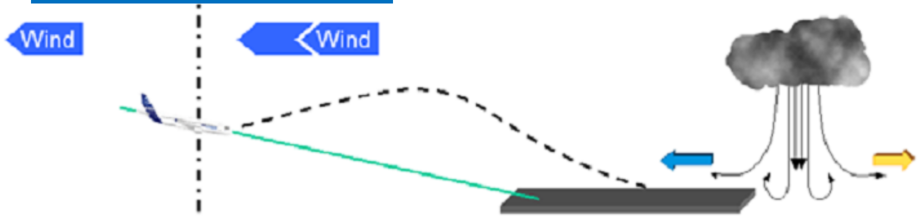
FIREFIGHTERS

MOVIE Firefighter

WHERE TO EXPECT HIGH LEVELS OF TURBULENCE (3/3)
MICROBURSTS



ZONE 1 : HEADWIND

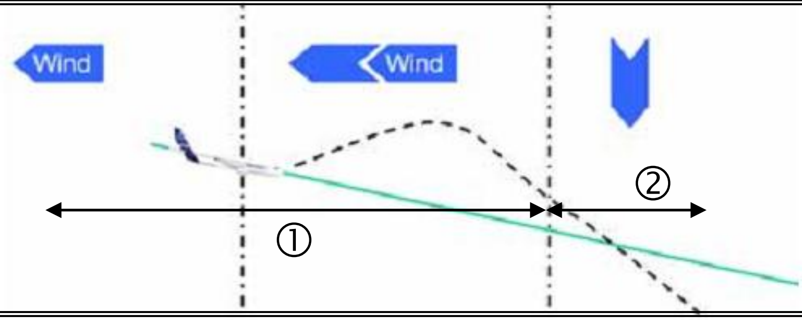


The crews do not always perceive an increase of the headwind as a risk. But such a headwind gust de-stabilizes the approach of the aircraft, which will tend to fly above path and/or accelerate, if the pilot does not react adequately.

If the headwind shear occurs at takeoff, the resulting aircraft performance will increase. Once out of the shear, the indicated airspeed decreases thus leading to an AOA increase which might trigger the alpha-floor protection and/or stick shaker activation.

TRANSITION FROM ZONE 1 TO 2 :

Vertical downdrafts are usually preceded by an increase of the headwind component If the pilot does not fully appreciate the situation, he/she will react to the headwind gust effects to regain the intended path by reducing the power and by pushing on the stick. At that point, a vertical downdraft will increase the aircraft sink rate, which will bring the aircraft below the intended path.



ZONE 3 : TAILWIND

In case of a sudden increase of the tailwind, the aircraft airspeed decreases instantaneously. The lift decreases and the aircraft tends to fly below the intended approach path.

If the pilots pulls on the stick to recapture the path without adding sufficient thrust, the AOA will increase significantly and the aircraft will sink down.

If sufficient thrust is set to regain the intended path, but the pilot's reaction is then slow to reduce the thrust once back on the path, the aircraft will fly above the path and/or will accelerate.

SO… UNLESS WARNING SYSTEMS DO DETECT SOME SEVERE CONDITIONS WITH RAINFALL, HAIL, AND SO ON…

SOME WIND-SHEAR CONDITIONS ARE NOT PREDICTED BY THE ON-BOARD SYSTEMS WHAT LEAD TO CONSIDER GUST/TURBULENCE ENCOUNTERING AS AN AIRCRAFT DESIGN CASE

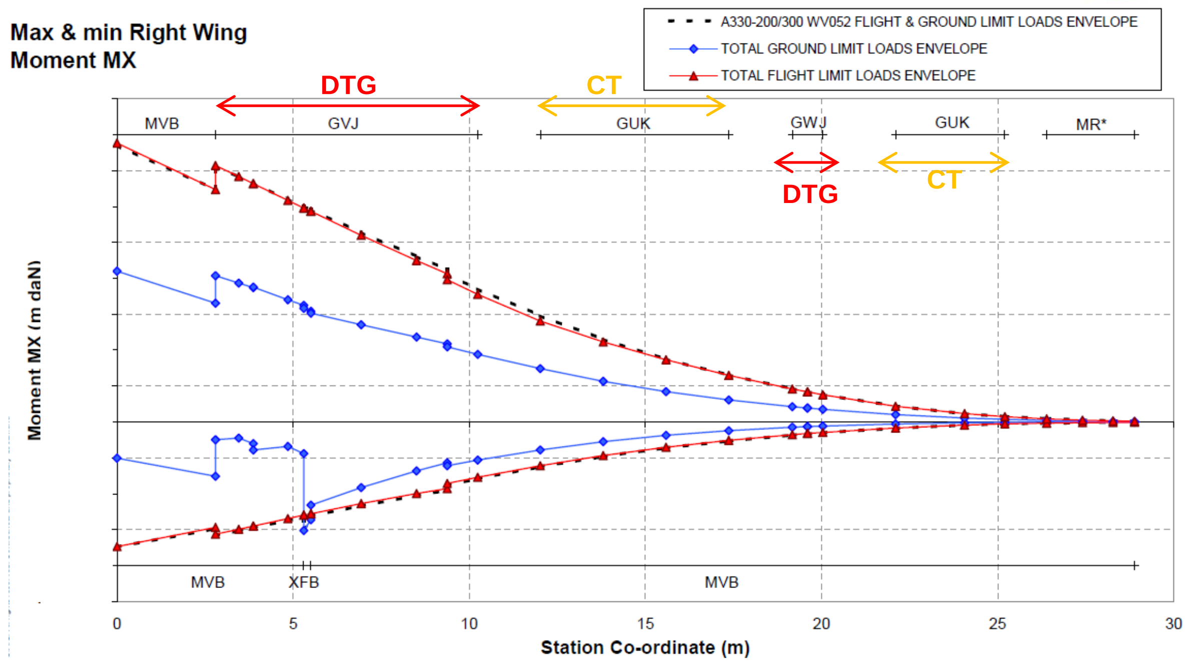
AND NOT ALL NEWS ARE BAD…

STRUCTURAL DYNAMICS AND AEROELASTICITY ENGINEERS WILL AVOID THE UNEMPLOYMENT LINE

RELEVANCE OF GUST/TURBULENCE LOADS IN THE DESIGN OF THE AIRCRAFT

- Wing UP-Bending design loads are mainly driven by Gust/Turbulence

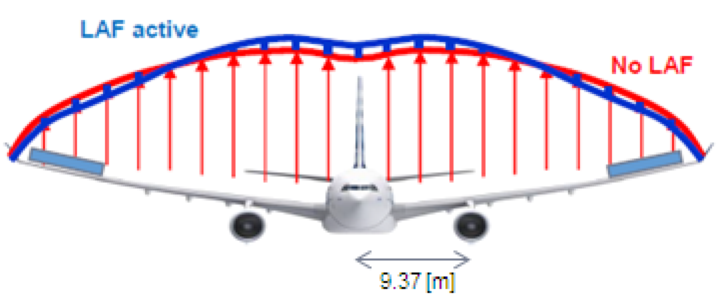
  (Figures extracted from A330-200 Freighter Loads Manual)

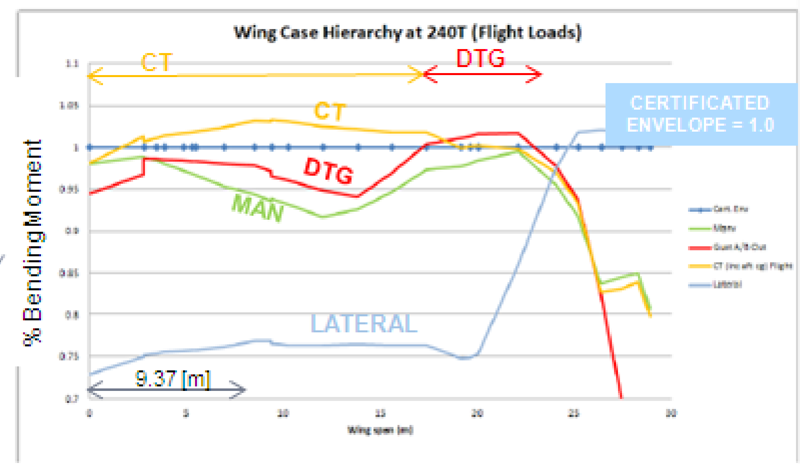


- Flight Control System to alleviate loads:

