AM - Prácticas 30-10-2020

6.1. $g(x) = \left\{ \begin{array}{ll} \frac{e^x-1}{x} & \text{si } x \neq 0 \\ 1 & \text{si } x = 0 \end{array} \right\}$

$\lim_{x\to 0} \frac{e^x-1}{x} = \lim_{x\to 0} \frac{e^x}{1} = 1 = g(0)$ Continua en $\mathbb{R}$

$g'(x) = \frac{e^x x - (e^x-1)}{x^2} = \frac{xe^x - e^x + 1}{x^2}$ si $x \neq 0$

$g'(0) = \lim_{h\to 0} \frac{g(h)-g(0)}{h} = \lim_{h\to 0} \frac{\frac{e^h-1}{h} - 1}{h} = \lim_{h\to 0} \frac{e^h-1-h}{h^2}$

$= \lim_{h\to 0} \frac{e^h-1}{2h} = \lim_{h\to 0} \frac{e^h}{2} = \frac{1}{2}$

$\lim_{x\to 0} g'(x) = \lim_{x\to 0} \frac{xe^x - e^x + 1}{x^2} = \lim_{x\to 0} \frac{\cancel{e^x} + xe^x - \cancel{e^x}}{2x} =$

$= \lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2} = g'(0) \Rightarrow g \in C^1(\mathbb{R})$

$\left(e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow \frac{e^x-1}{x} = \frac{1}{x}\left(\cancel{1} + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots - \cancel{1}\right)\right.$

$\left. = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} \right)$ convergente en todo $\mathbb{R}$

$\left(\sum_{n=1}^{\infty} f_n(x)\right)' \overset{?}{=} \sum_{n=1}^{\infty} f_n'(x)$ — x —

$x^2(e^x-1) - y^3 = 0$ Despejar $y(x)$ en $(0,0)$

Derivar con respecto a y: $-3y^2\big|_{(0,0)} = 0$

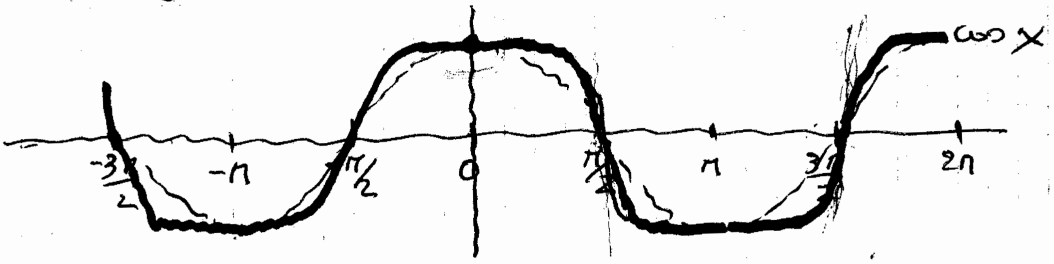
$y^3 = x^2(e^x-1) \Rightarrow y = x^{2/3}(e^x-1)^{1/3} = x\left(\frac{e^x-1}{x}\right)^{1/3}$

$f(x) = x\, g(x)^{1/3}$, $f(0) = 0 \cdot 1^{1/3} = 0$.

Comprobar que f' existe en todo x y es continua en $\mathbb{R}$.

b) $xy + \cos z = 1$. Tomar x, y cerca de $(0,0)$ con $xy < 0$. Entonces $\cos z = 1 - xy > 1$. Imposible.

c) $F = \cos x - y^3 = 0 \Rightarrow y = (\cos x)^{1/3} = f(x)$



$$\frac{\partial F}{\partial y} = -3y^2 \Rightarrow \frac{\partial F}{\partial y}(0,1) = -3 \neq 0 \quad \text{TF Implícita}$$

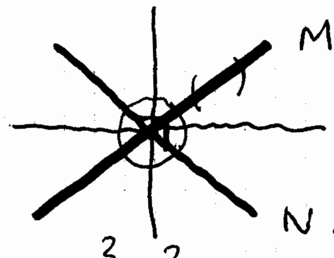
$$x_k = \frac{\pi}{2} + 2k\pi$$

$$f'(x_k) = \frac{1}{3}(\cos x_k)^{-2/3}(-\operatorname{sen} x_k) = \frac{-\frac{1}{3}\operatorname{sen} x_k}{(\cos x_k)^{2/3}} = -\infty$$

$$\lim_{h \to 0} \frac{f(x_k+h) - f(x_k)}{h} = -\infty$$

——— x ———

6.3. $M \cup N$, $M \cap N$ ¿subvariedades?

