1) $Q = \dfrac{W}{p_d + \rho g h - p_a}$ 2) $L^2 \dfrac{dh}{dt} = Q$ 3) $\dfrac{d}{dt}\left[\rho e V\right] + G h = p \dfrac{dh}{dt} L^2 = -p \dfrac{dV}{dt}$

$$\frac{1}{\gamma-1} p \frac{dV}{dt} + \frac{V}{\gamma-1}\frac{dp}{dt} - h \frac{d(V\rho)}{dt} = -p\frac{dV}{dt}$$

$$\cancel{\frac{1}{\gamma-1} p \frac{dV}{dt}} + \frac{V}{\gamma-1}\frac{dp}{dt} - \frac{\gamma}{\gamma-1}\frac{p}{\rho}\left[\cancel{\rho\frac{dV}{dt}} + V\frac{d\rho}{dt}\right] = \cancel{-p\frac{dV}{dt}} \rightarrow \boxed{\frac{dp}{p} = \gamma\frac{d\rho}{\rho}}$$

$$\frac{p_d}{p_a} = \left(\frac{\rho_d}{\rho_a}\right)^{\gamma}$$

4) si $\dfrac{p_d}{p_a} \leq \left(\dfrac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}} \simeq 1.893 \rightarrow G = \sqrt{\gamma p_a \rho_a}\, A \left(\dfrac{2}{\gamma-1}\right)^{1/2}\left[\left(\dfrac{p_d}{p_a}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]^{1/2}$

si $\dfrac{p_d}{p_a} \geq \left(\dfrac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \rightarrow G = \sqrt{\gamma p_a \rho_a}\, A \left(\dfrac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\left(\dfrac{p_d}{p_a}\right)^{\frac{\gamma+1}{2\gamma}}$

5) $\dfrac{d}{dt}\left[\rho_d (L-h) L^2\right] = -G$ $t=0,\ h=0,\ \rho_d = \rho_a$

6)

$1+2 \rightarrow L^2 \dfrac{dh}{dt} = \dfrac{W}{p_d + \rho g h - p_a}$

$4+5$

$$\rightarrow L^2 \frac{d}{dt}\left(\rho_d (L-h)\right) = -\sqrt{\gamma p_a \rho_a}\, A \begin{cases} \left(\frac{2}{\gamma-1}\right)^{1/2}\left[\left(\frac{p_d}{p_a}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]^{1/2} \\ \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{p_d}{p_a}\right)^{\frac{\gamma+1}{2\gamma}} \end{cases}$$

SI INTRODUCIMOS $\bar\rho = \dfrac{\rho_d}{\rho_a}$, $\bar h = \dfrac{h}{L}$ Y $\tau = \dfrac{Wt}{p_a L^3}$ Y USAMOS 3) LLEGAMOS FINALMENTE A

$$\frac{d\bar h}{d\tau} = \frac{1}{\bar\rho^{\gamma} - 1 + \Lambda \bar h} \quad \text{Y} \quad \frac{d}{d\tau}\left[\bar\rho(1-\bar h)\right] = -\Delta\, f(\gamma, \bar\rho) \qquad \boxed{\tau=0,\ \bar h = 0,\ \bar\rho = 1}$$

DONDE $\Lambda = \dfrac{\rho g L}{p_a}$, $\Delta = \dfrac{p_a A \sqrt{\gamma p_a/\rho_a}}{W}$ Y LA FUNCION

$f(\gamma,\bar\rho) = \left(\dfrac{2}{\gamma-1}\right)^{1/2}\left[\bar\rho^{\gamma-1} - 1\right]^{1/2}$, si $\bar\rho \leq \left(\frac{\gamma+1}{2}\right)$…

$f(\gamma,\bar\rho) = \left(\dfrac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \bar\rho^{\frac{\gamma+1}{2}}$, si $\bar\rho >$ …

7) $\underline{\Delta \ll 1 \text{ CON } \Lambda \sim O(1)} \rightarrow \bar\rho(1-\bar h) = 1 \rightarrow \dfrac{d\bar h}{d\tau} = \dfrac{1}{(1-\bar h)^{-\gamma} - 1 + \Lambda \bar h} \rightarrow \boxed{\int_0^{\bar h}\left[(1-\bar h)^{-\gamma} - 1 + \Lambda \bar h\right] d\bar h = \tau}$

$\hookrightarrow$ ES COMO SI NO EXISTIERA AGUJERO

8) $\underline{\Delta \gg 1 \text{ CON } \Lambda \sim O(1)}$ $\bar\rho = 1 \rightarrow \dfrac{d\bar h}{d\tau} = \dfrac{1}{\Lambda \bar h} \rightarrow \boxed{\bar h = \sqrt{\dfrac{2\tau}{\Lambda}}}$

$\hookrightarrow$ EL AGUJERO ES TAN GRANDE QUE ES COMO SI EL DEPOSITO ESTUVIERA ABIERTO