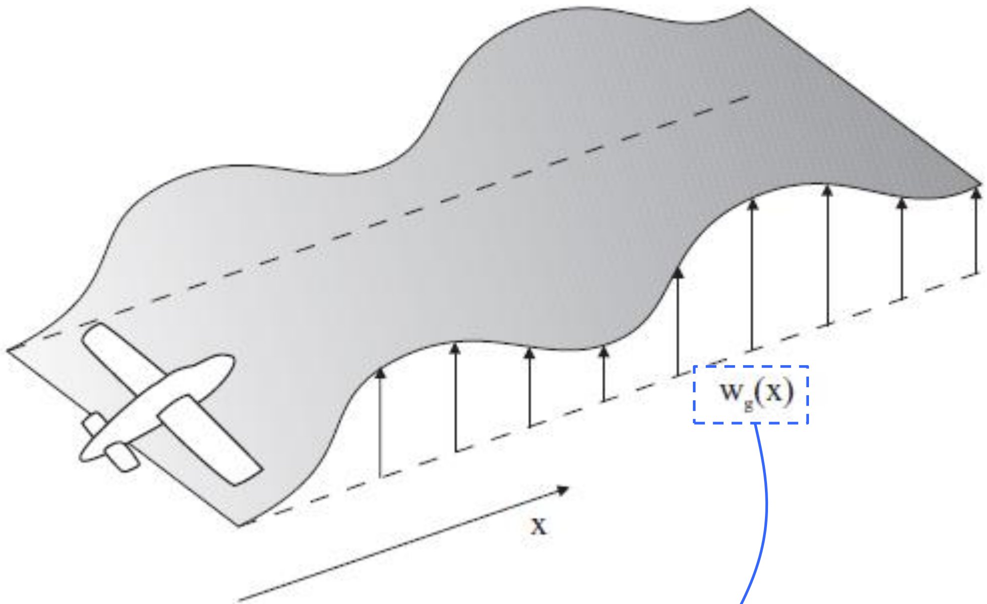
  (Example: A330-200/300 WV080)

• The E-LAF objective is to further reduce wing bending moment in these conditions by shifting loads inboard through aileron upward deflection



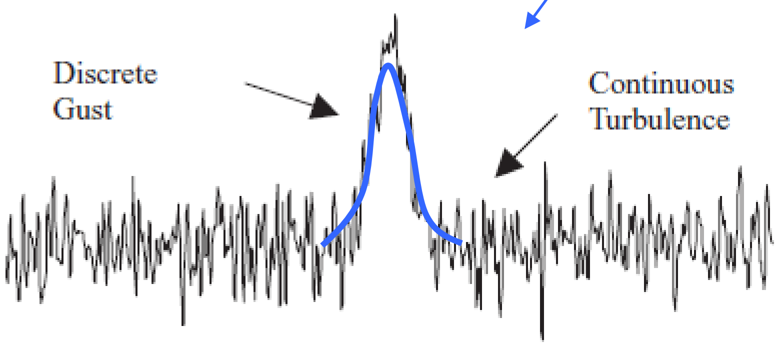


MOVIE [A400 Gust with FCS]



DISCRETE GUST

Gust velocity varies in a ***deterministic*** *manner, often in the form of a '1-cosine' shape*



CONTINUOUS TURBULENCE

Gust velocity is assumed to vary in a ***random*** *manner*

(*) Illustration extracted from "Introduction to Aircraft Aeroelasticity and Loads", Wright and Cooper

Nowadays A/Cs are designed with the risk of encountering a **SEVERE GUST** of

56 ft/s EAS SL for DTG

(85 ft/s TAS SL for CT)

with a probability of occurrence less than

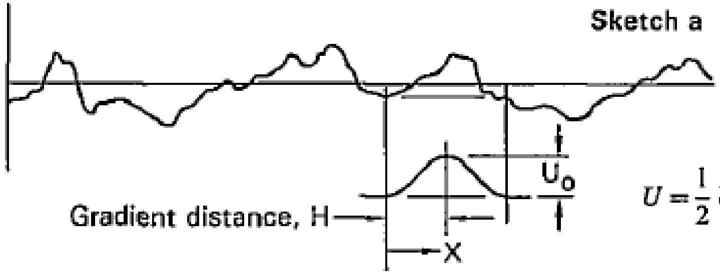
$2x10^{-5}$ per flight hour

EXTREME GUSTS occurs and they are covered by the 1.5 factor from LIMIT to ULTIMATE LOADS.

☝ DTG = Discrete Tuned Gust / CT = Continuous Turbulence

SEVERE GUST/TURBULENCE CONDITIONS

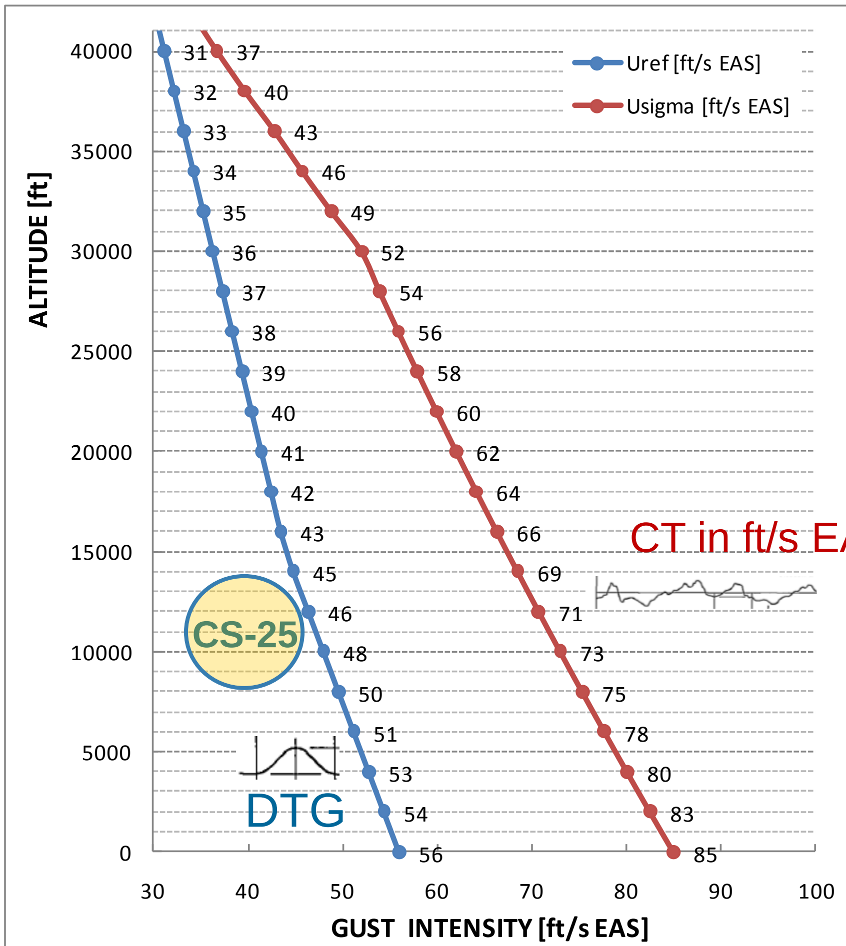
CS-25 Paragraph 25.0341 FOR VC/MC DESIGN CONDITIONS