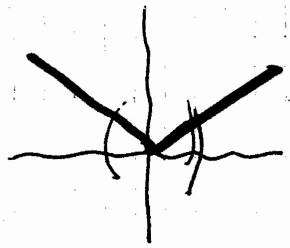


M $\quad f(x,y) = x^2 - y^2 \qquad F(x,y,z) = z - (x^2 - y^2)$

N $\quad z = 0$

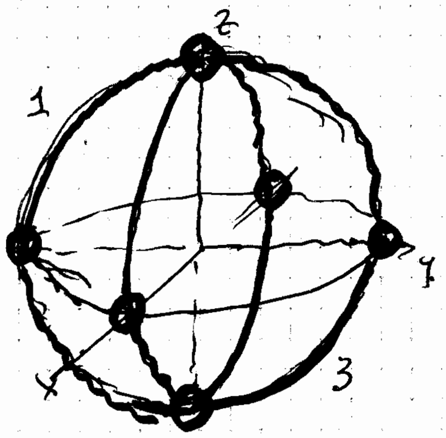
$M \cap N$ no es subvariedad.

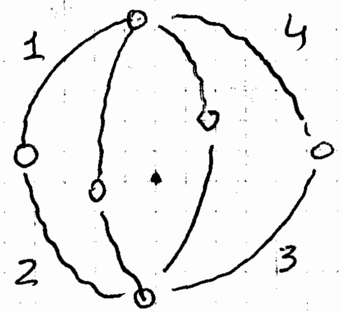
Clase $\quad x^2 - y^2 = 0,\ y \geq 0$



4. $M = \{(x,y,z)\in\mathbb{R}^3 : xy=0,\ x^2+y^2+z^2=1,\ z\neq 0,\pm 1\}$ (3)

$(x=0$ ó $y=0)$





$F(x,y,z) = (xy,\ x^2+y^2+z^2-1)$

Curva 1 $\quad F(x,y,z) = (x,\ y^2+z^2-1)$

Si $(x,y,z)\in$ Curva 1, $\quad F(x,y,z) = (0,0)$

y $DF(x,y,z)$ tiene que tener rango 2 en la curva 1

$$DF(x,y,z) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2y & 2z \end{pmatrix}$$

Si tuviera rango 1, $\begin{vmatrix} 1 & 0 \\ 0 & 2y \end{vmatrix} = 0 \iff y=0$ y

$\begin{vmatrix} 1 & 0 \\ 0 & 2z \end{vmatrix} = 0 \Rightarrow z=0$ y $(0,0)\notin$ curva 1.

Tiene rango 2.

Con $F(x,y,z) = (xy,\ x^2+y^2+z^2-1)$ se cumple $M = F^{-1}(\{(0,0)\})$

y $DF(x,y,z) = \begin{pmatrix} y & x & 0 \\ 2x & 2y & 2z \end{pmatrix}$ - x e y no pueden ser cero a la vez pq. si lo fueran $z^2-1=0 \iff z=\pm 1$. Luego rango $DF \geq 1$.

Si fuera 1, $\begin{vmatrix} y & 0 \\ 2x & 2z \end{vmatrix} = 0$ y $\begin{vmatrix} x & 0 \\ 2y & 2z \end{vmatrix} = 0 \iff yz=0$ y $xz=0$.

Como $z\neq 0$ tendríamos $y=0,\ x=0$ y $(x,y,z) = (0,0,z)$ no sería un punto de M.
