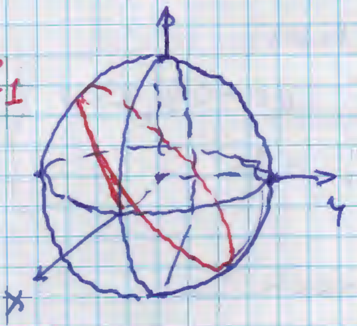
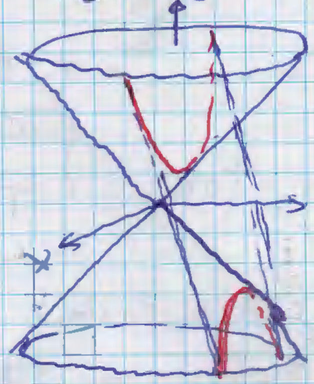
AM Prácticas 20/11/2020

6.7

$$C_1 : \begin{cases} x^2+y^2+z^2=1 \\ x+y+z=0 \end{cases} \qquad C_2 : \begin{cases} x^2+y^2=z^2 \\ x+y+z=1 \end{cases}$$

a) C_1





C_2

b) $F(x,y,z) = (x^2+y^2+z^2-1,\ x+y+z)$

$G(x,y,z) = (x^2+y^2-z^2,\ x+y+z-1)$

$$DG(x,y,z) = \begin{pmatrix} 2x & 2y & -2z \\ 1 & 1 & 1 \end{pmatrix}$$

c) Plano Normal a C_2 en $b' = (\frac{1}{2}, 0, \frac{1}{2})$

$DG(\frac{1}{2}, 0, \frac{1}{2}) = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, $\operatorname{Ker} DG(b) : \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{cases} x - z = 0 \\ x + y + z = 0 \end{cases}$ $T_{b'}(C_2) = \left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\rangle$, $N_{b'}(C_2) = \{x - 2y + z = 0\}$

Plano normal : $x - 2y + z = c$ con $\frac{1}{2} - 0 + \frac{1}{2} = c$

$$\boxed{x + 2y + z = 1}$$

d) Parametrizaciones de C_1 y C_2

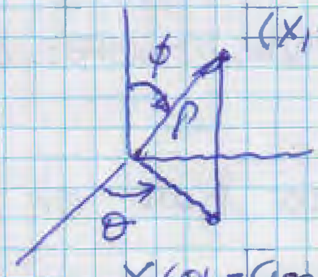
$x = \rho \cos\theta \operatorname{sen}\phi$

$y = \rho \operatorname{sen}\theta \operatorname{sen}\phi$

$z = \rho \cos\phi$

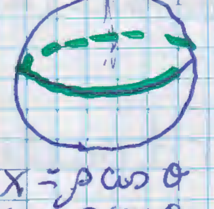
$0 \le \rho < \infty$

$0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$



$z = 0$

$x^2 + y^2 = \rho^2$



$x = \rho\cos\theta$

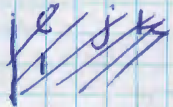
$y = \rho \operatorname{sen}\theta$

$z = 0$

$X(\theta) = (\cos\theta, \operatorname{sen}\theta, 0)$

$\vec{u}_3 = \frac{1}{\sqrt{3}}(1,1,1)$, $\vec{u}_2 = (1,-1,0)\frac{1}{\sqrt{2}}$, $\vec{u}_1 = \vec{u}_3 \times \vec{u}_2$

$= \frac{1}{\sqrt{6}}(+1,+1,-2)$ $\quad \mathcal{R} = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ $\quad X \sim (x', y', z')$



En el s. de ref $\mathcal{R}$ la ecuación de C_1 es:

$$\begin{cases} x' = \cos\theta \\ y' = \operatorname{sen}\theta \\ z' = 0 \end{cases} \qquad {}^{\mathcal{R}}X(\theta) = (\cos\theta, \operatorname{sen}\theta, 0) \quad 0 \le \theta \le 2\pi$$

Cambio de base

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} & & \end{pmatrix} \begin{pmatrix} \cos\theta \\ \operatorname{sen}\theta \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{6}}\cos\theta + \frac{1}{\sqrt{2}}\operatorname{sen}\theta \\ \frac{1}{\sqrt{6}}\cos\theta - \frac{1}{\sqrt{2}}\operatorname{sen}\theta \\ -\frac{2}{\sqrt{6}}\cos\theta \end{pmatrix}$$