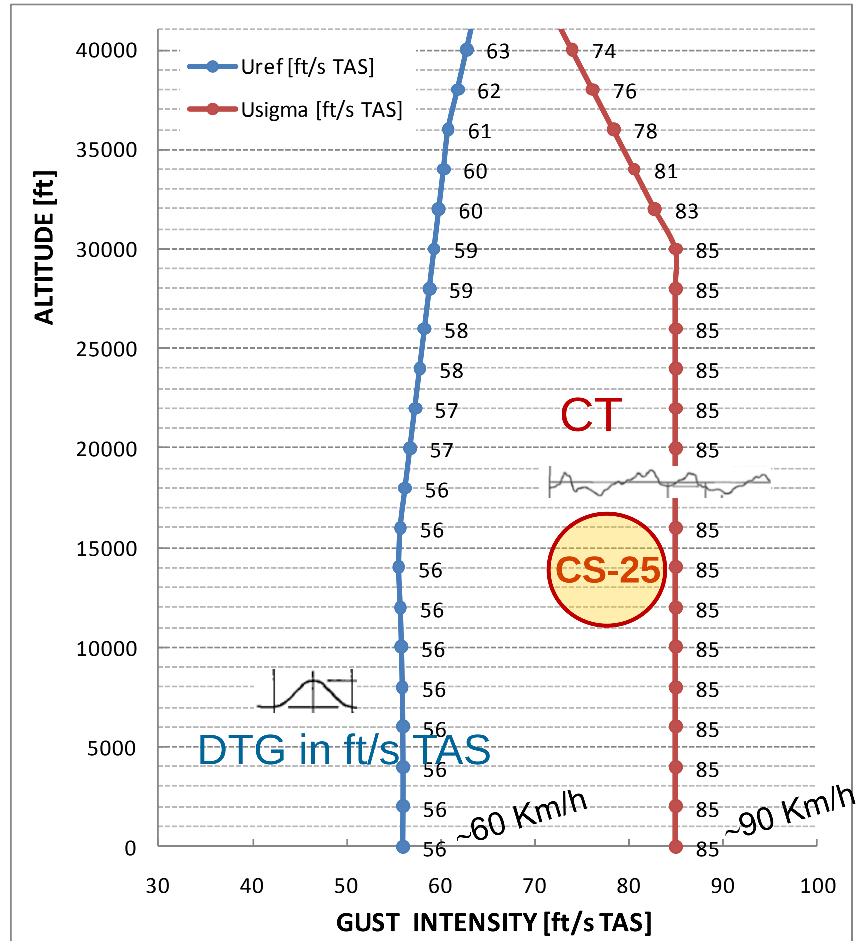


$$U = \frac{1}{2} U_0 \left(1 - \cos\frac{2\pi x}{2H}\right)$$

ft/s EAS

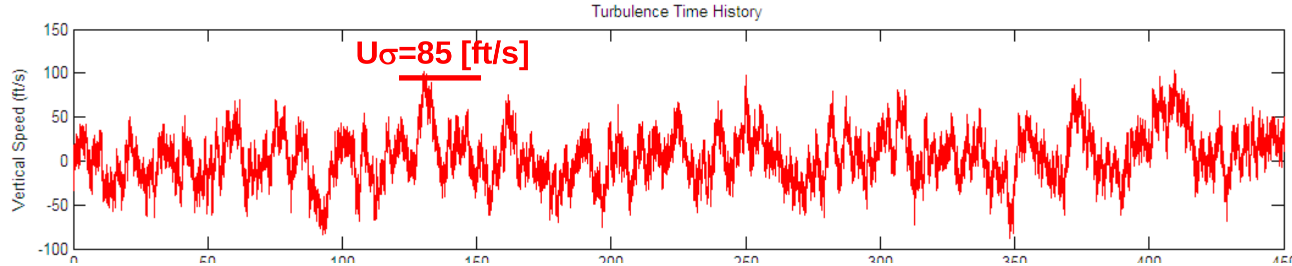
ft/s TAS

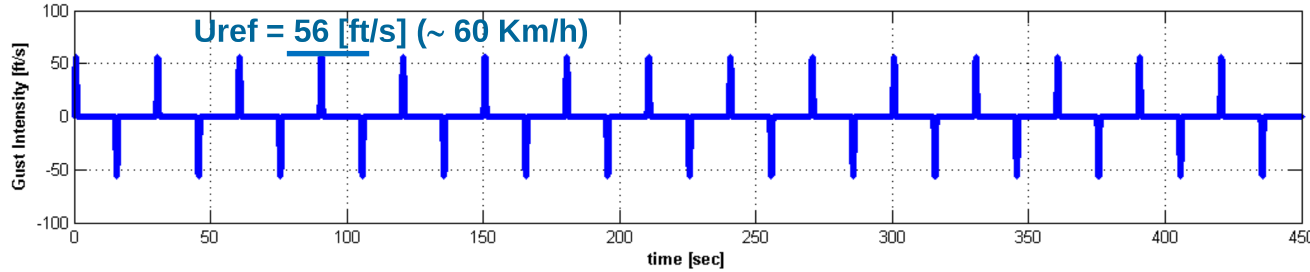


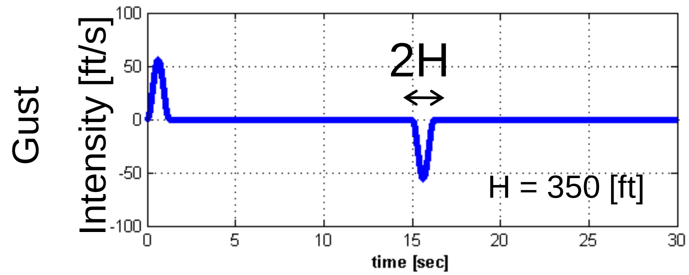


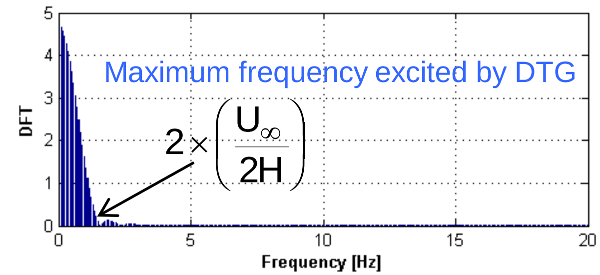
SEVERE GUST/TURBULENCE CONDITIONS
CT vs. DTG Analyses in terms of PSD Excitation

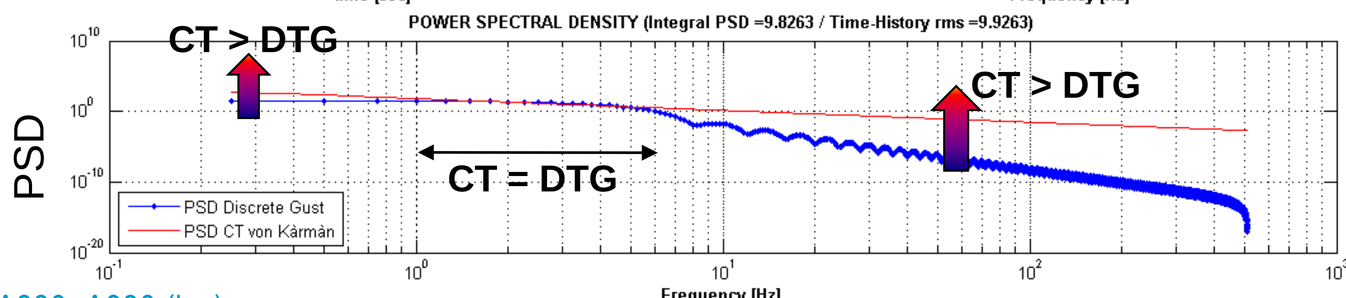
U_∞ = 330 KTAS at SL
(~ 600 Km/h)











MOVIE A330+A330 (xz) & A330+A330 (iso)

