AM Prácticas 27-11-2020 ①

7.7 b) $\sqrt[n]{a_1 \cdots a_n} \leq \frac{a_1 + \cdots + a_n}{n}$ $\quad (a_i = x_i^2)$

$f(x_1, \dots, x_n) = x_1^2 \times \cdots \times x_n^2$ sobre $x_1^2 + \cdots + x_n^2 = 1$

$F(x_1, \dots, x_n) = f(x_1, \dots, x_n) - \lambda (x_1^2 + \cdots + x_n^2 - 1)$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x_i} = 2x_i \prod_{j \neq i} x_j^2 - 2\lambda x_i = 0 \quad , i = 1, \dots, n \\ \frac{\partial F}{\partial \lambda} = 0 \iff x_1^2 + \cdots + x_n^2 = 1 \end{array}\right\}$$

Sumando las n-primeras ecuaciones $\left(\prod_{j=1}^{n} x_j^2 = \lambda x_i^2 \right)$ (1)

$$n \prod_{j=1}^{n} x_j^2 = \lambda \sum_{i=1}^{n} x_i^2 = \lambda$$

Sustituimos λ en en (1): $\cancel{\prod_{j=1}^{n} x_j^2} = \left(\cancel{\prod_{j=1}^{n} x_j^2}\right) n \cdot x_i^2$

$\Rightarrow x_i^2 = \frac{1}{n} \quad \Rightarrow x_i = \pm \frac{1}{\sqrt{n}}$

$X = \left(\pm \frac{1}{\sqrt{n}}, \dots \pm \frac{1}{\sqrt{n}}\right)$ son soluciones

Como $f(X) = f\left(\pm \frac{1}{\sqrt{n}}\right) = \prod_{j=1}^{n} \left(\pm \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n^n}$

Es decir, $f(x) = \prod_{i=1}^{n} x_i^2 \leq \frac{1}{n^n}$ cuando $\|X\|^2 = 1$ (2)

$X = (x_1, \dots, x_n) \in \mathbb{R}^n \setminus \{0\}$, $y = \frac{1}{\|X\|}(x_1, \dots, x_n)$ tiene $\|y\| = 1$

(2) $\Rightarrow$ $\prod_{i=1}^{n} \left(\frac{x_i}{\|X\|}\right)^2 \leq \frac{1}{n^n} \Rightarrow \prod_{i=1}^{n} x_i^2 \leq \frac{\|X\|^{2n}}{n^n} = \frac{\left(\sum_{i=1}^{n} x_i^2\right)^n}{n^n}$

$x_i^2 = a_i$

$a_1 \times \cdots \times a_n \leq \frac{(a_1 + \cdots + a_n)^n}{n^n} \Rightarrow \sqrt[n]{a_1 \times \cdots a_n} \leq \frac{a_1 + \cdots + a_n}{n}$

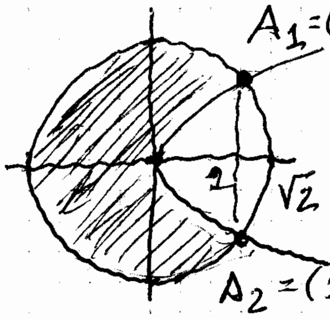
$\sqrt{a_1 a_2} \le \frac{a_1+a_2}{2} \iff 2\sqrt{a_1 a_2} \le a_1 + a_2 \iff$

$4 a_1 a_2 \le (a_1+a_2)^2 \iff 2a_1 a_2 \le a_1^2 + a_2^2 \iff$

$(a_1 - a_2)^2 \ge 0$ ✓

—— x ——

7.8 a) $f(x,y) = 2x + y^2$ sobre $K = \{x^2+y^2 \le 2,\ y^2 \ge x\}$



$A_1 = (1,\sqrt{2})$ $x^2 + x = 2$, $x = 1$ ($x = -2$ no vale)

En $\overset{\circ}{K}$, $\frac{\partial f}{\partial x} = 2 \ne 0$

No hay p. críticos en $\overset{\circ}{K}$

$A_2 = (1,-\sqrt{2})$ $A_1 = (1, 1)$, $A_2 = (1, -1)$

Lagrange para $f(x,y) = 2x + y^2$ en $x^2+y^2 = 2$ con $x \le 1$

$F(x,y,\lambda) = 2x + y^2 - \lambda(x^2+y^2-2)$

$$\left.\begin{array}{l} \frac{\partial F}{\partial x} = 2 - 2\lambda x = 0 \\ \frac{\partial F}{\partial y} = 2y - 2\lambda y = 0 \\ \frac{\partial F}{\partial \lambda} = 0 \iff x^2+y^2 = 2 \end{array}\right\} \begin{array}{l} \Rightarrow 1 = \lambda x \Rightarrow x \ne 0 \\ \Rightarrow y(1-\lambda) = 0 < \begin{array}{l} y = 0 \\ \lambda = 1 \end{array} \end{array}$$