$0 \le \theta \le 2\pi$ $\qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

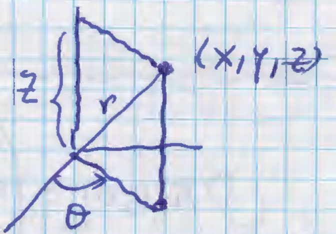
C_2

$x = r\cos\theta$

$y = r\operatorname{sen}\theta$

$z = z \in \mathbb{R}$

$0 \le \theta \le 2\pi$; $r > 0$



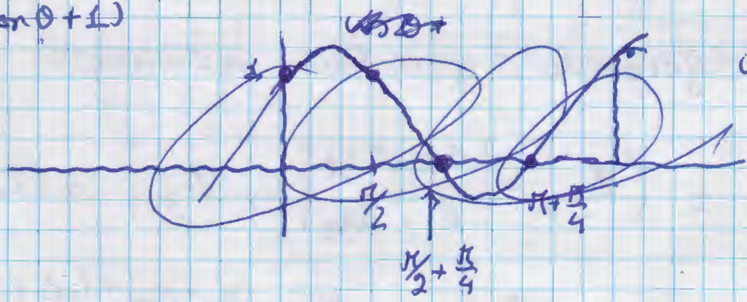
$$C_2: \begin{cases} x^2 + y^2 = z^2 \\ x + y + z = 1 \end{cases}$$

$r^2 = z^2 \Rightarrow z = \pm r$

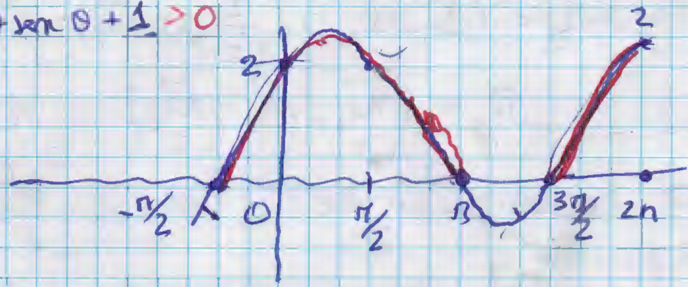
- $z = r > 0$, $x + y + z = 1 \Rightarrow r\cos\theta + r\operatorname{sen}\theta + r = 1 \Rightarrow$

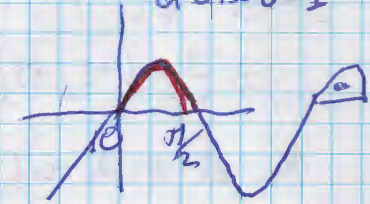
$r = \frac{1}{(\cos\theta + \operatorname{sen}\theta + 1)} > 0 \iff \cos\theta + \operatorname{sen}\theta + 1 > 0$

$\cos\theta + \operatorname{sen}\theta$



$f(\theta) = \cos\theta + \operatorname{sen}\theta + 1 > 0$





$-\frac{\pi}{2} < \theta \leq \pi$

$$\varphi(\theta) = \left(\frac{\cos\theta}{\cos\theta + \operatorname{sen}\theta + 1}, \frac{\operatorname{sen}\theta}{\cos\theta + \operatorname{sen}\theta + 1}, \frac{1}{\cos\theta + \operatorname{sen}\theta + 1} \right)$$

- $z = -r$ $\quad x + y + z = 1$ $\quad r = \dfrac{1}{\cos\theta + \operatorname{sen}\theta - 1} > 0$ y

tiene que ser $\cos\theta + \operatorname{sen}\theta - 1 \geq 0 \Rightarrow 0 < \theta < \frac{\pi}{2}$

$$\varphi(\theta) = \left(\frac{\cos\theta}{\cos\theta + \operatorname{sen}\theta - 1}, \frac{\operatorname{sen}\theta}{\cos\theta + \operatorname{sen}\theta - 1}, \frac{1}{\cos\theta + \operatorname{sen}\theta - 1} \right)$$

$$0 \leq \theta < \frac{\pi}{2}$$

6.9. $X(0,0) = (x_0, y_0, z_0)$, $\quad DX(0,0) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$

$$N_{(x_0,y_0,z_0)}(S) \equiv \frac{x - x_0}{\underbrace{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}_{\Delta_1}} = \frac{y - y_0}{\underbrace{\begin{vmatrix} a_{31} & a_{11} \\ a_{32} & a_{12} \end{vmatrix}}_{\Delta_2}} = \frac{z - z_0}{\underbrace{\begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}}_{\Delta_3}}$$

S/ $\quad T_{(x_0,y_0,z_0)}(S) = \operatorname{Img} DX(0,0) = \left\langle \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} \right\rangle$

Espacio normal generado = producto vectorial

$$\begin{vmatrix} i & j & k \\ a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{vmatrix} = \left(\Delta_1, -\begin{vmatrix} a_{11} & a_{31} \\ a_{12} & a_{32} \end{vmatrix}, \Delta_3 \right) = (\Delta_1, \Delta_2, \Delta_3)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + t \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} \quad \Leftarrow \quad \begin{aligned} x &= x_0 + t\Delta_1 \\ y &= y_0 + t\Delta_2 \\ z &= z_0 + t\Delta_3 \end{aligned}$$