SEVERE GUST/TURBULENCE CONDITIONS

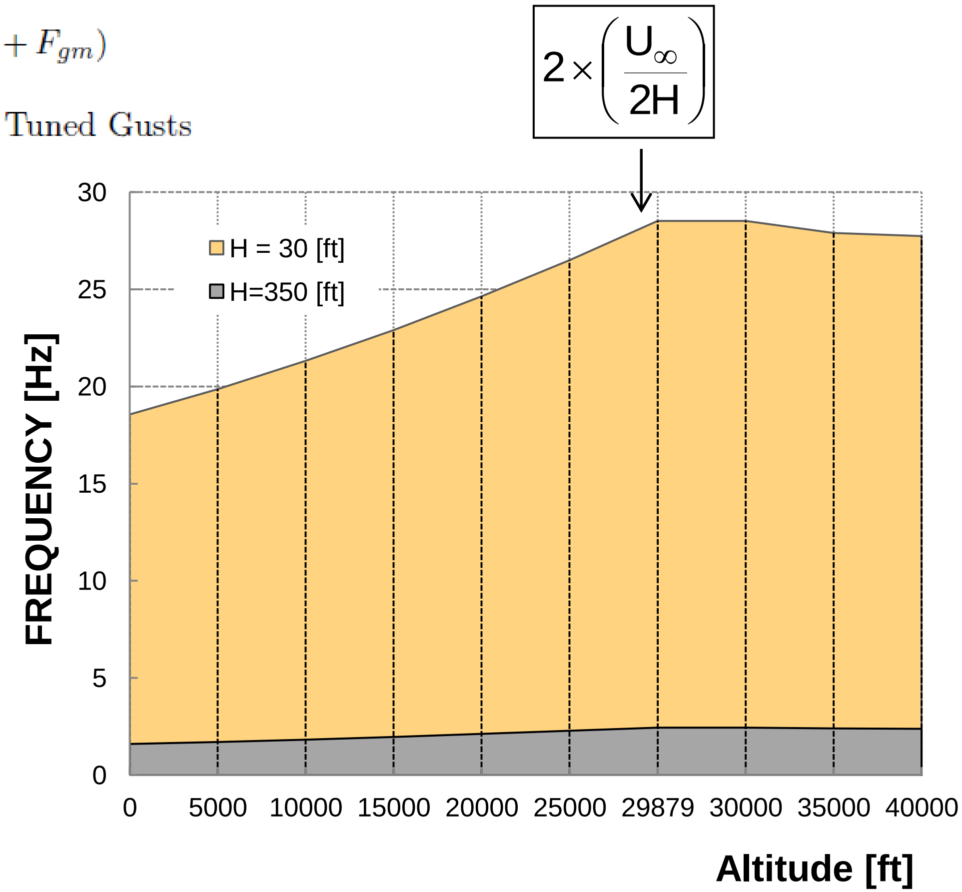
DTG Frequency Range Excitation as a function of Flight Altitude [ft]

$$U_g = U_{ds} \cdot \frac{1}{2}\left[1 - \cos\left(\frac{\pi s}{H}\right)\right] = U_{ref}F_g \cdot \frac{1}{2}\left[1 - \cos\left(\frac{\pi s}{H}\right)\right] = U_{ref}F_g \cdot \frac{1}{2}\left[1 - \cos\left(2\pi\frac{V}{2H}t\right)\right]$$

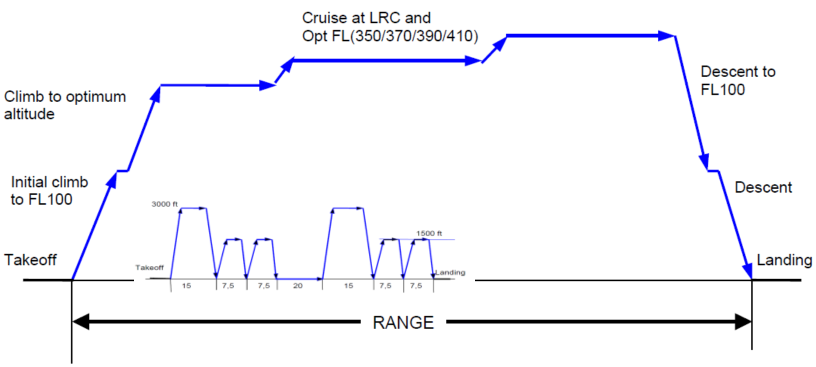
U_{ds}	Design Gust Velocity
s	Distance penetrated into the gust [feet]
H	Gust Gradient Distance [feet]
U_{ref}	Reference Gust Velocity
F_g	Flight Profile Alleviation Factor $F_g = 0{,}5\,(F_{gz} + F_{gm})$
V	Aircraft Flight Speed in fps
$V/2H$	Characteristic frequency in Hz of the Discrete Tuned Gusts

A330

Altitude [ft]	KCAS	KTAS	fps	Frequency [Hz] 2x (V/2H)	
				min	MAX
				H=350 ft	H= 30 ft
0	330	330	557	0.80	9.28
5000	330	353	596	0.85	9.93
10000	330	379	640	0.91	10.66
15000	330	407	687	0.98	11.45
20000	330	438	739	1.06	12.32
25000	330	471	795	1.14	13.25
29879	330	507	856	1.22	14.26
30000	329	507	856	1.22	14.26
35000	295	496	837	1.20	13.95
40000	263	493	832	1.19	13.87



LOW-ALTITUDE USAGE…



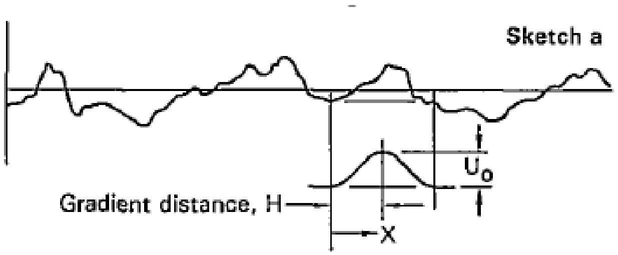
JAR regulations states severe gust intensities that works "fine" with typical civil A/C's usage with FL above 20000 [ft]. If the A/C missions change significantly to increase low-level operations….

… the low-altitude usage will lead to the same Limit Loads exceedences of $2x10^{-5}$ approx.?
If not… Could you evaluate the risk of the operation?

LESS-THAN-SEVERE TURBULENCE…

Specific operations (military AAR, combat off-loads, …) will be performed occasionally and during good weather conditions because of safety reasons, and the probability of encountering severe gust intensity decreases considerably.
Why not designing with a reduced gust intensity?

… the design loads calculated with reduced gust intensities do not violate the standard criteria of $2x10^{-5}$ Limit Loads Occurrences per Flight Hour?
If not … could you quantify the risk?



$$U = \frac{1}{2} U_0 \left(1 - \cos\frac{2\pi x}{2H}\right)$$

"A typical rule of thumb is that peak response occurs for gusts with a distance to maximum amplitude equal to 12.5 times the mean chord of the aircraft"

QINETIQ/FST/SMC/CR021764 Flight Loads Data for Gust Load Prediction, March 2002

$$\Delta L = \frac{\rho_\infty}{2} U_\infty^2 \cdot S \cdot C_{L\alpha} \cdot \Delta\alpha = \frac{\rho_\infty}{2} U_\infty^2 \cdot S \cdot C_{L\alpha} \cdot \frac{U_{gFTSTAS}}{U_\infty \times 3.28084} = \frac{\rho_\infty}{2} U_\infty U_{gFTSTAS} \cdot S \cdot C_{L\alpha} =$$

$$= \left(\frac{1}{2}\rho_0 \cdot 3.28084\right)\left(\sqrt{\frac{\rho_\infty}{\rho_0}} \cdot U_\infty\right)\left(\sqrt{\frac{\rho}{\rho_0}} \cdot U_{gFTSTAS}\right) \cdot S \cdot C_{L\alpha} = cte \cdot U_{\infty[KEAS]} \cdot U_{gFTSEAS}$$

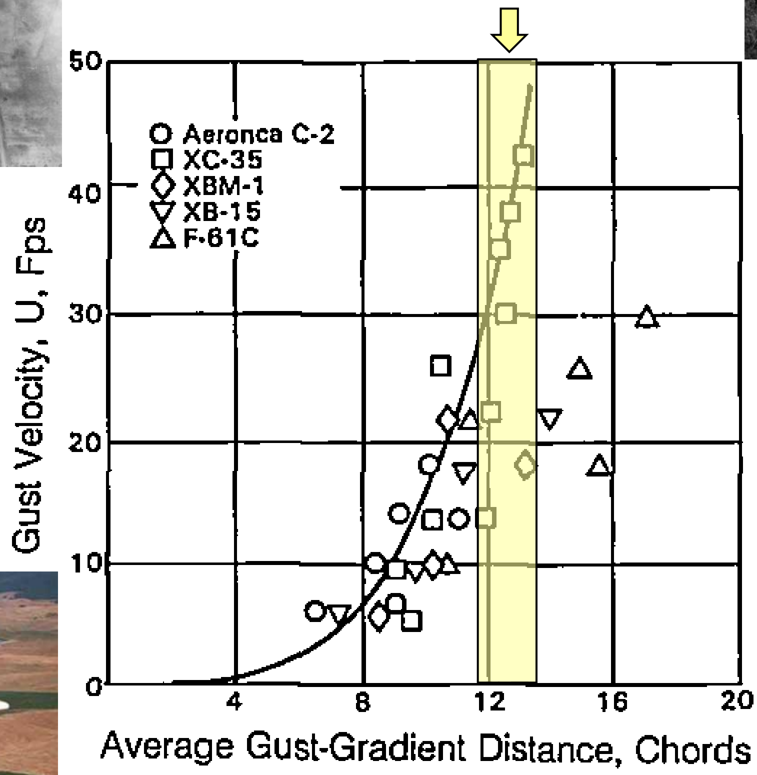
$$cte = \frac{1}{2}\rho_0 S C_{L\alpha} \times 3.28084 \times 0.514444 \approx \left(\frac{1}{2}\rho_0 \times 1.689\right) S C_{L\alpha}$$

$$\Delta n = \frac{\Delta L}{W} = \frac{U_{\infty[KEAS]} \cdot U_{gFTSEAS}}{\dfrac{1}{\dfrac{\rho_0}{2} \cdot 1.689} \dfrac{W}{S}} \cdot C_{L\alpha} \approx \frac{U_{\infty[KEAS]} \cdot U_{gFTSEAS}}{498 \cdot \dfrac{W}{S}} \cdot C_{L\alpha}$$