$\boxed{y=0} \Rightarrow x = \pm\sqrt{2} \Rightarrow x = -\sqrt{2}$ $\boxed{A_3 = (-\sqrt{2}, 0)}$

$\boxed{\lambda = 1} \Rightarrow x = 1,\ y = \pm 1 \to A_1, A_2$

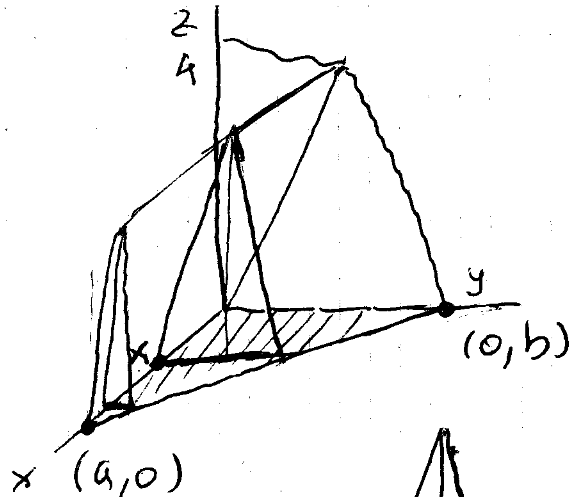
Lagrange para $f(x,y) = 2x + y^2$ con $y^2 - x = 0$

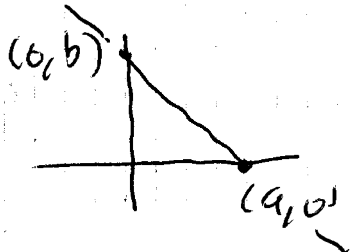
$G(x,y,\lambda) = 2x + y^2 - \lambda(y^2 - x) = 0$

$$\left.\begin{array}{l} 2 + \lambda = 0 \\ 2y - 2y\lambda = 0 \\ x = y^2 \end{array}\right\} \Rightarrow \lambda = -2 \Downarrow y(1+2) = 0 \Rightarrow y = 0 \Big\} \boxed{A_4 = (0,0)}$$

Máximo 3 en A_1 y A_2 y mínimo $-2\sqrt{2}$ en A_3

7.9. $ab(a+b)=1$

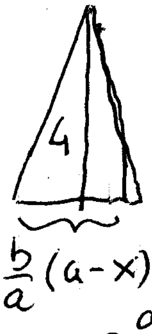




~~ay+bx=ab~~

$y = b - \frac{b}{a}x$

$y = +\frac{b}{a}(a-x)$



$$\text{Area} = \frac{\frac{b}{a}(a-x)\,4}{2} = \frac{2b}{a}(a-x)$$

Cavalieri: $V = \int_0^a \frac{2b}{a}(a-x)\,dx = \left[2bx - x^2\frac{b}{a}\right]_0^a$

$= 2ba - ab = ba$ $\qquad a, b > 0$

Máximo de $V = ba$ con la condición $ab(a+b)=1$

Multiplicadores de Lagrange: $a = b = \frac{1}{\sqrt[3]{2}}$

$\Rightarrow V = \frac{1}{\sqrt[3]{2}} \cdot \frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{4}}$

———— x ————

7.10. $f_\alpha(x,y) = x^4+y^4+\alpha(x^2+y^2)$, $\alpha \in \mathbb{R}$

a) f_α solo un máximo relativo

$$\frac{\partial f_\alpha}{\partial x} = 4x^3+2\alpha x = 0, \qquad \frac{\partial f_\alpha}{\partial y} = 4y^3+2\alpha y = 0$$

$$\left.\begin{array}{l} 2x(2x^2+\alpha)=0 \\ 2y(2y^2+\alpha)=0 \end{array}\right\} \quad \begin{array}{l} \to x=0 \\ \to x^2=-\alpha/2 \Rightarrow x=\pm\sqrt{\frac{-\alpha}{2}} \quad (\alpha\leq 0) \\ \to y=0 \\ \to y^2=-\alpha/2 \Rightarrow y=\pm\sqrt{\frac{-\alpha}{2}} \quad (\alpha\leq 0) \end{array}$$

$$Hf(x,y) = \begin{pmatrix} 12x^2+2\alpha & 0 \\ 0 & 12y^2+2\alpha \end{pmatrix}$$

Si $\alpha \geq 0$, Hf es def positiva $\Rightarrow$ Mínim relativ

$\alpha<0$ $\quad O=(0,0)$, $A=(0,\pm\sqrt{-\alpha/2})$, $B=(\pm\sqrt{\frac{\alpha}{2}},0)$