7.3. $X(u,v) = \left(\frac{2u}{1+u^2+v^2}, \frac{2v}{1+u^2+v^2}, \frac{u^2+v^2-1}{1+u^2+v^2} \right)$

a)

$$\|X(u,v)\|^2 = \frac{1}{(1+u^2+v^2)^2}\left(4u^2+4v^2+(u^2+v^2-1)^2\right) = 1$$

$$DX(u,v) = \frac{1}{(1+u^2+v^2)^2}\begin{pmatrix} 2(1+u^2+v^2)-4u^2 & -(2u)(2v) \\ -2v(2u) & 2(1+u^2+v^2)-4v^2 \\ 4u & 4v \end{pmatrix}$$

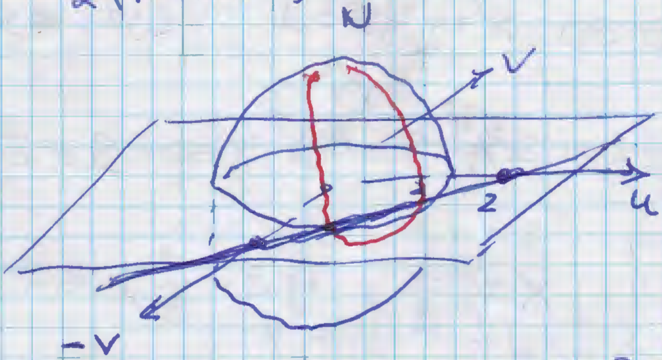
$$1 - \frac{2}{1+u^2+v^2} \qquad 2(1-u^2+v^2)$$

b)

$3v = u - 2$

$v=0 \Rightarrow u=2$

$u=0,\ v=-\frac{2}{3}$



c) $T_a(\Gamma)$ en $a = \left(\frac{10}{27}, \frac{2}{27}, \frac{25}{27}\right)$ $\quad v = \frac{u-2}{3}$

$$\frac{2u}{1+u^2+v^2} = \frac{10}{27} \quad (u,v)=(5,1) \quad , \quad \frac{2v}{1+u^2+v^2} = \frac{2}{27}$$

$(u=5,\ v=1)$

$$D\gamma(5) = \frac{1}{27}\begin{pmatrix} 66 \\ -6 \\ 24 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 22 \\ -2 \\ 8 \end{pmatrix} \qquad T_a(\Gamma) = \left\langle \frac{1}{9}\begin{pmatrix} 22 \\ -2 \\ 8 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10/27 \\ 2/27 \\ 25/27 \end{pmatrix} + \frac{t}{9}\begin{pmatrix} 22 \\ -2 \\ 8 \end{pmatrix} \qquad \frac{x-\frac{10}{27}}{11} = \frac{y-\frac{2}{27}}{-1} = \frac{z-\frac{25}{27}}{4}$$

7.5. $f(x,y,z) = x-2y+2z$ en $x^2+y^2+z^2=1$

S/ $F(x,y,z,\lambda) = x-2y+2z-\lambda(x^2+y^2+z^2-1)$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 1-2x\lambda=0 \\ \frac{\partial F}{\partial y} = -2-2y\lambda=0 \\ \frac{\partial F}{\partial z} = 2-2z\lambda=0 \end{array}\right\} \quad \lambda \neq 0 \quad x=\frac{1}{2\lambda},\ y=-\frac{1}{\lambda},\ z=\frac{1}{\lambda}$$

$$\Downarrow$$

$$\frac{\partial F}{\partial \lambda}=0 \Leftrightarrow x^2+y^2+z^2=1 \qquad \frac{1}{4\lambda^2}+\frac{1}{\cancel{4}\lambda^2}+\frac{1}{\cancel{4}\lambda^2}=1 \Rightarrow$$

$$\frac{9}{4\lambda^2}=1 \Rightarrow 9=4\lambda^2 \Rightarrow \lambda=\pm\frac{3}{2}$$

$\boxed{\lambda=\frac{3}{2}}$ $\quad x=\frac{1}{3},\ y=-\frac{2}{3},\ z=\frac{2}{3} \leftarrow A$

$\boxed{\lambda=-\frac{3}{2}}$ $\quad x=-\frac{1}{3},\ y=\frac{2}{3},\ z=-\frac{2}{3} \leftarrow B$

$f(A) = \frac{1}{3}-2\left(-\frac{2}{3}\right)+2\left(\frac{2}{3}\right)=3 \leftarrow$ Max

$f(B) = -\frac{1}{3}-2\left(\frac{2}{3}\right)+2\left(-\frac{2}{3}\right)=-3 \leftarrow$ Min