Load factor is shown to be proportional to the A/C *Equivalent Airspeed in KEAS* and the *Gust Intensity expressed in ft/s EAS*

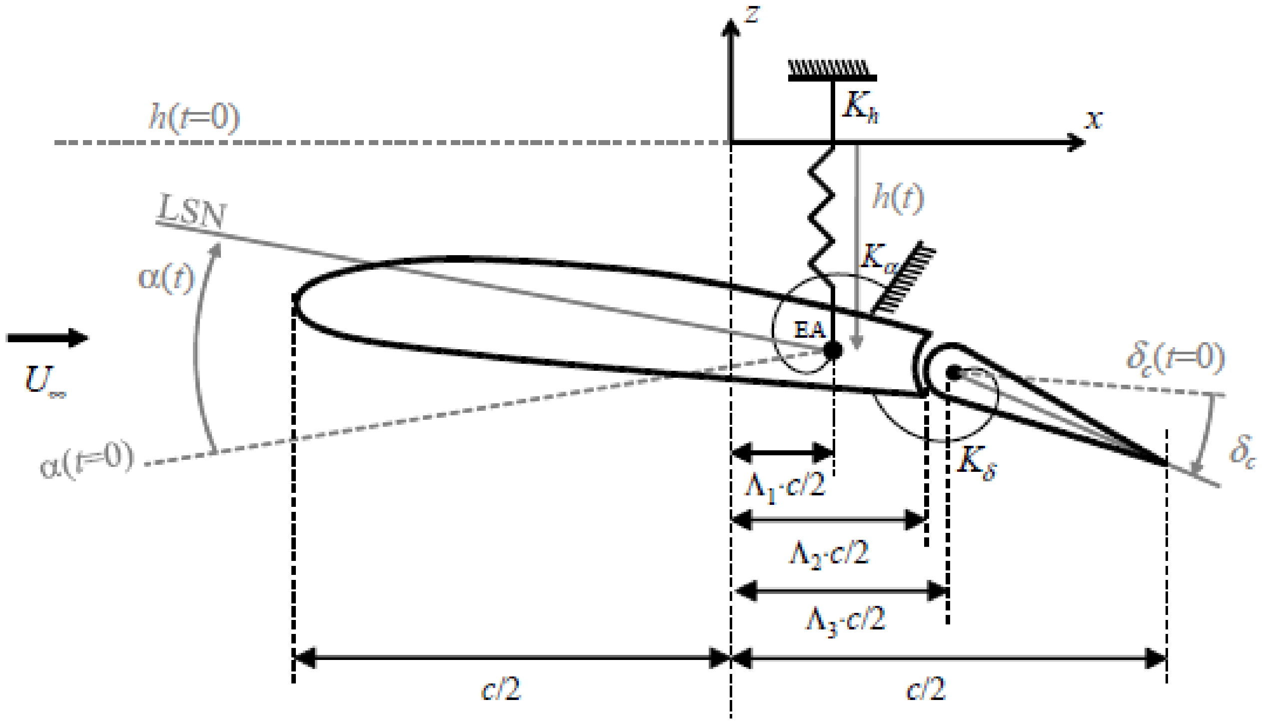
Gust Gradient H=12.5 MGC as
“representative for calculating Limit Load conditions”

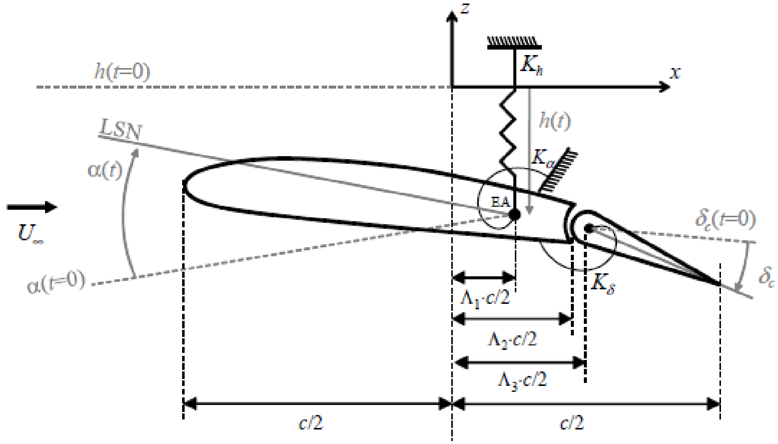
Gust Gradients of
Moderate/Severe Gust
Intensities are around 12.5
Mean Geometric Chord

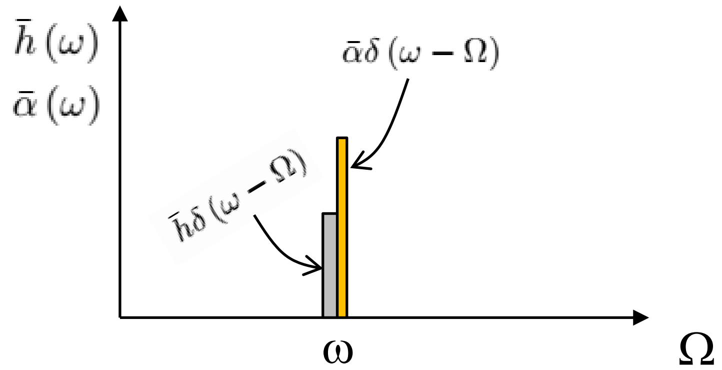


NOW… LET'S FIX CONCEPTS WITH OUR TYPICAL SECTION

THIS COURSE WILL BE FOCUSSED ONN "DYNAMIC RESPONSE". THE "DYNAMIC LOADS" IS OUT OF THE SCOPE ALTHOUGH, FOR MASOCHISTS, IT WILL BE INCLUDED IN SUBSEQUENT COURSES (MASTER)







Airfoil lift in the frequency-domain:

$$\frac{\tilde{Q}_h(k)}{q_\infty \frac{c}{2}} = -2\pi\left[\frac{t_0^2\ddot{h}}{c/2} + t_0\dot{\alpha} - \Lambda_1 t_0^2\ddot{\alpha}\right] - 4\pi C(k)\left[\frac{t_0\dot{h}}{c/2} + \alpha + \left(\frac{1}{2} - \Lambda_1\right)t_0\dot{\alpha}\right]$$

$$\frac{\tilde{Q}_h(\omega)}{q_\infty \frac{c}{2}} = -2\pi\left[t_0^2\frac{-\omega^2\tilde{h}}{c/2} + t_0 \mathrm{i}\omega\tilde{\alpha} - \Lambda_1 t_0^2\left(-\omega^2\tilde{\alpha}\right)\right] -$$

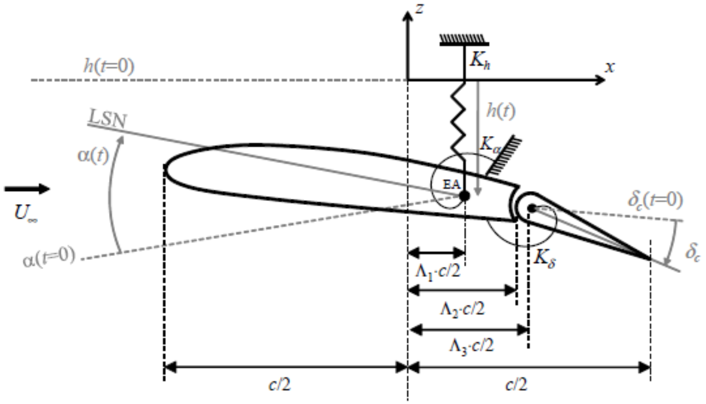
$$-4\pi C\left(\frac{\omega c}{2U_\infty}\right)\left[t_0\frac{\mathrm{i}\omega\tilde{h}}{c/2} + \alpha + \left(\frac{1}{2} - \Lambda_1\right)t_0\mathrm{i}\omega\tilde{\alpha}\right] = \tilde{Q}_h^{NC}(\omega) + \tilde{Q}_h^{C}(\omega)$$