$C=(\pm\sqrt{\frac{\alpha}{2}},\pm\sqrt{\frac{-\alpha}{2}})$

O: $\begin{pmatrix} 2\alpha & 0 \\ 0 & 2\alpha \end{pmatrix}$ Def Negat $\Downarrow$ Max rela

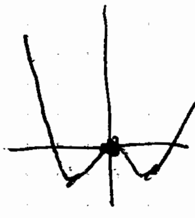
A: $\begin{pmatrix} 2\alpha & 0 \\ 0 & -4\alpha \end{pmatrix}$ No DP ni DN

B: ~~$12y^2=-6\alpha$~~ $\begin{pmatrix} -4\alpha & 0 \\ 0 & 2\alpha \end{pmatrix}$ No DP ni DN

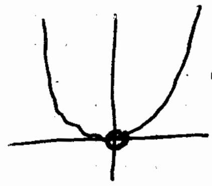
C: $\begin{pmatrix} -4\alpha & 0 \\ 0 & -4\alpha \end{pmatrix}$ $\quad -4\alpha>0$, $\quad 16\alpha^2>0$

Def pos $\Downarrow$ Min rela

$f(x) = x^4+\alpha x^2$



$\alpha<0$



$\alpha>0$

(5)

$f(x,y) = x^4 + y^4 + \alpha(x^2+y^2)$, $\alpha < 0$

Con x "cerca" de cero $x^4 < x^2$. Cerca de $(0,0)$

$$x^4+y^4+\alpha x^2+\alpha y^2 = (x^4+\alpha x^2)+(y^4+\alpha y^2)$$

$x^4+\alpha x^2<0$ cuando x cerca de cero $(x\neq 0)$

$x^4<-\alpha x^2 \Leftrightarrow x^2<-\alpha \Leftrightarrow$ Vale si $x\in(-\sqrt{-\alpha},\sqrt{-\alpha})$

Si $(x,y)\in(-\sqrt{-\alpha},\sqrt{-\alpha})\times(-\sqrt{-\alpha},\sqrt{-\alpha})$

$$f_\alpha(x,y)<0 \quad y \quad f_\alpha(0,0)=0$$

x

b) $5=\sqrt{\frac{-\alpha}{2}} \Rightarrow \alpha=-50$

x

c) Lagrange $A_1=(0,6)$, $A_2=(0,-6)$, $A_3=(6,0)$, $A_4=(-6,0)$

$A=(3\sqrt{2},3\sqrt{2})$, $B=(3\sqrt{2},-3\sqrt{2})$, $C=(-3\sqrt{2},3\sqrt{2})$

$D=(-3\sqrt{2},-3\sqrt{2})$

$f(A)=f(B)=-1152=f(C)=f(D)$ Mínimo ab

$f(A_1)=f(A_2)=f(A_3)=f(A_4)=6^4-50\times6^2=-504$ Max abs

x

9.11. $\omega = (1 - z e^{yz})\, dx \wedge dy + (1 - y e^{yz})\, dx \wedge dz$
$+ (2y + z + \operatorname{sen} z)\, dy \wedge dz$

$(1 - z e^{yz})\, dx \wedge dy \quad + (1 - y e^{yz})\, dx \wedge dz$

Hallar $a_2(x,y,z)$ y $a_3(x,y,z)$ t.q.

$$\frac{\partial a_2}{\partial x} = 1 - z e^{yz} \Rightarrow a_2(x,y,z) = \int (1 - z e^{yz})\, dx$$

$$\frac{\partial a_3}{\partial x} = 1 - y e^{yz} \Rightarrow a_3(x,y,z) = \int (1 - y e^{yz})\, dx$$

$$a_2(x,y,z) = x(1 - z e^{yz}), \quad a_3(x,y,z) = x(1 - y e^{yz})$$

$$\eta_1 = a_2\, dy + a_3\, dz = x(1 - z e^{yz})\, dy + x(1 - y e^{yz})\, dz$$

$\omega_1 = \omega - d\eta_1$; (no tiene ninguna x)

$$= (2y + z + \operatorname{sen} z)\, dy \wedge dz$$

Hallar $b_3(y,z)$ t.q. $\dfrac{\partial b_3}{\partial y} = 2y + z + \operatorname{sen} z$

$$b_3(y,z) = y^2 + yz + y \operatorname{sen} z$$

$$\eta_2 = b_3(y,z)\, dz = (y^2 + yz + y \operatorname{sen} z)\, dz$$

$\omega_2 = \omega_1 - d\eta_2 = 0 \quad \Rightarrow \quad \omega_1 = d\eta_2$

$$\omega = \omega_1 + d\eta_1 = d\eta_2 + d\eta_1 = d(\eta_2 + \eta_1)$$

$$\eta_1 + \eta_2 = x(1 - z e^{yz})\, dy + x(1 - y e^{yz})\, dz + (y^2 + yz + y \operatorname{sen} z)\, dz$$