PROPERTIES of the DFT

The index-domain signal is $x[n]$ for $n = 0, 1, 2, \ldots, N-1$; and the frequency domain values are $X[k]$ for $k = 0, 1, 2, \ldots, N-1$. Outside the range $[0, N-1]$, the values of $x[n]$ and $X[k]$ are periodic.

$$\text{Analysis Equation:} \qquad X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk} \tag{1}$$

$$\text{Synthesis Equation:} \qquad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, W_N^{-nk} \tag{2}$$

$$\text{Exponential:} \qquad W_N = e^{-j2\pi/N} \tag{3}$$

N-point signal: $x[n]$, $n = 0, 1, \ldots, N-1$	N-point DFT: $X[k]$, $k = 0, 1, \ldots, N-1$
$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$ (Linearity)
$x[n] = W_N^{-\ell_\circ n} = e^{+j2\pi\ell_\circ n/N}$	$X[k] = N\,\delta[(k-\ell_\circ) \bmod N]$
$y[n] = X[k]\,\vert_{k \leftarrow n}$	$Y[k] = N \cdot x[(-k) \bmod N]$ (Duality)
$y[n] = \frac{1}{N} \cdot X[(-n) \bmod N]$	$Y[k] = x[n]\,\vert_{n \leftarrow k}$ (Duality)
$x[(n-n_o) \bmod N]$	$W_N^{n_o k}\, X[k]$
$W_N^{-\ell_\circ n}\, x[n]$	$X[(k-\ell_\circ) \bmod N]$
$x[n] \circledN h[n] = \sum_{\ell=0}^{N-1} x[\ell] h[(n-\ell) \bmod N]$	$X[k] \cdot H[k]$
$x[n]\, w[n]$ (windowing)	$\frac{1}{N} \sum_{\ell=0}^{N-1} X[\ell] W[(k-\ell) \bmod N]$
$x^*[n]$	$X^*[(-k) \bmod N]$
$x^*[(-n) \bmod N]$	$X^*[k]$
$\Re e\{x[n]\}$	$X_{\text{cs}}[k] = \frac{1}{2}\{X[k \bmod N] + X^*[(-k) \bmod N]\}$
$j\,\Im m\{x[n]\}$	$X_{\text{cas}}[k] = \frac{1}{2}\{X[k \bmod N] - X^*[(-k) \bmod N]\}$
$x_{\text{cs}}[n] = \frac{1}{2}\{x[n \bmod N] + x^*[(-n) \bmod N]\}$	$\Re e\{X[k]\}$ (cs = conjugate-sym)
$x_{\text{cas}}[n] = \frac{1}{2}\{x[n \bmod N] - x^*[(-n) \bmod N]\}$	$j\,\Im m\{X[k]\}$ (cas = conj-anti-sym)
Parseval's Theorem: $N \sum_{n=0}^{N-1} \vert x[n]\vert^2 = \sum_{k=0}^{N-1} \vert X[k]\vert^2$	
The following properties apply when $x[n]$ is purely real:	
Conjugate Symmetry	$X[k] = X^*[(-k) \bmod N]$
Real part of $X[k]$ is even	$\Re e\{X[k]\} = \Re e\{X[(-k) \bmod N]\}$
Imaginary part of $X[k]$ is odd	$\Im m\{X[k]\} = -\Im m\{X[(-k) \bmod N]\}$
$\vert X[k]\vert$ (Magnitude) is even	
$\arg\{X[k]\}$ (Phase) is odd	