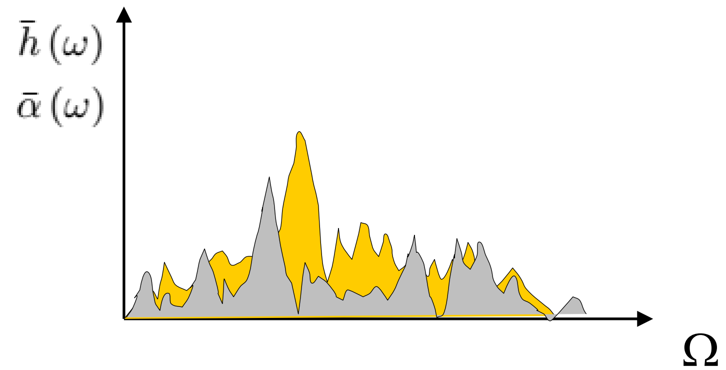
Time-domain force by inverse Fourier transform →

$$\frac{Q_h(t)}{q_\infty \frac{c}{2}} = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{Q}_h(\Omega)\,\mathrm{e}^{\mathrm{i}\Omega t}\,\delta(\omega - \Omega)\,\mathrm{d}\Omega$$

☝ The analysis is restricted to lift L although it can be extended to moments Q_α and Q_δ

2nd STEP: FROM OSCILLATORY MOTION δ(ω-Ω) TO ARBITRARY MOTION





Frequency-domain problem is formulated for motions with arbitrary Fourier transform

$$\bar{h}\delta(\omega-\Omega)\rightarrow\bar{h}(\omega)$$

$$\bar{\alpha}\delta(\omega-\Omega)\rightarrow\bar{\alpha}(\omega)$$

$$\frac{\tilde{Q}_h^{NC}(\omega)}{q_\infty\frac{c}{2}}=-2\pi\left[t_0^2\frac{-\omega^2\tilde{h}}{c/2}+t_0\mathrm{i}\omega\tilde{\alpha}-\Lambda_1t_0^2\left(-\omega^2\tilde{\alpha}\right)\right]$$

$$\frac{Q_h^{NC}(t)}{q_\infty\frac{c}{2}}=-2\pi\left[t_0^2\frac{\ddot{h}}{c/2}+t_0\dot{\alpha}-\Lambda_1t_0^2\ddot{\alpha}\right]$$

$$\frac{Q_h^C}{q_\infty\frac{c}{2}}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}-4\pi C\left(\frac{\Omega c}{2U_\infty}\right)\left\{t_0\frac{\mathrm{i}\omega\tilde{h}}{c/2}+\alpha+\left(\frac{1}{2}-\Lambda_1\right)t_0\mathrm{i}\omega\tilde{\alpha}\right\}\mathrm{e}^{\mathrm{i}\Omega t}\delta(\Omega-\omega)\,\mathrm{d}\Omega=$$

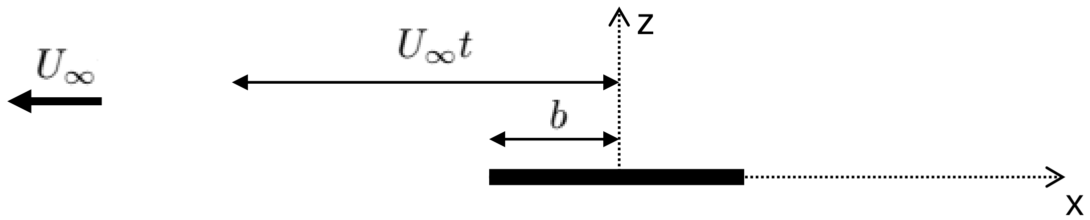
$$=4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty}C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)\,\delta(\Omega-\omega)}{U_\infty}\mathrm{e}^{\mathrm{i}\Omega t}\mathrm{d}\Omega\right],$$

Fourier transform of the instantaneous vertical velocity of the fluid particle in contact with the three-quarter chord point of the airfoil

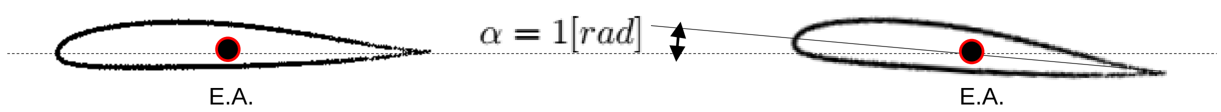
$$\frac{Q_h(t)}{q_\infty\frac{c}{2}}=-2\pi\left(t_0^2\frac{\ddot{h}}{c/2}+t_0\dot{\alpha}-t_0^2\Lambda_1\ddot{\alpha}\right)-4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty}C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)}{U_\infty}\mathrm{e}^{\mathrm{i}\Omega t}\mathrm{d}\Omega\right]$$

NON-CIRCULATORY TERMS

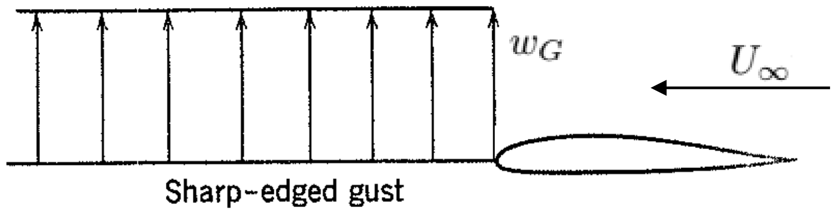
CIRCULATORY TERMS



- Non-dimensional time : $s = \dfrac{U_\infty t}{b}$
- Wagner's function $\Phi(s)$
  - Dimensionless lift development due to an impulsive change of AoA from 0 to 1 rad



- Küssner's function $\Psi(s)$
  - Dimensionless lift development due to a 1-rad AoA sharp-edged gust



$$\frac{w_G}{U_\infty} = 1$$

STEP CHANGE IN ANGLE OF ATTACK: WAGNER'S FUNCTION Φ

- Impulsive change of AoA from 0 to 1 [rad]:

$$\frac{Q_h(t)}{q_\infty \frac{c}{2}} = -2\pi\left(t_0^2\frac{\ddot{h}}{c/2} + t_0\dot{\alpha} - t_0^2\Lambda_1\ddot{\alpha}\right) - 4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty} C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)}{U_\infty}\mathrm{e}^{\mathrm{i}\Omega t}\mathrm{d}\Omega\right]$$

$$\frac{w_{\frac{3}{4}c}(t)}{U_\infty} = 0 \text{ if } t \leq 0$$

$$\frac{w_{\frac{3}{4}c}(t)}{U_\infty} = -1 \text{ if } t > 0$$

$$\frac{\bar{w}_{\frac{3}{4}c}(\omega)}{U_\infty} = \int_0^\infty (-1)\, e^{-i\omega t}dt = -\frac{1}{i\omega}$$

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{\bar{w}_{\frac{3}{4}c}(\omega)}{U_\infty}e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{e^{i\omega t}}{i\omega}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{e^{iks}}{ik}dk = \Phi(s)$$

$$\frac{Q_h(t)}{q_\infty \frac{c}{2}} = -2\pi\left(t_0^2\frac{\ddot{h}}{c/2} + t_0\dot{\alpha} - t_0^2\Lambda_1\ddot{\alpha}\right) - 4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty} C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)}{U_\infty}\mathrm{e}^{\mathrm{i}\Omega t}\mathrm{d}\Omega\right] = -4\pi\Phi(s)$$

