FINANCIAL MATHEMATICS

Course: 2018/ 2019

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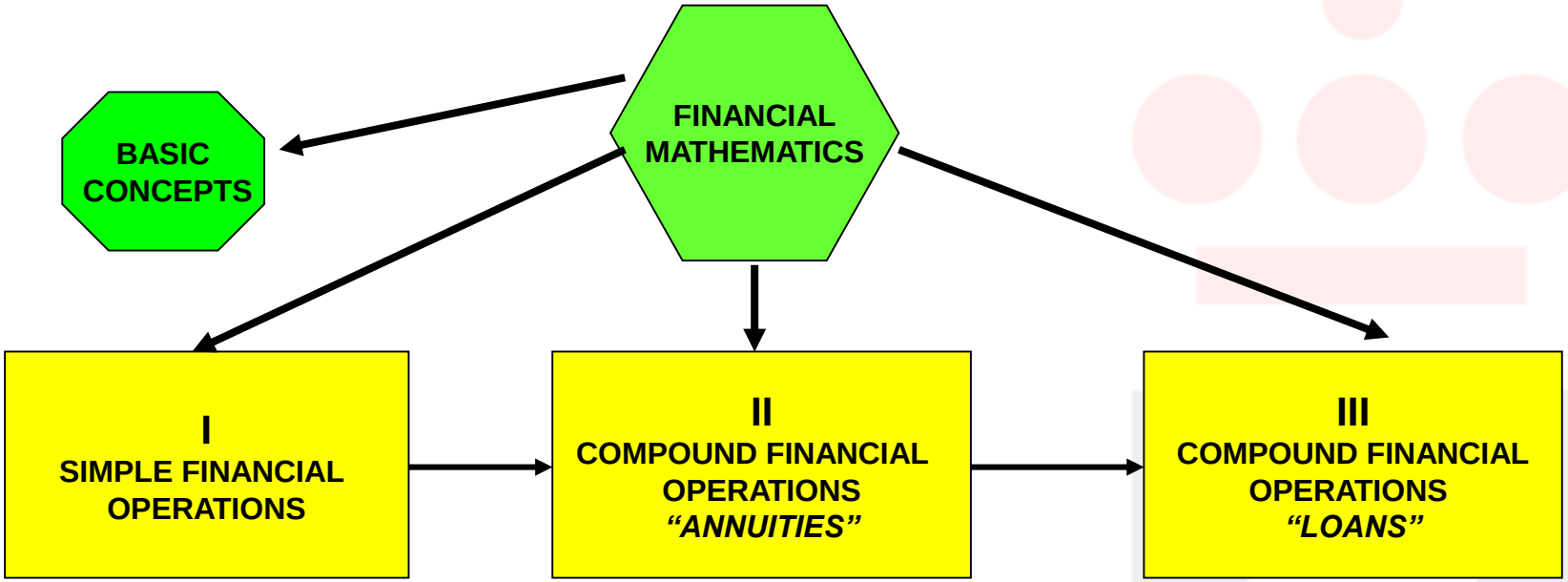
COURSE DIAGRAM



BASIC CONCEPTS

- FINANCIAL CAPITAL
- FINANCIAL OPERATION
- FINANCIAL LAW

FINANCIAL PHENOMENON

- ***The time factor plays an important role when fixing the value of capital. It is not the same to have 6,000 € available today than within 1 year as the money will depreciate due to inflation.***

- ***Therefore , 6,000 € in this moment will be equivalent to 6,000€ plus an additional amount within 1 year***

- ***This additional amount is what compensates the loss of value suffered by the money during this period.***