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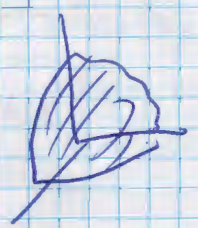
7.7. $f(x,y,z)=\log x+\log y+3\log z$

en $x^2+y^2+z^2=5r^2$, $x>0$, $y>0$, $z>0$

Probar $abc^3 \leq 27\left(\frac{a+b+c}{5}\right)^5$



5/ $\frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x}$, $\frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y}$, $\frac{\partial f}{\partial z} = \lambda \frac{\partial g}{\partial z}$

$$\left.\begin{array}{l} \frac{1}{x} = 2\lambda x \\ \frac{1}{y} = 2\lambda y \\ \frac{3}{z} = 2\lambda z \\ x^2+y^2+z^2 = 5r^2 \end{array}\right\} \quad \lambda \neq 0; \quad \begin{array}{l} x^2 = y^2 \\ z^2 = 3y^2 \end{array}$$

$$y^2+y^2+3y^2 = 5r^2 \Rightarrow 5y^2 = 5r^2$$

$y = \pm r$. Como $y > 0$, $y = r$

$$A = (r, r, r\sqrt{3})$$

$$f(A) = \log r + \log r + 3\log(r\sqrt{3}) = 5\log r + 3\log\sqrt{3}$$

$f(x,y,z) \leq 5\log r + 3\log\sqrt{3}$

$r^2 = \frac{x^2+y^2+z^2}{5}$

$r = \left(\frac{x^2+y^2+z^2}{5}\right)^{1/2}$

$\log xyz^3 \leq \log(r^5\sqrt{27})$

$$= \log\left(\sqrt{27}\left(\frac{x^2+y^2+z^2}{5}\right)^{5/2}\right)$$

$$xyz^3 \leq \left[27\left(\frac{x^2+y^2+z^2}{5}\right)^5\right]^{1/2}$$

$x^2 = a$, $y^2 = b$, $z^2 = c$

$$\sqrt{a}\sqrt{b}(\sqrt{c})^3 \leq \left[27\left(\frac{a+b+c}{5}\right)^5\right]^{1/2}$$

$$abc^3 \leq 27\left(\frac{a+b+c}{5}\right)^5$$

———— x ————

$f(x) = x_1^2 \cdot x_2^2 \cdot \dots \cdot x_n^2$, $x_1^2 + x_2^2 + \dots + x_n^2 = 1$ g

$$\frac{\partial f}{\partial x_i} = \lambda \frac{\partial g}{\partial x_i} \qquad i = 1, 2, \dots n$$

$$2x_i \prod_{\substack{j=1 \\ j \neq i}}^{n} x_j^2 = \lambda 2 x_i \qquad i = 1, \dots, n \qquad (x_i \neq 0)$$

Suma todas las ecuaciones $i = 1 \dots n$

$$\prod_{j=1}^{n} x_j^2 = \lambda x_i^2 \overset{\text{sum}}{\Longrightarrow} n \prod_{j=1}^{n} x_j^2 = \lambda \sum_{i=1}^{n} x_i^2 = 1 \cdot \lambda$$

$$\frac{\lambda}{n} = \lambda x_i^2 \Rightarrow x_i^2 = \frac{1}{n} \Rightarrow x_i = \frac{\pm 1}{\sqrt{n}} = \pm \frac{1}{\sqrt{n}}$$

$$A = \left(\pm \frac{1}{\sqrt{n}}, \pm \frac{1}{\sqrt{n}}, \dots, \pm \frac{1}{\sqrt{n}} \right)$$

$$f(A) = \left(\pm \frac{1}{\sqrt{n}} \right)^{2n} = \frac{1}{n^n} \quad \text{Max.}$$

$$f(x) = \prod_{i=1}^{n} x_i^2 \leq \frac{1}{n^n} \quad \text{cuando } \|x\| = 1$$

$x \in \mathbb{R}^n$, $x \neq 0$ $\qquad y = \frac{1}{\|x\|}(x_1, \dots, x_n)$

$$f(y) \leq \frac{1}{n^n} ; \quad \frac{x_1^2}{\|x\|^2} \cdot \frac{x_2^2}{\|x\|^2} \cdots \frac{x_n^2}{\|x\|^2} \leq \frac{1}{n^n}$$

$$\prod_{i=1}^{n} x_i^2 \leq \frac{1}{n^n} \|x\|^{2n} = \frac{1}{n^n} \left(\sum_{i=1}^{n} x_i^2 \right)^n$$

$x_i^2 = a_i$ $\qquad \prod_{i=1}^{n} a_i \leq \frac{1}{n^n} \left(\sum_{i=1}^{n} a_i \right)^n$ $\qquad$ Tomar raíces