- In many applications it is convenient to use Φ(s) itself when writing the circulatory lift and moment due to arbitrary motion.
- See approximations of Wagner's function in "Introducción a la Aeroelasticidad" (Garceta Editorial)
- For any other arbitrary motion:

$$\begin{aligned} Q_h(t) &= -\pi\rho_\infty\left(\frac{c}{2}\right)^2\left[\ddot{h} + U_\infty\dot{\alpha} - \frac{c}{2}\Lambda_1\ddot{\alpha}\right] \\ &\quad + 2\pi\rho_\infty U_\infty\frac{c}{2}\left[w_{\frac{3}{4}c}(0)\,\Phi(s) + \int_0^s \frac{\mathrm{d}w_{\frac{3}{4}c}}{\mathrm{d}\sigma}(\sigma)\,\Phi(s-\sigma)\,\mathrm{d}\sigma\right] \end{aligned}$$

LIFT DEVELOPMENT DUE TO A SHARP-EDGE GUST: KÜSSNER'S FUNCTION Ψ

- Lift due to 1-rad AoA sharp-edged gust (w_G / U_∞ = 1 [rad])

$$\frac{Q_h(t)}{q_\infty \frac{c}{2}} = -2\pi\left(t_0^2 \frac{\ddot{h}}{c/2} + t_0\dot{\alpha} - t_0^2\Lambda_1\ddot{\alpha}\right) - 4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty} C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)}{U_\infty} e^{i\Omega t} d\Omega\right]$$

$$h(t) = \alpha(t) = 0 \text{ if } t > 0$$

$$\frac{w_{\frac{3}{4}c}}{U_\infty} = 0 \text{ if } t \leq \frac{3b/2}{U_\infty}$$

$$\frac{w_{\frac{3}{4}c}}{U_\infty} = -\frac{w_G}{U_\infty} = -1 \text{ if } t > \frac{3b/2}{U_\infty}$$

$$\frac{\bar{w}_{\frac{3}{4}c}}{U_\infty} = \int_{\frac{3b/2}{U_\infty}}^{\infty}\left(-\frac{w_G}{U_\infty}\right)e^{-i\omega t}dt = -\frac{w_G}{U_\infty}\left.\frac{e^{-i\omega t}}{-i\omega}\right|_{\frac{3b/2}{U_\infty}}^{\infty} = -\frac{w_G}{U_\infty}\frac{e^{-i\omega\frac{3b}{2U_\infty}}}{i\omega} = -\frac{w_G}{U_\infty}\frac{e^{-i\frac{3}{2}k}}{i\omega}$$

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{\bar{w}_{\frac{3}{4}c}(\omega)}{U_\infty}e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{w_G}{U_\infty}\frac{e^{-i\frac{3}{2}k}}{i\omega}e^{i\omega t}d\omega = \frac{w_G}{U_\infty}\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} C(k)\frac{e^{ik\left(s-\frac{3}{2}\right)}}{ik}dk = \frac{w_G}{U_\infty}\Psi(s)$$

$$\frac{Q_h(t)}{q_\infty \frac{c}{2}} = -2\pi\left(t_0^2 \frac{\ddot{h}}{c/2} + t_0\dot{\alpha} - t_0^2\Lambda_1\ddot{\alpha}\right) - 4\pi\left[-\frac{1}{2\pi}\int_{-\infty}^{+\infty} C\left(\frac{\Omega c}{2U_\infty}\right)\frac{\tilde{w}_{\frac{3}{4}c}(\Omega)}{U_\infty} e^{i\Omega t} d\Omega\right] = -4\pi\Psi(s)$$

- If an arbitrary $w_G(s)$ is defined, Duhamel's integral gives the following "lift" (positive sign) associated to the gust:

$$\frac{L_G(s)}{q_\infty \frac{c}{2}} = 4\pi\int_0^s \hat{w}_G(s)\,\dot{\Psi}(s-\sigma)\,d\sigma.$$

- See approximations of Küssner's function in "Introducción a la Aeroelasticidad" (Garceta Editorial)

WAGNER vs KÜSSNER FUNCTIONS

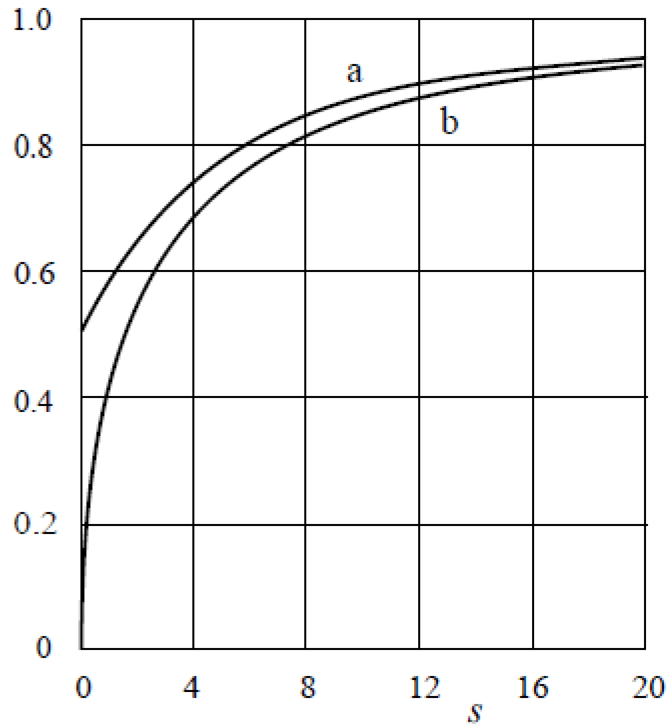
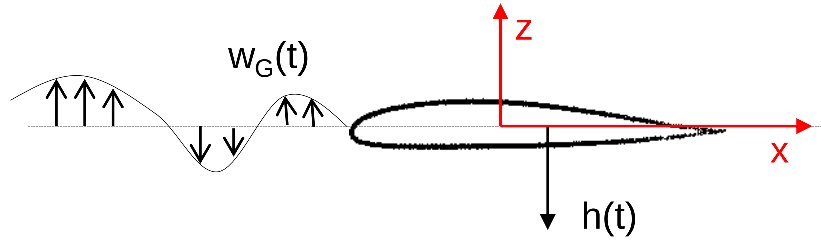


Figura 5.1. Función de Wagner $\Phi(s)$ (a) y función de Küssner $\Psi(s)$ (b) (adaptada de Bisplinghoff, 1996).



$$\frac{L_G(s)}{q_\infty \frac{c}{2}} = 4\pi \int_0^s \hat{w}_G(s)\,\dot{\Psi}(s-\sigma)\,\mathrm{d}\sigma$$

$$M\ddot{h}(t) = -L_M(t) - L_G(t) \qquad \left(\frac{2U_\infty}{c}\right)^2 M\ddot{h}(s) = -L_M - L_G$$

$$\frac{L_M(s)}{q_\infty \frac{c}{2}} = 2\pi \left[\frac{\ddot{h}(s)}{c/2} + 2\int_0^s \frac{\ddot{h}(\sigma)}{c/2}\Phi(s-\sigma)\,\mathrm{d}\sigma\right]$$

$$\left(\frac{2U_\infty}{c}\right)^2 M\ddot{h}(s) = -2\pi q_\infty \frac{c}{2}\left\{2\int_0^s \hat{w}_G(s)\,\dot{\Psi}(s-\sigma)\,\mathrm{d}\sigma + \right.$$
$$\left. + \frac{\ddot{h}(s)}{c/2} + 2\int_0^s \frac{\ddot{h}(\sigma)}{c/2}\Phi(s-\sigma)\,\mathrm{d}\sigma\right\},$$