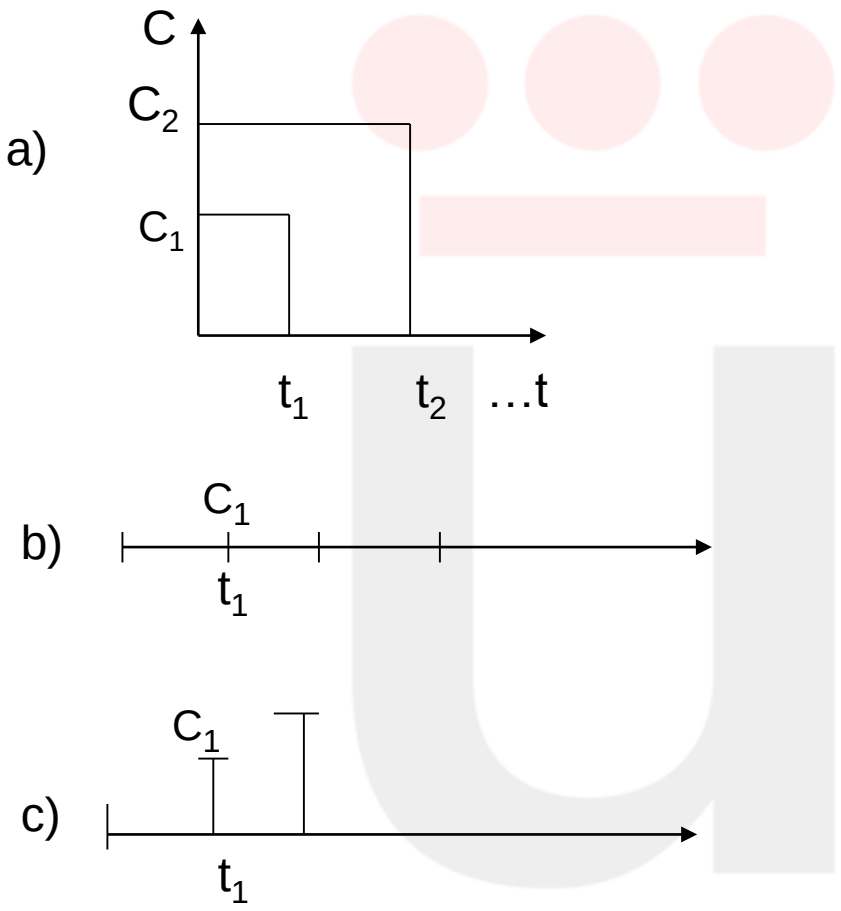
FINANCIAL CAPITAL

- ***Financial capital is represented by the pair (C , t), where "C" represents a positive quantity expressed in monetary units where "t" represents the moment of availability or the maturity , expressed in concrete dates (days, months, years, ...)***

  C: Quantity

  t: Maturity

- **WAYS OF REPRESENTING FINANCIAL CAPITAL**



TIME VALUE OF MONEY

- Two basic rules:

1. *Regarding two capitals of the same quantity in different moments, the one closest in time is preferred*

   PRINCIPLE OF UNDER ESTIMATION OF FUTURE NECESSITIES OR PREFERENCES FOR LIQUIDITY

   Maturity is an economic asset, however, it is negative

2. *Regarding two capitals in the same moment, but of different quantity the highest quantity is preferred*

   Quantity is a positive economic asset

FINANCIAL OPERATIONS

- **All exchange of financial capitals available in different moments**
- **(every exchange not simultaneous between capitals)**
- **(Cash purchases: financial operations do not exist, however, instalment purchases do)**
-

- In reality, all financial operations are loans. Someone – the lender- gives a quantity of money as a loan to a borrower on the condition that the loan is returned including a predetermined amount of interest

- ***ELEMENTS OF A FINANCIAL OPERATION***
- **ORIGIN:** coincides with maturity (moment of availability) of the first capital.
- **End :** coincides with the maturity of the last capital.
- **DURATION:** time between the origin and the end.
- **LENDER:** a **person** or entity which hands over or lends first capital.
- **DEBTOR :** a person or entity which receives capital.

FINANCIAL OPERATIONS

Loan amount :

A group of financial capitals given by the inital lender

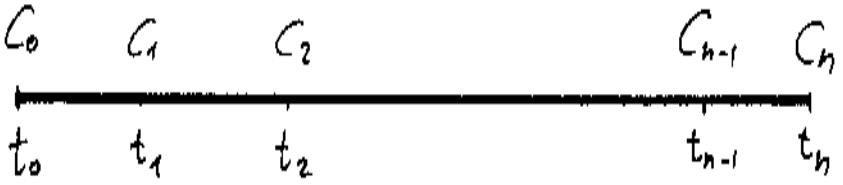
$$P \equiv \{(C_0, t_0), (C_1, t_1), (C_2, t_2), ..., (C_n, t_n)\}$$

The second part of the simple loan:

A group of financial capitals returned by the initial debtor

$$CP \equiv \{(C_0', t_0'), (C_1', t_1'), (C_2', t_2'), ..., (C_m', t_m')\}$$

(P)



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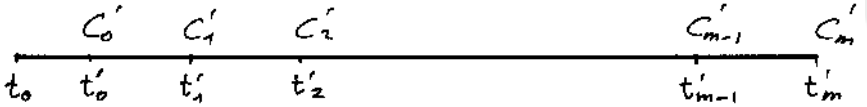


Simple Financial Operation

- A financial operation is considered to be simple only if two capitals are exchanged
- Both parts are formed by one single capital

$$(\mathrm{P})\ \{(C_0, 0)\}$$

$$(\mathrm{CP})\ \{(C_n, n)\}$$

FINANCIAL LAW

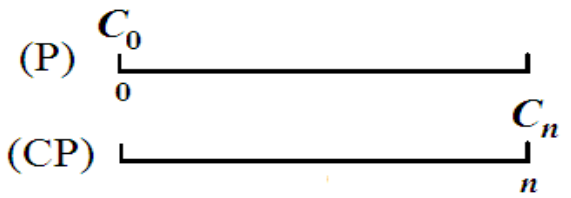
- **All financial operations implies an inherent agreement between both parts regarding the capitals to be exchanged. In the case of simple financial operations, the agreement must have a relationship with the initial and final capital. This may be expressed mathematically by means of a Function.**

CLASIC FINANCIAL LAWS

- SIMPLE INTEREST LAW
- COMPOUND INTEREST LAW

- DISCOUNTING
  - SIMPLE
    - COMMERCIAL
    - RATIONAL OR MATHEMATICAL DISCOUNTING
  - COMPOUND

Financial Notation



Elements to consider:

C_0 : The principal, or initial capital at time point "0", or present value of Cn. (At time point 0)

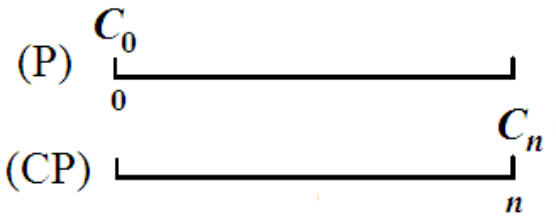
C_n : The amount, or accumulated value of Co, or future value of Co , or the maturity value of Co **(** At time point "n")

n : Duration of the operation from 0 to n (time)

i = simple interest rate

$I_{(0, n)}$: = C_n - C_0 : Total Interest expressed in monetary unit

Simple Interest



The interest is proportional to the invested capital (C_0) and to the duration of the operation (n)

At simple interest, the interest is computed on the principal the whole time

Calculating the final capital:

$$C_n = C_0 + C_0 \cdot i \cdot n$$

$$I_{0,n} = C_n - C_0 = i \cdot C_0 \cdot n$$

$$C_n = C_0 (1 + i \cdot n)$$

This equation is known as the Simple Interest Law

i: Simple Interest Rate

Calculating “i” from the previous equation

$$C_n = C_0\left(1+i\cdot n\right) \quad \Rightarrow i\cdot n = \frac{C_n}{C_0} - 1 \Rightarrow$$

$$i = \frac{C_n - C_0}{C_0 n}$$

i means the ratio of interest produced by each monetary unit in one time unit on the principal.

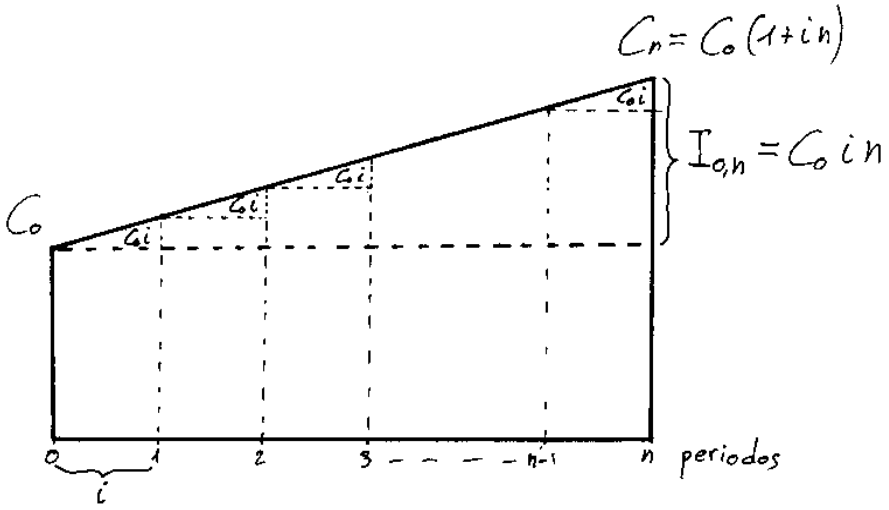
i is expressed as a percentage

NOTE: A temporary correspondence must exist between n and i , as both must be expressed using the same units (time)

FUTURE VALUE AT SIMPLE INTEREST

$$C_1 = C_0 + C_0 \cdot i$$
$$C_2 = C_0 + C_0 \cdot i \cdot 2$$
$$C_3 = C_0 + C_0 \cdot i \cdot 3$$
$$\dots$$
$$C_n = C_0 + C_0 \cdot i \cdot n$$

$$C_1 - C_0 = C_0 \cdot i$$
$$C_2 - C_1 = C_0 \cdot i$$
$$C_3 - C_2 = C_0 \cdot i$$
$$\dots$$
$$C_n - C_{n-1} = C_0 \cdot i$$



Simple Interest: :
Interest paid or earned on only the principal or initial amount or principal, borrowed or lent

With simple interest, the key assumption is that you withdraw the interest from your bank account as soon as it is paid (and presumably spent it). Therefore the interest of each period is constant and equal to Co i

Simple Equivalent Interest Rates

Equivalent Interest Rates

Two types of interest rates are said to be equivalent or indifferent using whichever chosen, they will produce the same final value investing the same amount of money for the same period of time

SIMPLE EQUIVALENT INTEREST RATE

In Simple Interest the equivalent interest rates are proportional

$$i = i_m.m \quad \Rightarrow \quad i_m = \frac{i}{m}$$

i: Simple Interest Rate

The Simple Interest Law is usually applied to a short term operations.
In finance practice "i " is called Annual Interest Rate. When the time is given in days, there are two different varieties of simple interest in use:

Exact interest: The fraction of the year is expressed by $t = k/365$, where k is the number of days that the capital remains invested and "i" is the interest rate applied.
The year is taken as 365 days.

Ordinary interest: The fraction of the year is expressed by $t = k/360$
The year is taken as 360 days.