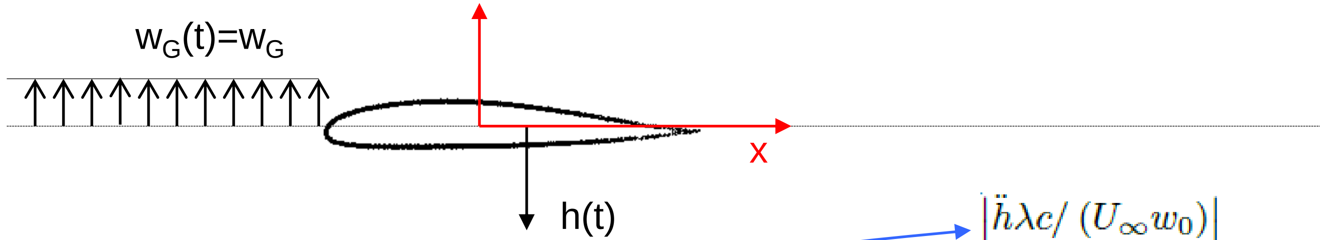
y dividiendo por $2\pi q_\infty$

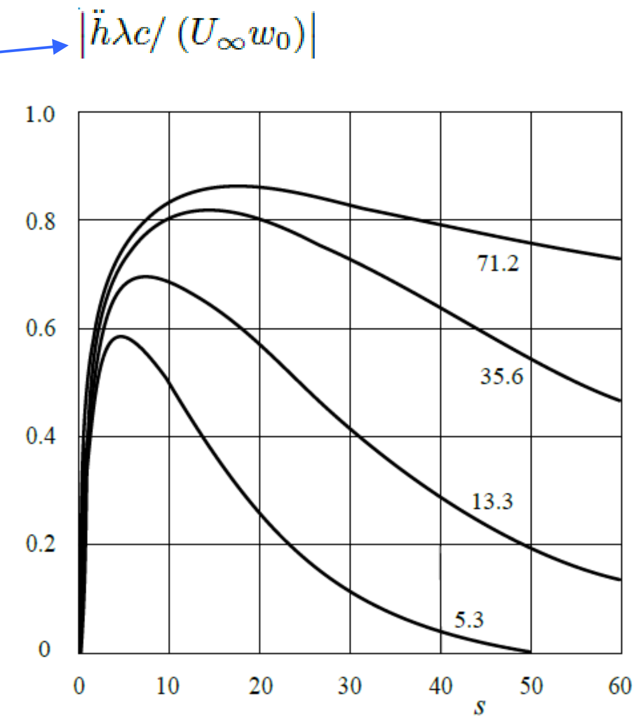
$$4\lambda\frac{\ddot{h}(s)}{c/2} + \frac{\ddot{h}(s)}{c/2} + 2\int_0^s \frac{\ddot{h}(\sigma)}{c/2}\Phi(s-\sigma)\,\mathrm{d}\sigma = -2\int_0^s \hat{w}_G(s)\,\dot{\Psi}(s-\sigma)\,\mathrm{d}\sigma. \qquad \lambda = M/\left(\pi\rho_\infty c^2\right)$$

$$4\lambda p^2\frac{\bar{h}(p)}{c/2} + p^2\frac{\bar{h}(p)}{c/2} + 2p^2\frac{\bar{h}(p)}{c/2}\bar{\Phi}(p) = -2\frac{\bar{w}_G}{U_\infty}p\bar{\Psi}(p)$$

$$\frac{\bar{h}(p)}{c/2} = -\frac{\dfrac{\bar{w}_G(p)}{U_\infty}\bar{\Psi}(p)}{2p\left[\lambda + \dfrac{1}{4} + \dfrac{1}{2}\bar{\Phi}(p)\right]}$$

2D AIRFOIL INCOMPRESSIBLE FLOW
h-MOTION DOF + GUST ENCOUNTERING





$$\frac{w_G(s)}{U_\infty} = \frac{w_G}{U_\infty} \rightarrow \mathcal{L}\{\frac{w_G}{U_\infty}\} = \frac{w_G}{U_\infty}\frac{1}{p}$$

$$\Psi(s) \approx 1 - \frac{1}{2}e^{-0{,}130s} - \frac{1}{2}e^{-s} \rightarrow \mathcal{L}\{\Psi\} = \frac{1}{p} - \frac{1}{2(p+0{,}130)} - \frac{1}{2(p+1)}$$

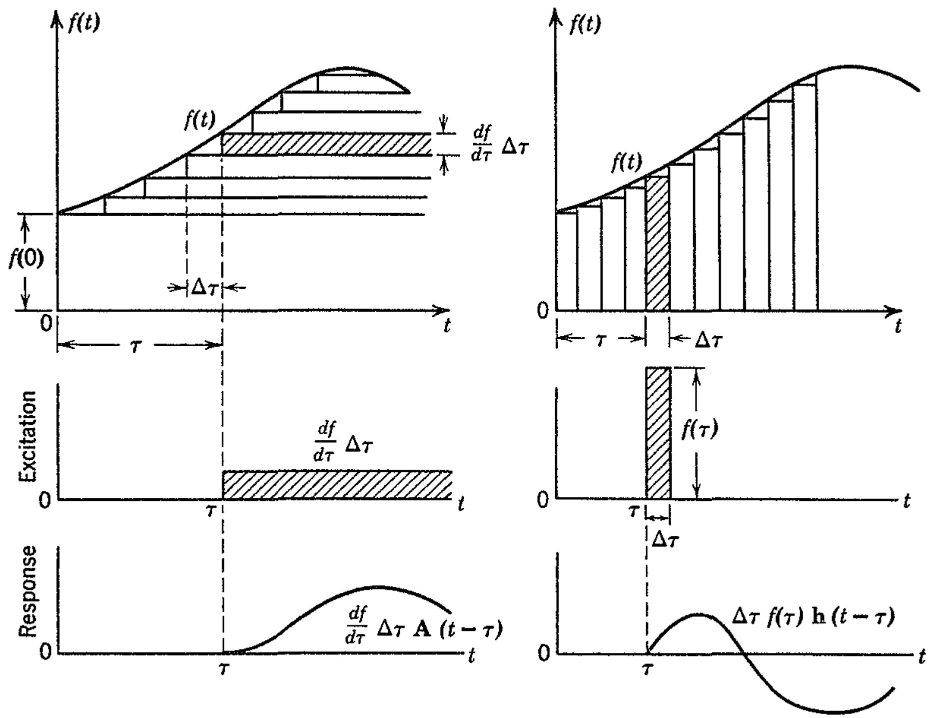
$$\Phi(s) \approx 1 - 0{,}165e^{-0{,}0455s} - 0{,}335e^{-0{,}300s} \rightarrow \mathcal{L}\{\Phi\} = \frac{1}{p} - \frac{0{,}165}{p+0{,}0455} - \frac{0{,}335}{p+0{,}3}$$

$$\frac{p^2\mathcal{L}\{h\}}{\frac{c}{4\lambda}\frac{w_G}{U_\infty}} = \frac{\mathcal{L}\{\Psi\}}{1+\frac{1}{4\lambda}+\frac{1}{2\lambda}\mathcal{L}\{\Phi\}} \Rightarrow \frac{\ddot{h}(s)}{\frac{c}{4\lambda}\frac{w_G}{U_\infty}} = \frac{\frac{d^2h}{dt^2}(s)\left(\frac{dt}{ds}\right)^2}{\frac{c}{4\lambda}\frac{w_G}{U_\infty}} = \frac{\frac{d^2h}{dt^2}(s)}{\frac{w_G U_\infty}{\lambda c}} = \mathcal{L}^{-1}\{\frac{\mathcal{L}\{\Psi\}}{1+\frac{1}{4\lambda}+\frac{1}{2\lambda}\mathcal{L}\{\Phi\}}\}$$

$$\mathcal{L}^{-1}\{\frac{\mathcal{L}\{\Psi\}}{1+\frac{1}{4\lambda}+\frac{1}{2\lambda}\mathcal{L}\{\Phi\}}\} = \mathcal{L}^{-1}\{\frac{\frac{1}{p} - \frac{1}{2(p+0{,}130)} - \frac{1}{2(p+1)}}{1+\frac{1}{4\lambda}+\frac{1}{2\lambda}\left(\frac{1}{p} - \frac{0{,}165}{p+0{,}0455} - \frac{0{,}335}{p+0{,}3}\right)}\}$$

DUHAMEL'S INTEGRAL
PRINCIPLE OF SUPERPOSITION

x(t) response of the physical system to the input **f(t)**



A(t)

"Indicial Admittance" = response to unit-step function

$$x(t) = f(0)\,A(t) + \int_0^t \frac{df}{dt}(\tau)\,A(t-\tau)\,d\tau$$

h(t)

response to unit-step function

$$x(t) = \int_0^t f(\tau)\,h(t-\tau)\,d\tau$$

$$h(t) = A(0)\,\delta(t) + \frac{dA}{dt}(t)$$

LAPLACE TRANSFORM
CONVOLUTION THEOREM

$$\mathcal{L}\left\{f\left(t\right)\right\} = \int_0^\infty f(t)e^{-st}dt$$

$$\mathcal{L}^{-1}\{F(s)G(s)\} = (f * g)(t)$$

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - x)g(x)\mathrm{d}x$$

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